Homework 5
Due Mon., Oct. 27

CS 4511: Automata ...

Fall, 2003

1 Let L be the language $\{0^m1^n \mid m, n \in \mathcal{N}, m \geq n\}$ over $\{0, 1\}$. Use the corollary to Theorem 3.2 to show that L is not accepted by any FA.

2 Show that there are at least two distinct strings that are indistinguishable wrt the language L from the previous problem.

3 Given an FA $M = (Q, \Sigma, q_0, A, \delta)$, let

$$Q_r = \{q \in Q \mid \textit{ there is an } x \in \Sigma^* \textit{ s.t. } \delta^*(q_0, x) = q \ \} .$$

Show that for all $q \in Q_r$ and $a \in \Sigma$, $\delta(q, a) \in Q_r$.

4 Show that the unreachable states can be eliminated from any FA without affecting the language that it accepts. More precisely, given an FA $M = (Q, \Sigma, q_0, A, \delta)$, let $M_r = (Q_r, \Sigma, q_0, A \cap Q_r, \delta_r)$ where Q_r is defined as in the previous problem, and for all $q \in Q_r$ and $a \in \Sigma$, $\delta_r(q, a) = \delta(q, a)$. Show that $L(M) = L(M_r)$.

(Hint: Begin by showing that for all $x \in \Sigma^*$, $\delta^*(q_0, x) = \delta_r^*(q_0, x)$. If your proof uses some kind of induction, specify clearly what the IH is, what you need to show, and where you use the IH.)

5 Let M_1 and M_2 be the FA's shown below. Draw the FA for $L(M_1) - L(M_2)$ given by the construction discussed in class. (In light of the result from the previous problem, you may omit any unreachable states.)

6 For any NFA $M = (Q, \Sigma, q_0, A, \delta)$, show that for any $q \in Q$ and $a \in \Sigma$, $\delta^*(q, a) = \delta(q, a)$.

7 Draw an NFA to recognize $(a + aa)(ba)^*$.

8 Give an example of an NFA $M = (Q, \Sigma, q_0, A, \delta)$ for which the NFA $M' = (Q, \Sigma, q_0, Q - A, \delta)$ is such that $L(M) \cup L(M') \neq \Sigma^*$. Draw M', and give an example of a string not in $L(M) \cup L(M')$.

9 Give an example of an NFA $M = (Q, \Sigma, q_0, A, \delta)$ for which the NFA $M' = (Q, \Sigma, q_0, Q - A, \delta)$ is such that $L(M) \cap L(M') \neq \emptyset$. Draw M' and give an example of a string in $L(M) \cap L(M')$.

10 True or False? (In each case, if your answer is false, justify it by giving a counterexample.) For any NFA's $M_1 = (Q_1, \Sigma, q_1, A_1, \delta_1)$, $M_2 = (Q_2, \Sigma, q_2, A_2, \delta_2)$, and $M = (Q_1 \times Q_2, \Sigma, (q_1, q_2), A, \delta)$ where for all $q \in Q_1$, $r \in Q_2$ and $a \in \Sigma$,

$$\delta((q,r),a) = \delta_1(q,a) \times \delta_2(r,a) \; :$$

(a) If $A = \{(p,q) \in Q_1 \times Q_2 \mid p \in A_1 \textit{ or } q \in A_2\}$, then $L(M) = L(M_1) \cup L(M_2)$?

(b) If $A = \{(p,q) \in Q_1 \times Q_2 \mid p \in A_1 \textit{ and } q \in A_2\}$, then $L(M) = L(M_1) \cap L(M_2)$?

(c) If $A = \{(p,q) \in Q_1 \times Q_2 \mid p \in A_1 \textit{ and } q \notin A_2\}$, then $L(M) = L(M_1) - L(M_2)$?