

Recommended problems 7 — CS3512 — Fall '09

From the textbook exercises for Section 3.1, you should be able to do problems *like* 1,2,3a,4,6–10,12–20. Similarly, for Section 3.2 you should be able to do problems *like* 1–6,8–12,16–19.

Additional problems (with selected solutions)

1 Give a concise characterization of the language L over $\{0, 1\}$ defined inductively as follows.

- *Basis*: $0, 1 \in L$.
- *Induction*: If $x \in L$, then $xx \in L$.

$$L = \{0^{2^n} \mid n \in \mathcal{N}\} \cup \{1^{2^n} \mid n \in \mathcal{N}\}$$

2 Give a concise characterization of the language L over $\{0, 1\}$ defined inductively as follows.

- *Basis*: $\Lambda, 0, 1 \in L$.
- *Induction*:
 - For all $x \in \{0, 1\}^*$, if $0x \in L$, then $10x \in L$.
 - For all $x \in \{0, 1\}^*$, if $1x \in L$, then $01x \in L$.

L is the set of binary strings in which 0's and 1's alternate.

That is, $L = \{0, \Lambda\}\{10\}^*\{1, \Lambda\}$.

3 Describe in words the following recursively defined function from $\text{lists}(\mathcal{N}) - \{\langle \rangle\}$ to \mathcal{N} .

$$f(\langle x \rangle) = x$$
$$f(x :: y :: L) = \begin{cases} f(x :: L) & , \text{ if } x \leq y \\ f(y :: L) & , \text{ otherwise} \end{cases}$$

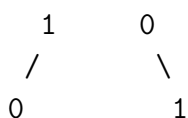
$f(L)$ is a minimal member of the nonempty list L of natural numbers.

4 Let BT be the set of binary trees over $\{0, 1\}$ defined inductively in the usual way, and consider the function $f : BT \rightarrow \{0, 1\}^*$ defined recursively as follows.

$$\begin{aligned} f(\langle \rangle) &= \Lambda \\ f(\langle L, x, R \rangle) &= f(L)xf(R) \end{aligned}$$

a) What is $|f^{-1}(\{01\})|$? 2

Indeed, the pre-image of $\{01\}$ under f has the following two elements:

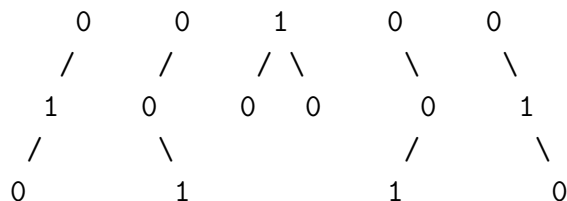


b) What is $|f^{-1}(\{00, 01, 10, 11\})|$? 8

(Do you see why?)

c) What is $|f^{-1}(\{010\})|$? 5

Indeed, the pre-image of $\{010\}$ under f has the following five elements:



The next problem here is completely optional – but it can help you understand the closure condition on inductive set definitions...

5 An *operator* on a set A is a function whose arguments and values are from A . For instance, a function from A to A is a *unary* operator on A , and a function from $A \times A$ to A is a *binary* operator on A . And so forth. Given a subset S of A and an n -ary operator f on A , we say S is *closed under f* if, for all $x_1, \dots, x_n \in A$, if $x_1, \dots, x_n \in S$, then $f(x_1, \dots, x_n) \in S$. (That is, applying f to elements of S yields only elements of S .) These definitions allow us to make

precise the idea of an “inductive definition” of a subset S of A : the “basis” of the definition is a subset of A —the “initial elements” of S —and the “induction” part of the definition is a set of operators (on A) that S is closed under. Show that, for any subset I of A , and any set Γ of operators on A , the intersection S of all subsets of A that both contain I and are closed under the operators Γ is itself a superset of I that is closed under the operators Γ . (Indeed, this intersection S is precisely the set that is defined by the inductive definition.)