Signal – A function of time

System – Processes input signal (excitation) and produces output signal (response)
1. Types of signals

2. Going from analog to digital world

3. An example of a system

4. Mathematical representation of signals
## Types of Signals

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<tr>
<th>Time</th>
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<td>3</td>
<td>Discrete</td>
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- **Analog**
- **Digital**
Figure 1.3
Examples of continuous-time and discrete-time signals.

Figure 1.4
Examples of continuous-time and digital signals.
Types of Signals

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Role of Noise!
Figure 1.5
Examples of noise and a noisy digital signal.
Figure 1.8
Noisy digital ASCII signal.
Advantage of Digital World

![Graph showing noiseless, noisy, and filtered digital signals with bit detection threshold and bit error.](image)

**Figure 1.9**
Use of a filter to reduce bit error rate in a digital signal.
Going from Analog to Digital World

Three Step Process

Sampling

Quantization

Encoding
Figure 1.6
Sampling, quantization, and encoding of a signal to illustrate various signal types.
Figure 1.7
Asynchronous serial binary ASCII-encoded voltage signal for the message "SIGNAI"
Example of System

A simple system example – *Sound Recording System*

*What constitutes a sound recording system?*

[Diagram showing the process of sound recording system with labels: Acoustic pressure variation → Microphone → Electronics → ADC → Computer memory]
Recorded Sound as a Signal Example

- “s” “i” “gn” “al”
Representation of Signals

Time Domain

Frequency Domain

Figure 1.17
Three sounds in the word signal and their associated power spectral densities.
Another Example of System

A very complex system example – Human Brain

Figure 1.18
Communication between two people involving signals and signal processing by systems.
**Sampling a CT Signal to Create a DT Signal**

- **Sampling is acquiring the values of a CT signal at discrete points in time**

- **$x(t)$ is a CT signal --- $x[n]$ is a DT signal**

\[
x[n] = x(nT_s)
\]

where \( T_s \) is the time between samples
Mathematical Representation of Signals

**Continuous Time**

\[ x(t) = A \sin(\omega_0 t) \]

\[ = A \sin(2\pi f_0 t) \]

**Discrete Time**

\[ x[nT_s] = A \sin[\omega_0 nT_s] \]

\[ = A \sin[2\pi f_0 nT_s] \]
Continuous Time Signals

**Sinusoidal Signal:** \[ g(t) = A \sin(\omega_o t) \]

\[ = A \sin(2\pi f_o t) \]

\[ = A \sin\left(\frac{2\pi t}{T_o}\right) \]

**General Form:** \[ g(t) = C \cos(2\pi f_o t + \theta) \]
Review of Euler’s Identity – Complex valued sinusoidal signals

**Euler’s Identity**

\[ e^{j2\pi f_F t} = \cos(2\pi f_F t) + j\sin(2\pi f_F t) \]

Also

\[ e^{-j2\pi f_F t} = \cos(2\pi f_F t) - j\sin(2\pi f_F t) \]

\[ \Rightarrow \cos(2\pi f_F t) = \frac{e^{j2\pi f_F t} + e^{-j2\pi f_F t}}{2} \]

and

\[ \Rightarrow \sin(2\pi f_F t) = \frac{e^{j2\pi f_F t} - e^{-j2\pi f_F t}}{2j} \]
**Review of Euler’s Identity – Complex valued sinusoidal signals**

**Euler’s Identity**

\[ Ce^{j(2\pi f_F t + \theta)} = C \cos(2\pi f_F t + \theta) + jC \sin(2\pi f_F t + \theta) \]

Also

\[ Ce^{-j(2\pi f_F t + \theta)} = C \cos(2\pi f_F t + \theta) - jC \sin(2\pi f_F t + \theta) \]

\[ \Rightarrow C \cos(2\pi f_F t + \theta) = \frac{Ce^{j(2\pi f_F t + \theta)} + Ce^{-j(2\pi f_F t + \theta)}}{2} \]

and

\[ \Rightarrow C \cos(2\pi f_F t + \theta) = \frac{Ce^{j\theta}}{2} e^{j2\pi f_F t} + \frac{Ce^{-j\theta}}{2} e^{-j2\pi f_F t} \]
Three ways to represent a sinusoidal frequency $f_F$

**Method 1:**

$$Ce^{j\theta} \frac{e^{j2\pi f_F t}}{2} + Ce^{-j\theta} \frac{e^{-j2\pi f_F t}}{2}$$

**Method 2:**

$$C \cos(2\pi f_F t + \theta)$$

**Method 3:**

$$A \cos(2\pi f_F t) + B \sin(2\pi f_F t)$$

**Where**

$$C = \sqrt{A^2 + B^2} \quad \theta = \tan^{-1} - \frac{B}{A}$$
Exponential Functions

\[ g(t) = Ae^{\sigma t} \]

\[ g(t) = Ae^{-\sigma t} \]

Complex valued exponential signal:

\[ g(t) = Ae^{(\sigma + j\omega)t} = Ae^{\sigma t} [\cos \omega t + j \sin \omega t] \]

Where do these functions occur in real life?
Discontinuity of a function

Definition: \[ \lim_{\varepsilon \to 0} g(t + \varepsilon) \neq \lim_{\varepsilon \to 0} g(t - \varepsilon) \]

Simple words:

If the value of function is different at time \( t_0 \) when approached at \( t_0 \) by decreasing and increasing time, then the function is discontinuous at time \( t_0 \)

Examples:
Unit Step Function

Definition:

\[ u(t) = \begin{cases} 
  1 & t > 0 \\
  1/2 & t = 0 \\
  0 & t < 0 
\end{cases} \]

Real Physical Phenomenon: Switching
Signum Function

**Definition:**

\[
\text{sgn}(t) = \begin{cases} 
1 & \text{if } t > 0 \\
0 & \text{if } t = 0 \\
-1 & \text{if } t < 0 
\end{cases}
\]

\[
= 2u(t) - 1
\]
Ramp Function

Definition:

\[ ramp(t) = \begin{cases} 
  t & t > 0 \\
  0 & t \leq 0
\end{cases} \]

\[ = tu(t) \]

\[ = \int_{-\infty}^{t} u(x) \, dx \]

Can you generate this function?
Unit Impulse Function

\[ \lim_{a \to 0} \text{Area} = \frac{1}{a} (a) = 1 \]
Definition:
\[\delta(t) = 0 \quad t \neq 0\]
\[\int_{-\infty}^{\infty} \delta(t) dt = 1\]

Can you represent \( u(t) \) in terms of unit impulse function?

\[u(t) = \int_{-\infty}^{t} \delta(x) dx\]
Another Important Fact about Unit Impulse Function!

\[
\int_{-\infty}^{\infty} \delta(t) dt = 1
\]

\[
\Rightarrow \int_{-\infty}^{\infty} g(t) \delta(t) dt = g(0)
\]

Isn’t it Sampling?
Unit Comb

\[ \text{comb}(t) = \sum_{n=-\infty}^{n=\infty} \delta(t - n) \]
Rectangular Function

\[ rect(t) = \begin{cases} 
1 & |t| < 1/2 \\
1/2 & |t| = 1/2 \\
0 & |t| > 1/2 
\end{cases} \]
**Triangular Function**

\[ tri(t) = \begin{cases} 
1 - |t| & |t| < 1 \\
0 & |t| \geq 1 
\end{cases} \]
Unit Sinc Function

\[ \sin c(t) = \frac{\sin(\pi t)}{\pi t} \]
Combinations of Functions

\[ g(t) = \sin c(t) \cos(20\pi t) \]

\[ g(t) = Ae^{-t} \cos 20\pi t \]

\[ g(t) = u(t) + \text{ramp}(t) \]

\[ g(t) = \text{sgn}(t) \sin(2\pi t) \]
Some More Examples

\[ [\sin(4\pi t) + 2] \cos(40\pi t) \]

\[ e^{-2t} \cos(10\pi t) \]

\[ \text{sinc}(4t) = \frac{\sin(4\pi t)}{4\pi t} \]

\[ \cos(20\pi t) + \cos(22\pi t) \]
Amplitude Transformations of Functions

**Amplitude Shifting**

\[ g(t) \rightarrow A + g(t) \]

**Amplitude Scaling**

\[ g(t) \rightarrow Ag(t) \]
Time Transformations of Functions

**Time Shifting**

\[ g(t) \rightarrow g(t - a) \]

**Time Scaling**

\[ g(t) \rightarrow g\left(\frac{t}{a}\right) \]
Multiple Transformations

Case 1

\[ g(t) \rightarrow Ag\left(\frac{t - t_o}{a}\right) \]

Case 2

\[ g(t) \rightarrow Ag(bt - t_o) \]
Example of Case 1

A sequence of amplitude scaling, time scaling, and time shifting a function.

Figure 2.35
Example of Case 2

Figure 2.36
A sequence of amplitude scaling, time shifting, and time scaling a function.
Some More Examples

\[ 3 \text{rect} \left( \frac{t+1}{4} \right) \]

\[ -5 \text{ramp}(0.1t) \]

\[ 2 \text{sinc}(5t) \]

\[ -3 \text{sgn}(2t) \]

\[ -7 \text{tri} \left( \frac{t-4}{8} \right) \]

\[ 4u(3-t) \]
Differentiation and Integration of Functions

Differentiation: Slope of the function at time $t$

\[ \frac{dg(t)}{dt} \]

Integration: Accumulative area under the curve

\[ \int_{-\infty}^{t} g(x) \, dx \]
Differentiation – A kind of Transformation of a Signal
Integration – A kind of Transformation of a Signal
Even and Odd Functions

Function is Even if \( g(t) = g(-t) \)

Example: \( \cos(\omega t) \)

Function is Odd if \( g(t) = -g(-t) \)

Example: \( \sin(\omega t) \)
Even and Odd Components of a Function

If function is neither even nor odd, then

\[ g(t) = g_e(t) + g_o(t) \]

Where

\[ g_e(t) = \frac{g(t) + g(-t)}{2} \]

\[ g_o(t) = \frac{g(t) - g(-t)}{2} \]
Products of Even and Odd CT Functions

Even $\times$ Even = Even

$g_1(t)$

$g_2(t)$

$g_1(t)g_2(t)$
Products of Even and Odd CT Functions

Even x Odd = Odd
Products of Even and Odd CT Functions

Even x Odd = Odd
Products of Even and Odd CT Functions

Odd x Odd = Even
Integrals of Even and Odd CT Functions

Even Function

\[ \int_{-a}^{a} g(t) \, dt = 2 \int_{0}^{a} g(t) \, dt \]

Odd Function

\[ \int_{-a}^{a} g(t) \, dt = 0 \]
Continuous Time Periodic Functions

Function is periodic with period $T$, if

$$g(t) = g(t + nT)$$

What is the effect on periodic function of time shifting by $nT$?
Examples of Periodic Signals

\[ g(t) = 3 \sin(400\pi t) \]

\[ g(t) = 2 + t^2 \]

\[ g(t) = \sin(12\pi t) + \sin(6\pi t) \]

\[ g(t) = \sin(\pi t) + \sin(6\pi t) \]
Discrete Time Signals

**Continuous Time**

\[ x(t) = A \sin(2\pi f_0 t) \]

**Discrete Time**

\[ x[nT_s] = A \sin[2\pi f_0 nT_s] \]

\[ = A \sin[2\pi f_0 T_s n] \]

\[ = A \sin\left(\frac{2\pi f_0}{f_s} n\right) \]
\[ x[nT_s] = A \sin\left(\frac{2\pi f_o}{f_s} n\right) \]

\[ x[n] = A \sin\left(2\pi \frac{f_o}{f_s} n\right) \]

\[ x[n] = A \sin\left(2\pi \frac{T_s}{T_o} n\right) \]

\[ x[n] = A \sin\left(2\pi Kn\right) \]

To be periodic, “\(Kn\)” has to be an integer for some “\(n\)”

\[ \Rightarrow K \text{ has to be a ratio of integers} \]

\[ x[n] = A \sin\left(2\pi \frac{p}{q} n\right) \]

\[ \text{Period} = q \]
Discrete-Time Sinusoids

Periodic Sinusoids
How Many CT periodic Cycles are Present in One DT Periodic Cycle

\[ x[n] = A \sin[2\pi \frac{p}{q} n] \quad \text{Period} = q \]

\[ x[n] = A \sin[2\pi \frac{n}{q}] \]

\[ x[n] = A \sin[2\pi p] \quad \text{When} \ n = q \quad \text{one DT period} \]

=> There are \( p \) cycles of CT periodic sinusoidal function per one cycle of DT periodic sinusoidal function
Examples

\[ g_1[n] = \cos \left[ \frac{2\pi}{5} n \right] \quad \text{Period} = 5 \]

\[ g_2[n] = \cos \left[ \frac{12\pi}{5} n \right] \quad \text{Period} = 5 \]
$$g_1[n] = \cos\left(\frac{2\pi n}{5}\right)$$

$$g_2[n] = \cos\left(\frac{12\pi n}{5}\right)$$
Two Discrete Sinusoids could be similar?

Two different-looking DT sinusoids,

\[ g_1[n] = A \cos(2\pi K_1 n + \theta) \quad \text{and} \quad g_2[n] = A \cos(2\pi K_2 n + \theta) \]

may actually be the same. If

\[ K_2 = K_1 + m, \quad \text{where} \ m \ \text{is an integer} \]

then (because \( n \) is discrete time and therefore an integer),

\[ A \cos(2\pi K_1 n + \theta) = A \cos(2\pi K_2 n + \theta) \]

(Example on next slide)
Examples

\[ g_1[n] = \cos\left(\frac{2\pi}{5} n\right) \quad \text{Period} = 5 \]

\[ g_2[n] = \cos\left(\frac{12\pi}{5} n\right) \quad \text{Period} = 5 \]

\[ g_3[n] = \cos\left(\frac{16\pi}{5} n\right) \quad \text{Period} = 5 \]
$$\cos\left(\frac{2\pi}{5} n\right)$$

$$\cos\left(\frac{12\pi}{5} n\right)$$

$$\cos\left(\frac{16\pi}{5} n\right)$$
Other Discrete Functions

Unit Impulse Function

\[ \delta[n] = \begin{cases} 
1 & n = 0 \\
0 & n \neq 0
\end{cases} \]

Please note:

\[ \delta[n] = \delta[an] \]

\[ \sum_{n=-\infty}^{\infty} x[n] \delta[n] = x[0] \]  
\[ \sum_{n=-\infty}^{\infty} x[n] \delta[n-n_0] = x[n_0] \]

Discrete Sampling
**Unit Step Function**

\[ u[n] = \begin{cases} 
1 & n \geq 0 \\
0 & n < 0 
\end{cases} \]

**Unit Ramp Function**

\[ \text{ramp}[n] = \begin{cases} 
n & n \geq 0 \\
0 & n < 0 
\end{cases} \]
Rectangular Function

\[ rect_{N_w}[n] = \begin{cases} 
1 & |n| \leq N_w \\
0 & |n| > N_w 
\end{cases} \]

Please note that

\[ rect_{N_w}[n] = u[n + N_w] - u[n - N_w - 1] \]
Transformations on Discrete Time Functions

Amplitude Shifting

\[ g[n] \rightarrow A + g[n] \]

Amplitude Scaling

\[ g[n] \rightarrow Ag[n] \]
**Time Shifting**

\[ g[n] \rightarrow g[n-a] \]

*Same as continuous time*

**Time Scaling**

\[ g[n] \rightarrow g\left[\frac{n}{a}\right] \]

*Tricky! Isn’t it?*
Example of Time Shifting

Figure 2.68
Graphical definition of a DT function $g[n]$, where $g[n] = 0$ and $|n| > 15$.

Figure 2.69
Graph of $g[n + 3]$ illustrating the time-shifting functional transformation.
Example of Time Compression

Figure 2.70
Time compression for a DT function.
Discrete Time Even and Odd Functions

Function is Even if \( g[n] = g[-n] \)

Function is Odd if \( g[n] = -g[-n] \)

If function is neither even nor odd, then

\[
g[n] = g_e[n] + g_o[n]
\]

Where

\[
g_e[n] = \frac{g[n] + g[-n]}{2}
\]

\[
g_o[n] = \frac{g[n] - g[-n]}{2}
\]
Differencing and Accumulation

\[ \Delta g[n] = g[n + 1] - g[n] \]

\[ \sum_{n=-\infty}^{\infty} g[n] \]
Energy of a Signal

For Continuous Time Signals

\[ E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt \]

For Discrete Time Signals

\[ E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2 \]
Visual Example of Energy of a Signal – CT Signal

\[ |x(t)|^2 \]

Area = Signal Energy
Visual Example of Energy of a Signal – DT Signal

\[ x[n] \]

\[ |x[n]|^2 \]

\[ \sum |x[n]|^2 \]

Signal Energy
Power of a Signal

Some signals have infinite signal energy. In that case
It is more convenient to deal with average signal power.

For Continuous Time Signals

\[ P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt \]

For Discrete Time Signals

\[ P_x = \lim_{N \to \infty} \frac{1}{2N} \sum_{n=-N}^{N-1} |x[n]|^2 \]
Power of a Periodic Signal

For a periodic CT signal, $x(t)$, the average signal power is

$$P_x = \frac{1}{T} \int_{T} |x(t)|^2 \, dt$$

where $T$ is any period of the signal.

For a periodic DT signal, $x[n]$, the average signal power is

$$P_x = \frac{1}{N} \sum_{n=\langle N \rangle} |x[n]|^2$$

where $N$ is any period of the signal.
Energy and Power Signals

A signal with finite signal energy is called an energy signal.

A signal with infinite signal energy and finite average signal energy is called a power signal.