Recommended problems 7 — CS3512 —Fall '09

From the textbook exercises for Section 3.1, you should be able to do problems *like* 1,2,3a,4,6–10,12–20. Similarly, for Section 3.2 you should be able to do problems *like* 1–6,8–12,16–19.

Additional problems (with selected solutions)

1 Give a concise characterization of the language L over $\{0,1\}$ defined inductively as follows.

- *Basis*: $0, 1 \in L$.
- *Induction*: If $x \in L$, then $xx \in L$.

$$L = \{\, 0^{2^n} \mid n \in \mathcal{N} \,\} \cup \{\, 1^{2^n} \mid n \in \mathcal{N} \,\}$$

2 Give a concise characterization of the language L over $\{0,1\}$ defined inductively as follows.

- *Basis*: $\Lambda, 0, 1 \in L$.
- *Induction*:

  For all $x \in \{0,1\}^*$, if $0x \in L$, then $10x \in L$.

  For all $x \in \{0,1\}^*$, if $1x \in L$, then $01x \in L$.

L is the set of binary strings in which 0's and 1's alternate.

That is, $L = \{0, \Lambda\}\{10\}^*\{1, \Lambda\}$.

3 Describe in words the following recursively defined function from lists($\mathcal{N}$) $- \{\langle\rangle\}$ to $\mathcal{N}$.

$$\begin{aligned} f(\langle x \rangle) &= x \\ f(x :: y :: L) &= \begin{cases} f(x :: L) & \text{, if } x \leq y \\ f(y :: L) & \text{, otherwise} \end{cases} \end{aligned}$$

$f(L)$ is a minimal member of the nonempty list L of natural numbers.

4 Let BT be the set of binary trees over $\{0,1\}$ defined inductively in the usual way, and consider the function $f : BT \to \{0,1\}^*$ defined recursively as follows.

$$\begin{aligned} f(\langle\rangle) &= \Lambda \\ f(\langle L, x, R\rangle) &= f(L)xf(R) \end{aligned}$$

a) What is $|f^{-1}(\{01\})|$? 2

Indeed, the pre-image of $\{01\}$ under f has the following two elements:

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  1     0
 /       \
0         1
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b) What is $|f^{-1}(\{00, 01, 10, 11\})|$? 8

(Do you see why?)

c) What is $|f^{-1}(\{010\})|$? 5

Indeed, the pre-image of $\{010\}$ under f has the following five elements:

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    0     0     1      0     0
   /     /     / \      \     \
  1     0     0   0      0     1
 /       \              /       \
0         1            1         0
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The next problem here is completely optional – but it can help you understand the closure condition on inductive set definitions...

5 An *operator* on a set A is a function whose arguments and values are from A. For instance, a function from A to A is a *unary* operator on A, and a function from $A \times A$ to A is a *binary* operator on A. And so forth. Given a subset S of A and an n-ary operator f on A, we say S is *closed under* f if, for all $x_1, \ldots, x_n \in A$, if $x_1, \ldots, x_n \in S$, then $f(x_1, \ldots, x_n) \in S$. (That is, applying f to elements of S yields only elements of S.) These definitions allow us to make precise the idea of an "inductive definition" of a subset S of A: the "basis" of the definition is a subset of A—the "initial elements" of S—and the "induction" part of the definition is a set of operators (on A) that S is closed under. Show that, for any subset I of A, and any set Γ of operators on A, the intersection S of all subsets of A that both contain I and are closed under the operators Γ is itself a superset of I that is closed under the operators Γ. (Indeed, this intersection S is precisely the set that is defined by the inductive definition.)