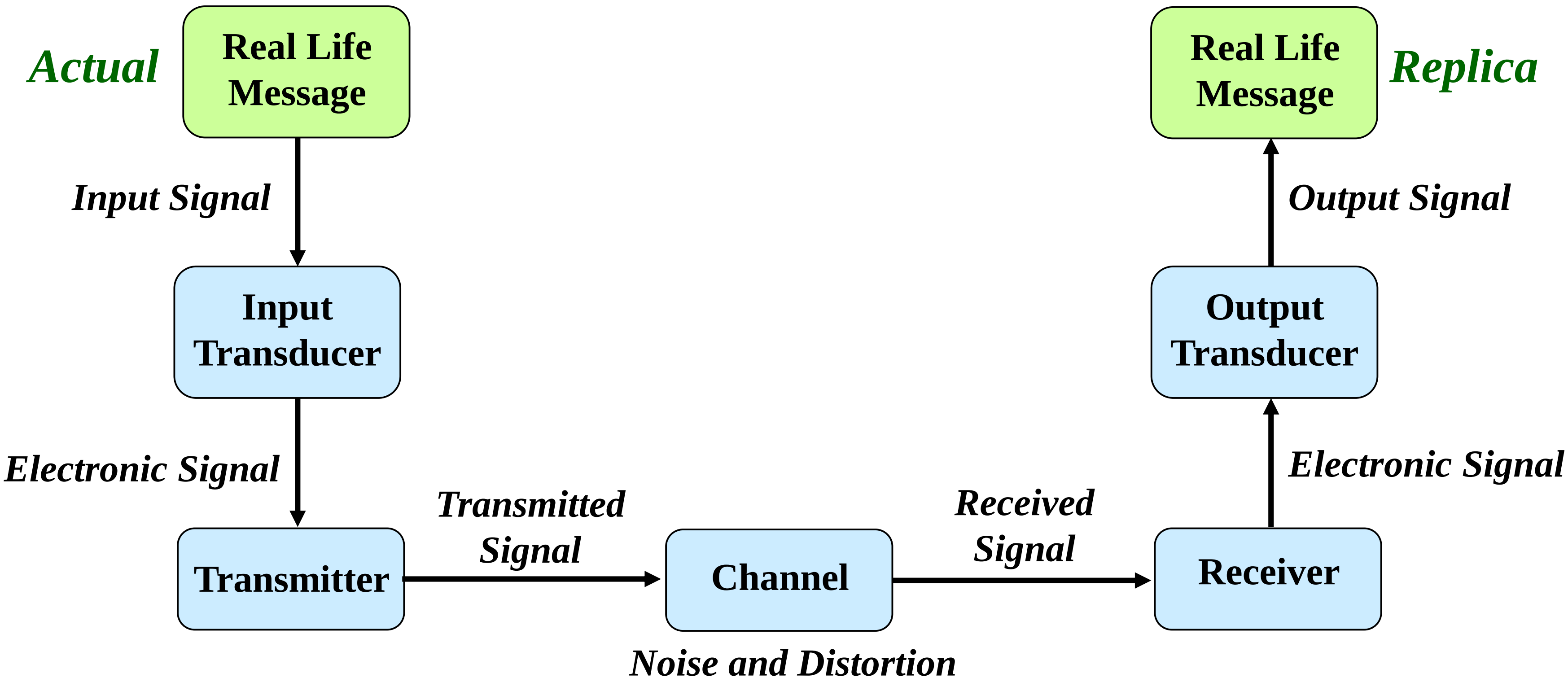
What is a Communications System?



Modulation/Coding
Redundancy

Signal to Noise Ratio (SNR)
Capacity

Demodulation
Error Correction

Classification of Signals

- *Continuous-time and Discrete-time Signals*
- *Analog and Digital Signals*
- *Deterministic and Random Signals*
- *Periodic and Aperiodic Signals*
- *Energy and Power Signals*

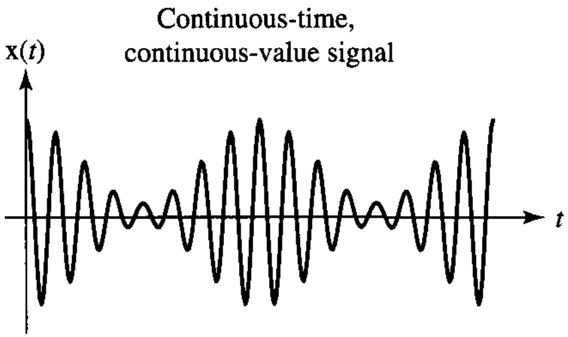
Continuous vs. Discrete Time
Analog vs. Digital

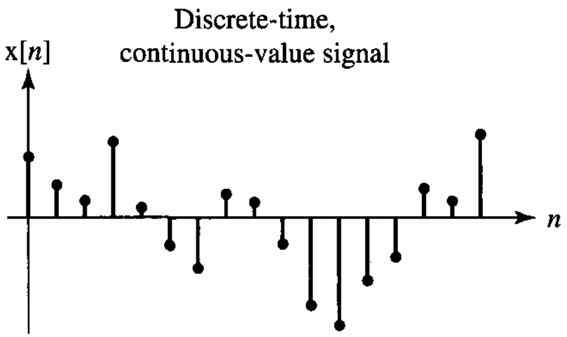
	Time	***Value***
1	*Continuous*	*Continuous*
2	*Continuous*	*Discrete*
3	*Discrete*	*Continuous*
5	*Discrete*	*Discrete*

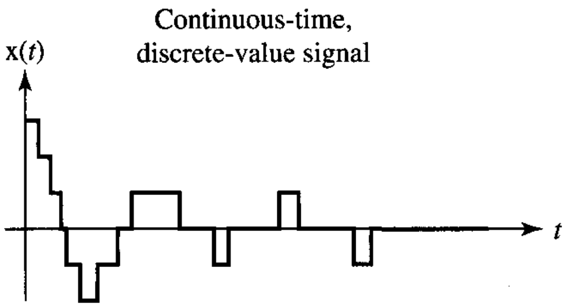
← ***Analog*** (row 1)

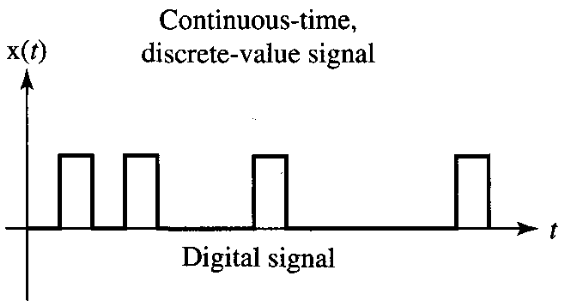
← ***Digital*** (row 2)

Some Examples

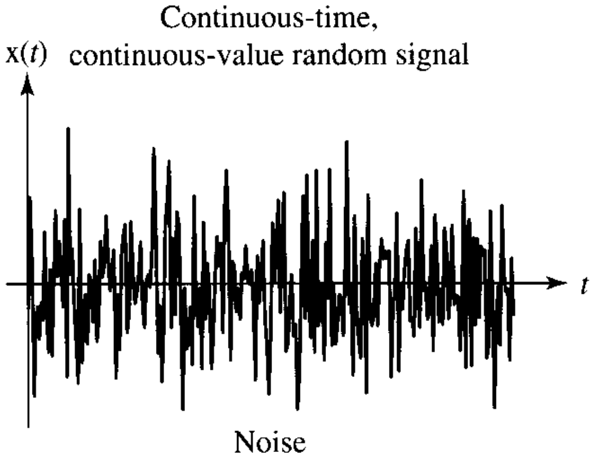


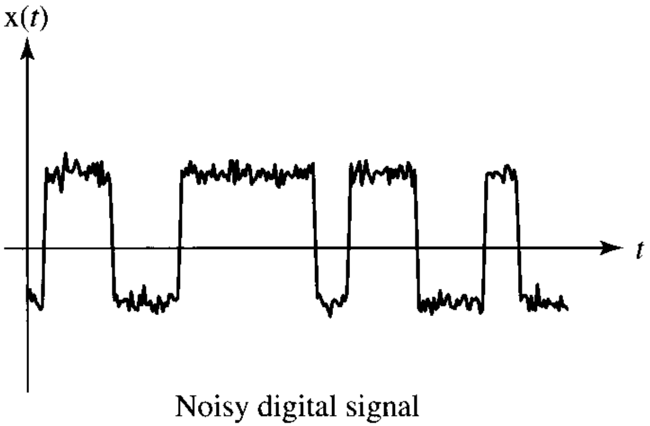






Deterministic vs. Random Signals



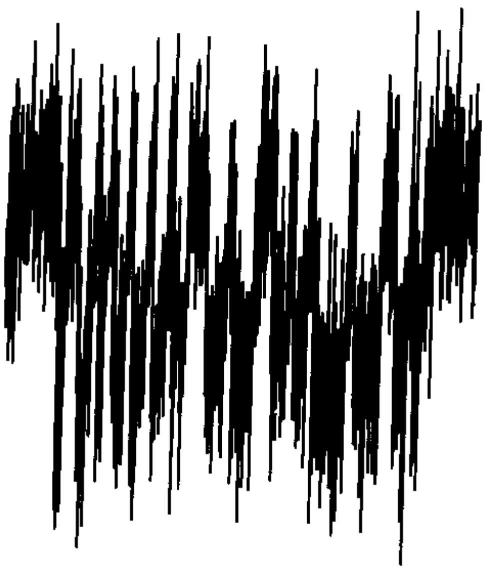


Effect of SNR

Signal-to-noise ratio = 311.6159

Signal-to-noise ratio = 51.3176

Signal-to-noise ratio = 12.6983



Signal-to-noise ratio = 3.2081

Periodic vs. Aperiodic Signals

Signal is Periodic if :

$$g(t) = g(t + nT)$$

Period of the Periodic Signal	**= T**
Frequency of the Periodic Signal	**= f = 1/T**

If the signal is not Periodic, it will be Aperiodic

Some Examples

$$g(t) = 3\sin(400\pi t)$$

$$g(t) = 2 + t^2$$

$$g(t) = \sin(12\pi t) + \sin(6\pi t)$$

Size of the Signal?

Energy of Signal

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt$$

Power of Signal (Time Average Energy)

$$P_g = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt$$

RMS value of Signal $= \sqrt{P_g}$

Example of Energy and Power Signals

Neither $$g(t) = e^{-t/2}$$

Energy Signal $$g(t) = u(t)e^{-t/2}$$

Power Signal $$g(t) = C\cos(\omega_0 t + \theta_0)$$

Power Signal $$g(t) = C_1\cos(\omega_1 t + \theta_1) + C_2\cos(\omega_2 t + \theta_2)$$

Example 1 – neither energy nor power signal

$$g(t) = e^{-t/2}$$

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt$$

$$= \int_{-\infty}^{\infty} \left|e^{-t/2}\right|^2 dt$$

$$= \int_{-\infty}^{\infty} e^{-t} dt = \left|-e^{-t}\right|_{-\infty}^{\infty}$$

$$= 0 + \infty$$

Example 2 – energy signal

$$g(t) = u(t)e^{-t/2}$$

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 \, dt$$

$$= \int_{0}^{\infty} \left|e^{-t/2}\right|^2 dt$$

$$= \int_{0}^{\infty} e^{-t} dt = \left|-e^{-t}\right|_0^{\infty}$$

$$= 0 + 1 = 1$$

Example 3 – power signal

$$g(t) = C\cos(\omega_0 t + \theta_0)$$

$$P_g = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}|g(t)|^2\,dt$$

$$= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}C^2\cos^2(\omega_0 t + \theta_0)dt$$

$$= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\frac{C^2}{2}[1+\cos(2\omega_0 t + 2\theta_0)]dt$$

$$= \lim_{T\to\infty}\frac{C^2}{2T}\int_{-T/2}^{T/2}dt + \lim_{T\to\infty}\frac{C^2}{2T}\int_{-T/2}^{T/2}\cos(2\omega_0 t + 2\theta_0)]dt$$

$$= \lim_{T\to\infty} \frac{C^2}{2T} \int_{-T/2}^{T/2} dt + \lim_{T\to\infty} \frac{C^2}{2T} \int_{-T/2}^{T/2} \cos(2\omega_0 t + 2\theta_0)]dt$$

$$= \lim_{T\to\infty} \frac{C^2}{2T}(T) + \lim_{T\to\infty} \frac{C^2}{2T}(0)$$

$$= \frac{C^2}{2}$$

$$\Rightarrow P_g = \frac{C^2}{2}$$

For a sinosoidal signal regardless of frequency and phase shift

$$RMS \quad value = C/\sqrt{2}$$

Example 4 – power signal

$$g(t) = C_1 \cos(\omega_1 t + \theta_1) + C_2 \cos(\omega_2 t + \theta_2)$$

$$P_g = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} C_1{}^2 \cos^2(\omega_1 t + \theta_1) dt + \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} C_2{}^2 \cos^2(\omega_2 t + \theta_2) dt$$

$$+ \lim_{T\to\infty} \frac{2C_1C_2}{T} \int_{-T/2}^{T/2} \cos(\omega_1 t + \theta_1) \cos(\omega_2 t + \theta_2) dt$$

$$= \frac{C_1^2}{2} + \frac{C_2^2}{2} + \lim_{T\to\infty} \frac{2C_1C_2}{T} \int_{-T/2}^{T/2} \cos(\omega_1 t + \theta_1) \cos(\omega_2 t + \theta_2) dt$$

$$= \frac{C_1^2}{2} + \frac{C_2^2}{2} + \lim_{T\to\infty} \frac{2C_1C_2}{T} \int_{-T/2}^{T/2} \cos(\omega_1 t + \theta_1)\cos(\omega_2 t + \theta_2)dt$$

Hint -> $\cos(a+b)+\cos(a-b)=2\cos a\cos b$

$$\Rightarrow P_g = \frac{C_1^2}{2} + \frac{C_2^2}{2} + \lim_{T\to\infty} \frac{C_1C_2}{T} \int_{-T/2}^{T/2} \cos[(\omega_1+\omega_2)t + \theta_1 + \theta_2]dt$$

$$+ \lim_{T\to\infty} \frac{C_1C_2}{T} \int_{-T/2}^{T/2} \cos[(\omega_1-\omega_2)t + \theta_1 - \theta_2]dt$$

$$\Rightarrow P_g = \frac{C_1^2}{2} + \frac{C_2^2}{2}$$

Power of the sum of the two sinosoid signals with distinct frequencies is equal to the sum of the powers of the individual signals

Generalizing the Result

$$g(t) = \sum_{n=1}^{\infty} C_n \cos(\omega_n t + \theta_n)$$

$$P_g = \sum_{n=1}^{\infty} \frac{C_n^2}{2}$$

It is called Parseval's Theorem

Some Important Operations on Signals

Time Shifting $$x(t) = g(t-a)$$

Time Scaling $$x(t) = g(\frac{t}{a})$$

Time Inversion $$x(t) = g(-t)$$

Unit Impulse Function

Definition:

$$\begin{cases} \delta(t) = 0 \quad t \neq 0 \\ \int_{-\infty}^{\infty} \delta(t) = 1 \end{cases}$$

Sampling Property

$$\int_{-\infty}^{\infty} g(t)\delta(t)dt = g(0)$$

or

$$\int_{-\infty}^{\infty} g(t)\delta(t-a)dt = g(a)$$

Unit Step Function

Definition:

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Can you represent u(t) in terms of unit impulse function?

$$u(t) = \int_{-\infty}^{t} \delta(x)dx$$

Components of a Signal

Let's approximate a signal with another signal

$$g(t) \cong cx(t) \qquad t_1 < t < t_2$$

Error of approximation is:

$$e(t) = g(t) - cx(t) \qquad t_1 < t < t_2$$

Energy (size) of error is

$$E_e = \int_{t_1}^{t_2} e^2(t)dt$$

$$= \int_{t_1}^{t_2} [g(t) - cx(t)]^2 dt$$

For best approximation, energy needs to be minimized

To minimize energy, a necessary condition is $\frac{dE_e}{dc} = 0$

$$\Rightarrow \frac{d}{dc}\int_{t_1}^{t_2}[g(t) - cx(t)]^2 dt = 0$$

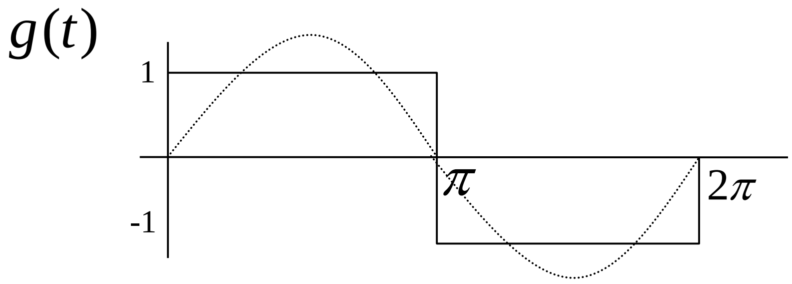
or $$\frac{d}{dc}\int_{t_1}^{t_2} g^2(t)dt + \frac{d}{dc}\int_{t_1}^{t_2} c^2 x^2(t)dt - \frac{d}{dc}\int_{t_1}^{t_2} 2cg(t)x(t)dt = 0$$

$$2c\int_{t_1}^{t_2} x^2(t)dt - 2\int_{t_1}^{t_2} g(t)x(t)dt = 0$$

$$\Rightarrow c = \frac{\int_{t_1}^{t_2} g(t)x(t)dt}{\int_{t_1}^{t_2} x^2(t)dt} = \frac{1}{E_x}\int_{t_1}^{t_2} g(t)x(t)dt$$

cx(t) is the projection of g(t) on x(t)

Example 1



$$g(t) = c\sin t \qquad 0 \le t \le 2\pi \qquad c = ?$$

We know

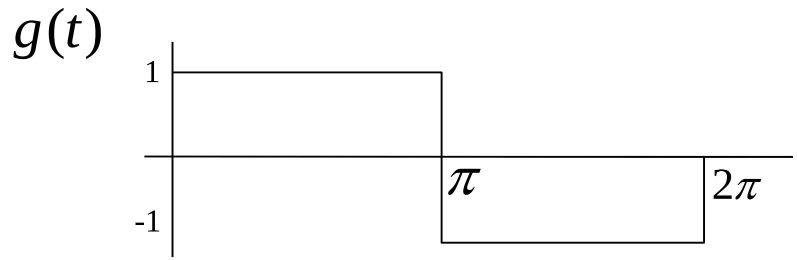
$$c = \frac{1}{E_x}\int_{t_1}^{t_2} g(t)x(t)dt$$

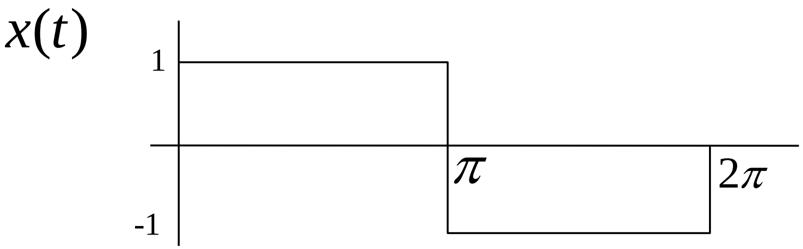
$$E_x = \int_0^{2\pi} \sin^2 t dt = \left| \frac{t}{2} - \frac{\sin 2t}{4} \right|_0^{2\pi} = \pi$$

$$\Rightarrow c = \frac{1}{\pi}\left[\int_0^{2\pi} g(t)\sin t dt\right] = \frac{1}{\pi}\left[\int_0^{\pi}\sin t dt - \int_{\pi}^{2\pi}\sin t dt\right]$$

$$= \frac{1}{\pi}\left[(1+1)-(-1-1)\right] = \frac{4}{\pi}$$

Example 2





$$g(t) = cx(t) \quad 0 \le t \le 2\pi$$

$$c = ?$$

We know

$$c = \frac{1}{E_x}\int_{t_1}^{t_2} g(t)x(t)dt$$

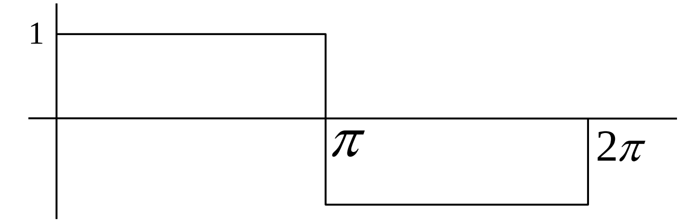
$$E_x = \int_0^{2\pi} 1dt = |t|_0^{2\pi} = 2\pi$$

$$c = \frac{1}{2\pi}\left[\int_0^{\pi} 1dt + \int_{\pi}^{2\pi} 1dt\right] = 1$$

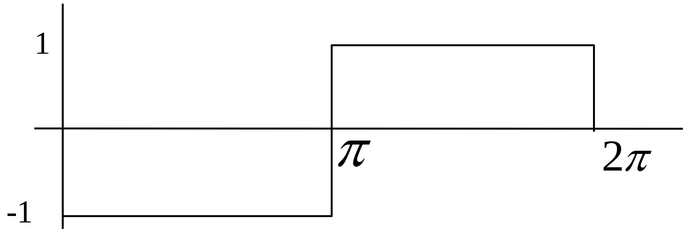
g(t) has a full projection on x(t) – both are same

Example 3

$g(t)$



$x(t)$



$$g(t) = cx(t) \qquad 0 \le t \le 2\pi$$

$$c = ?$$

We know

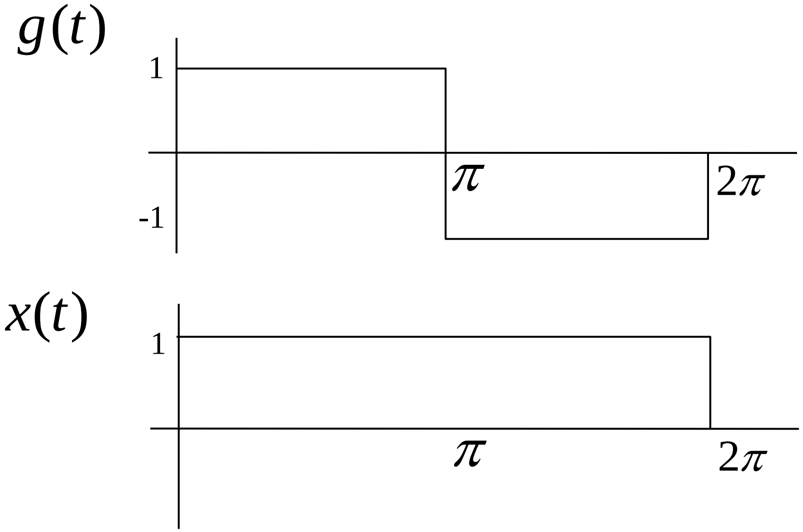
$$c = \frac{1}{E_x} \int_{t_1}^{t_2} g(t)x(t)dt$$

$$E_x = \int_0^{2\pi} 1dt = |t|_0^{2\pi} = 2\pi$$

$$c = \frac{1}{2\pi} \left[\int_0^{\pi} (-1)dt + \int_{\pi}^{2\pi} (-1)dt \right] = -1$$

g(t) has a full negative projection on x(t)
or g(t) and x(t) are opposite to each other

Example 4



$$g(t) = cx(t) \qquad 0 \le t \le 2\pi$$

$$c = ?$$

We know

$$c = \frac{1}{E_x}\int_{t_1}^{t_2} g(t)x(t)dt$$

$$E_x = \int_0^{2\pi} 1dt = \left|t\right|_0^{2\pi} = 2\pi$$

$$c = \frac{1}{2\pi}\left[\int_0^{\pi} 1dt + \int_{\pi}^{2\pi} (-1)dt\right] = 0 \qquad ?$$

g(t) does not have any projection on x(t)
or g(t) and x(t) are orthogonal to each other

Correlation between Signals

We already know, if $g(t) \cong cx(t) \qquad t_1 < t < t_2$

$$c = \frac{1}{E_x} \int_{t_1}^{t_2} g(t)x(t)dt$$

Let's define a correlation co-efficient between two functions, g(t) and x(t), as

$$c_n = \frac{1}{\sqrt{E_g E_x}} \int_{t_1}^{t_2} g(t)x(t)dt \qquad t_1 < t < t_2$$

Or to generalize

$$c_n = \frac{1}{\sqrt{E_g E_x}} \int_{-\infty}^{\infty} g(t)x(t)dt \qquad -1 < c_n < 1$$

$c_n = -1 \qquad 0 \qquad 1$

Opposite ***Orthogonal*** ***Itself***

Energy of Sum of Orthogonal Signals

$$z(t) = g(t) + x(t) \qquad t_1 \le t \le t_2$$

If g(t) and x(t) are orthogonal, then

$$E_z = E_g + E_x$$

By definition:

$$E_z = \int_{t_1}^{t_2} [g(t) + x(t)]^2 dt$$

$$= \int_{t_1}^{t_2} g^2(t)dt + \int_{t_1}^{t_2} x^2(t)dt + 2\int_{t_1}^{t_2} g(t)x(t)dt$$

$$= E_g + E_x$$

Orthogonal Signal Set

Let's define a signal set $x_1(t)$, $x_2(t)$, …., $x_N(t)$, such that

$$\int_{t_1}^{t_2} x_m(t)x_n(t)dt = \begin{cases} 0 & m \neq n \\ E_n & m = n \end{cases} \qquad t_1 < t < t_2$$

Then, $x_1(t)$, $x_2(t)$, …., $x_N(t)$ are called orthogonal signal set and if E_n is = 1, then the set is called orthonormal

Now let's assume

$$g(t) \cong c_1x_1(t) + c_2x_2(t) + c_3x_3(t) + \ldots\ldots + c_Nx_N(t) \;= \sum_{n=1}^{N} c_nx_n(t)$$

Then for best approximation i.e., minimized error energy

$$c_n = \frac{\int_{t_1}^{t_2} g(t)x_n(t)dt}{\int_{t_1}^{t_2} x_n^2(t)dt} = \frac{1}{E_n}\int_{t_1}^{t_2} g(t)x_n(t)dt$$

If the orthogonal set is complete, then error energy -> 0, i.e., approximation changes to equality

$$g(t) = c_1 x_1(t) + c_2 x_2(t) + c_3 x_3(t) + \ldots\ldots + c_N x_N(t) \;\; = \sum_{n=1}^{N} c_n x_n(t)$$

The above equation is called generalized Fourier Series

What about energy of g(t)? – Parseal's Theorem

$$E_g = c_1^2 E_1 + c_2^2 E_2 + c_3^2 E_3 + \ldots\ldots + c_N^2 E_N = \sum_{n=1}^{N} c_n^2 E_n$$

An Example of Complete Orthogonal Signal Set

$$\{1, \cos\omega_0 t, \cos 2\omega_0 t, \cos 3\omega_0 t.....$$

$$\sin\omega_0 t, \sin 2\omega_0 t, \sin 3\omega_0 t, ...\}$$

$$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$$

Follows by:

$$\int_{T_0} \cos n\omega_0 t \cos m\omega_0 t dt = \begin{cases} 0 & m \neq n \\ T_0/2 & m = n \neq 0 \end{cases}$$

$$\int_{T_0} \sin n\omega_0 t \sin m\omega_0 t dt = \begin{cases} 0 & m \neq n \\ T_0/2 & m = n \neq 0 \end{cases}$$

$$\int_{T_0} \sin n\omega_0 t \cos m\omega_0 t dt = 0 \quad for \quad all \quad m \quad and \quad n$$

Let's assume:

$$g(t) = a_0 + a_1 \cos\omega_0 t + a_2 \cos 2\omega_0 t + a_3 \cos 3\omega_0 t + ...$$

$$+ b_1 \sin\omega_0 t + b_2 \sin 2\omega_0 t + b_3 \sin 3\omega_0 t + ... \qquad t_1 < t < t_2 + T_0$$

$$= a_0 + \sum_{n=1}^{\infty} [a_n \cos n\omega_0 t + b_n \sin n\omega_0 t] \qquad \omega_0 = \frac{2\pi}{T_0}$$

Remember: $$c_n = \frac{1}{E_n} \int_{t_1}^{t_2} g(t) x_n(t) dt$$

Therefore,

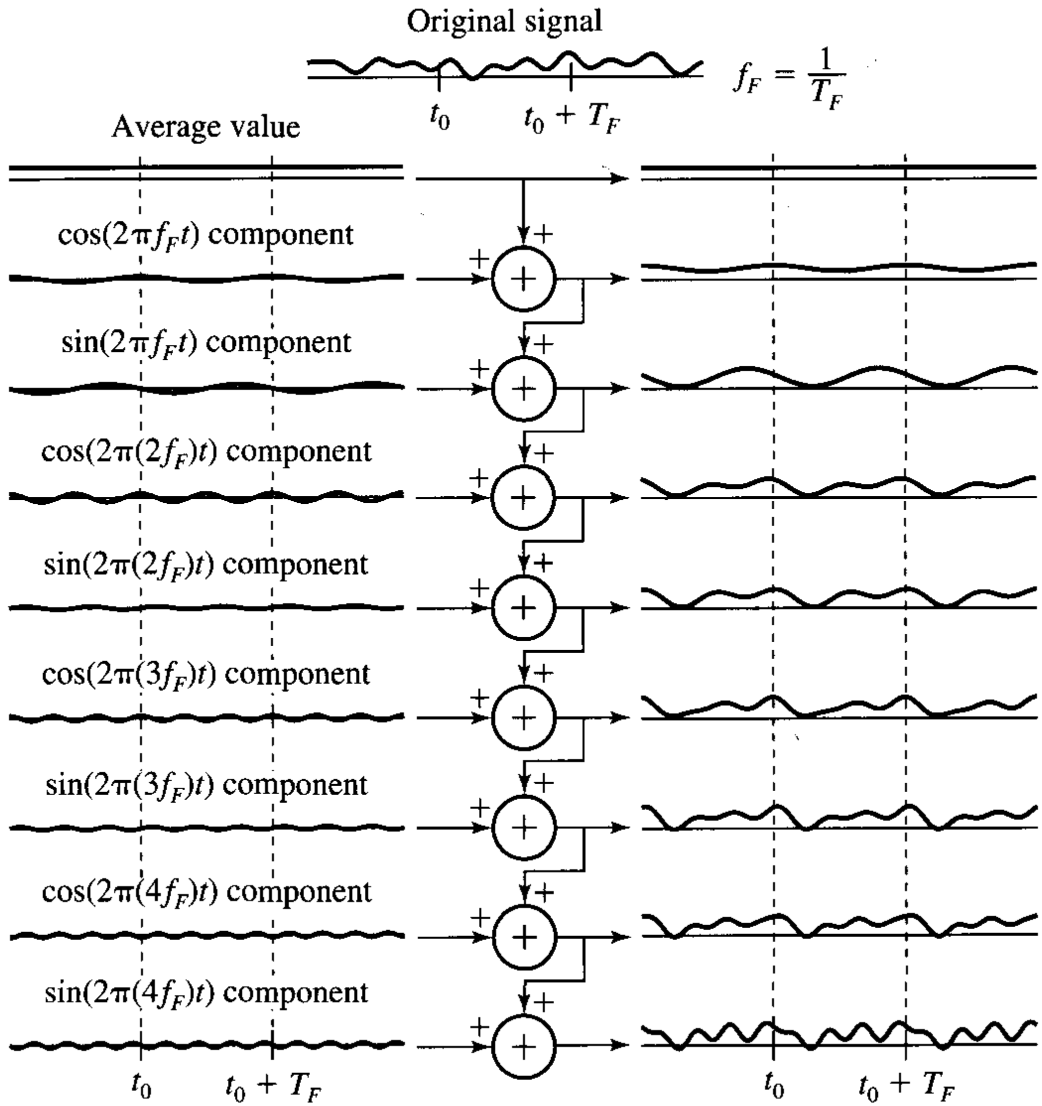
$$a_n = \frac{2}{T_0} \int_{t_1}^{t_1+T_0} g(t) \cos n\omega_0 t dt \qquad n = 1,2,3...$$

$$a_0 = \frac{1}{T_0} \int_{t_1}^{t_1+T_0} g(t) dt$$

$$b_n = \frac{2}{T_0} \int_{t_1}^{t_1+T_0} g(t) \sin n\omega_0 t dt \qquad n = 1,2,3...$$

Trigonometric Fourier Series

Concept of representing a periodic signal with a summation of sinusoids



Compact Trigonometric Fourier Series

$$g(t) = a_0 + \sum_{n=1}^{\infty}[a_n \cos n\omega_0 t + b_n \sin n\omega_0 t]$$

$$= C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$$

Compact Trigonometric Fourier Series

Where

$$C_n = \sqrt{a_n^2 + b_n^2}$$

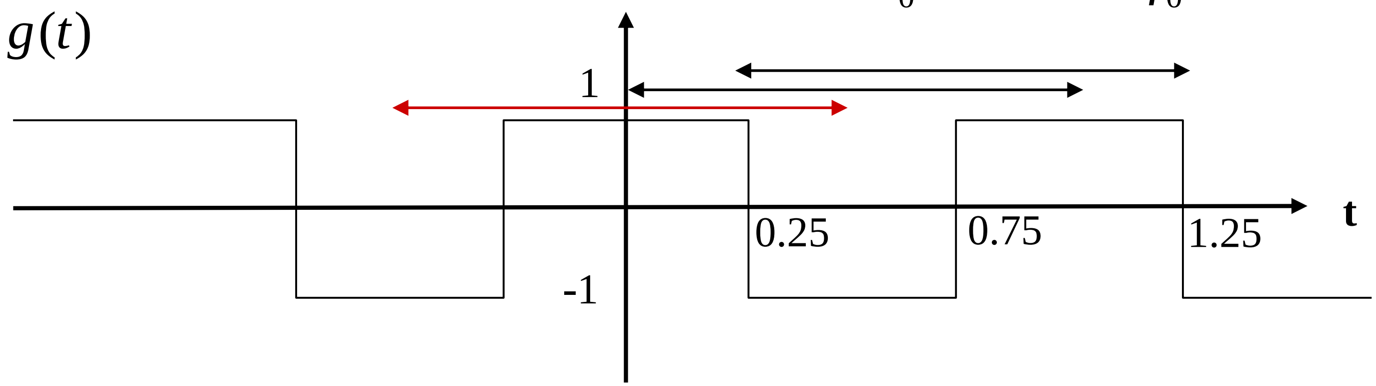
$$\theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right)$$

$$C_0 = a_0$$

Also called Fourier Spectrum of periodic signal

Example 1

$$T_0 = 1\sec \Rightarrow f_0 = 1Hz \Rightarrow \omega_0 = 2\pi$$



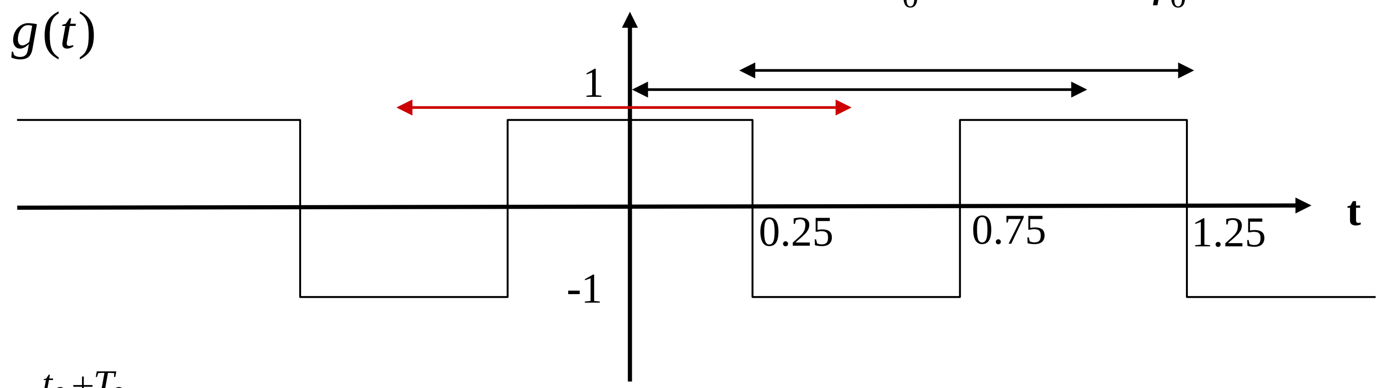
Without any mathematical calculations,

$$a_0 = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} g(t)dt = 0$$

$$b_n = \frac{2}{T_0}\int_{t_0}^{t_0+T_0} g(t)\sin(n\omega_0 t)dt = 0$$

Example 1 … Cont.

$$T_0 = 1\sec \Rightarrow f_0 = 1Hz \Rightarrow \omega_0 = 2\pi$$



$$a_n = \frac{2}{T_0}\int_{t_0}^{t_0+T_0} g(t)\cos(n\omega_0 t)dt$$

$$a_n = 2\int_{-0.5}^{0.5} g(t)\cos(2\pi nt)dt = 2x2\int_{0}^{0.5} g(t)\cos(2\pi nt)dt$$

$$= 4\int_{0}^{0.25}\cos(2\pi nt)dt + 4\int_{0.25}^{0.5}(-1)\cos(2\pi nt)dt$$

$$= 4\int_{0}^{0.25}\cos(2\pi nt)dt + 4\int_{0.25}^{0.5}(-1)\cos(2\pi nt)dt$$

$$= 4\int_{0}^{0.25}\cos(2\pi nt)dt - 4\int_{0.25}^{0.5}\cos(2\pi nt)dt$$

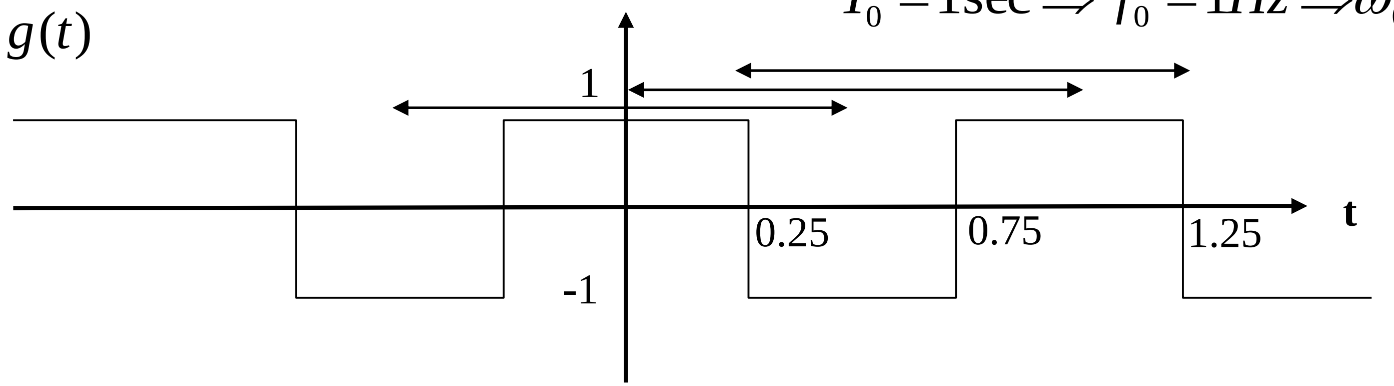
$$= \left|\frac{4\sin(2\pi nt)}{2\pi n}\right|_{0}^{0.25} - \left|\frac{4\sin(2\pi nt)}{2\pi n}\right|_{0.25}^{0.5}$$

$$= \frac{2}{\pi n}\sin(\frac{\pi n}{2}) - (-\frac{2}{\pi n}\sin(\frac{\pi n}{2})) = \frac{4}{\pi n}\sin(\frac{\pi n}{2})$$

$$\Rightarrow a_n = \frac{4}{\pi n}\sin(\frac{\pi n}{2})$$

Example 1 … Cont.

$$T_0 = 1\sec \Rightarrow f_0 = 1Hz \Rightarrow \omega_0 = 2\pi$$



$$a_n = \frac{4}{\pi n}\sin\left(\frac{\pi n}{2}\right)$$

$$a_1 = \frac{4}{\pi}$$

$$a_2 = 0$$

$$a_3 = -\frac{4}{3\pi}$$

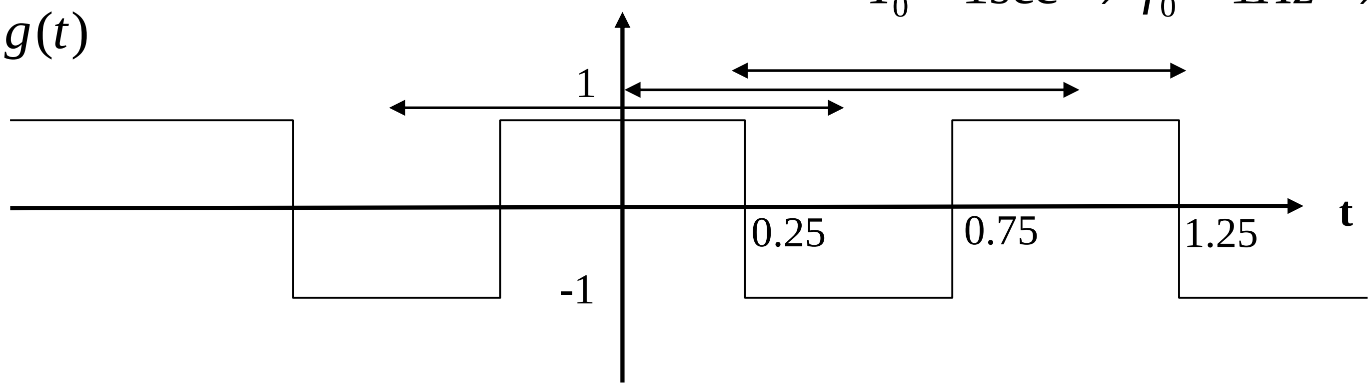
$$a_4 = 0$$

$$a_5 = \frac{4}{5\pi}$$

$$a_6 = 0$$

Example 1 … Cont.

$$T_0 = 1\sec \Rightarrow f_0 = 1Hz \Rightarrow \omega_0 = 2\pi$$

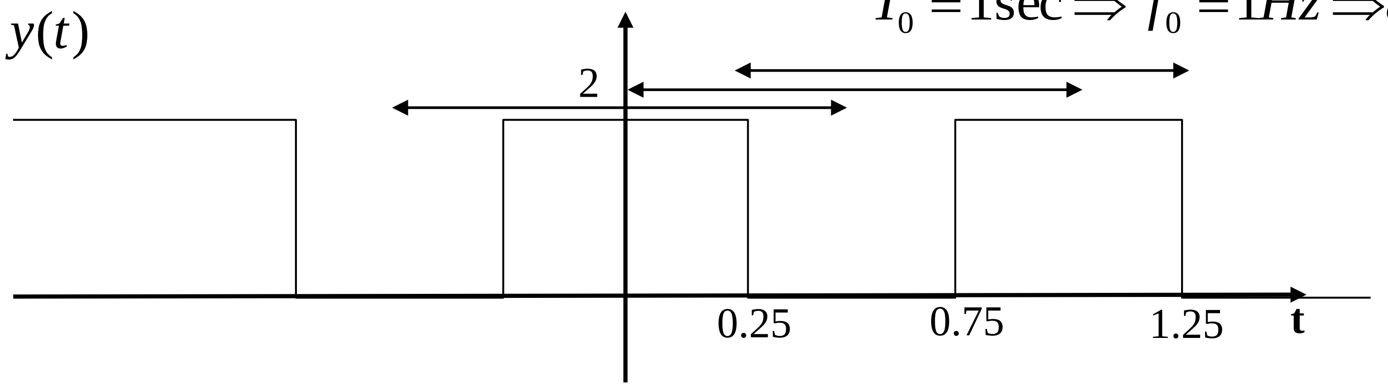


$$g(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos n\omega_0 t + b_n \sin n\omega_0 t]$$

$$\Rightarrow g(t) = \frac{4}{\pi}\cos(\omega_0 t) - \frac{4}{3\pi}\cos(3\omega_0 t) + \frac{4}{5\pi}\cos(5\omega_0 t) - \frac{4}{7\pi}\cos(7\omega_0 t) + \ldots$$

$$= \frac{4}{\pi}\cos((2\pi)t) - \frac{4}{3\pi}\cos(3(2\pi)t) + \frac{4}{5\pi}\cos(5(2\pi)t) - \frac{4}{7\pi}\cos(7(2\pi)t) + \ldots$$

Example 2



$$T_0 = 1\sec \Rightarrow f_0 = 1Hz \Rightarrow \omega_0 = 2\pi$$

$$y(t) = 1 + g(t)$$

We know that

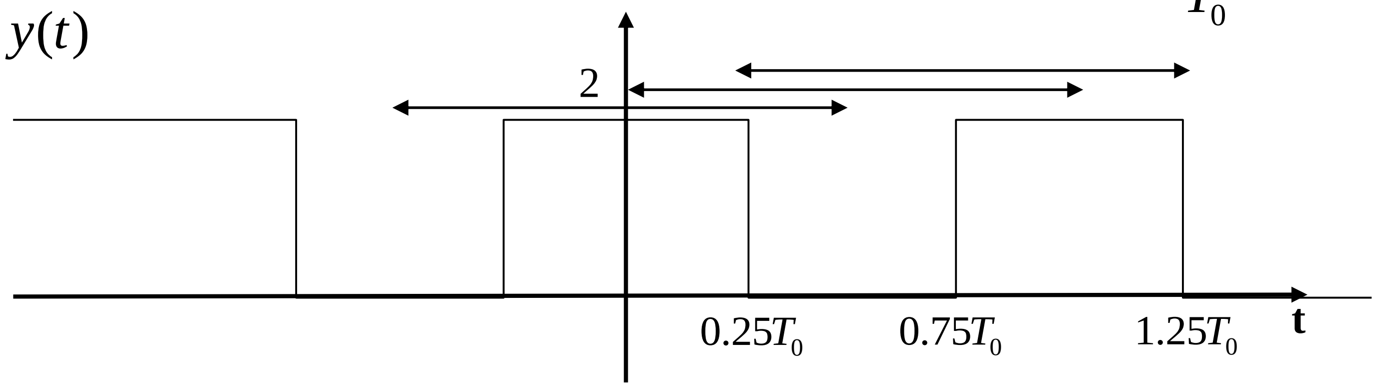
$$g(t) = \frac{4}{\pi}\cos((2\pi)t) - \frac{4}{3\pi}\cos(3(2\pi)t) + \frac{4}{5\pi}\cos(5(2\pi)t) - \frac{4}{7\pi}\cos(7(2\pi)t) + \dots$$

Therefore,

$$y(t) = 1 + \frac{4}{\pi}\cos((2\pi)t) - \frac{4}{3\pi}\cos(3(2\pi)t) + \frac{4}{5\pi}\cos(5(2\pi)t) - \frac{4}{7\pi}\cos(7(2\pi)t) + \dots$$

Example 3

$$T_0 \Rightarrow f_0 = \frac{1}{T_0} \Rightarrow \omega_0 = \frac{2\pi}{T_0}$$



We know that with time period = 1 sec

$$y(t) = 1 + \frac{4}{\pi}\cos((2\pi)t) - \frac{4}{3\pi}\cos(3(2\pi)t) + \frac{4}{5\pi}\cos(5(2\pi)t) - \frac{4}{7\pi}\cos(7(2\pi)t) + \ldots$$

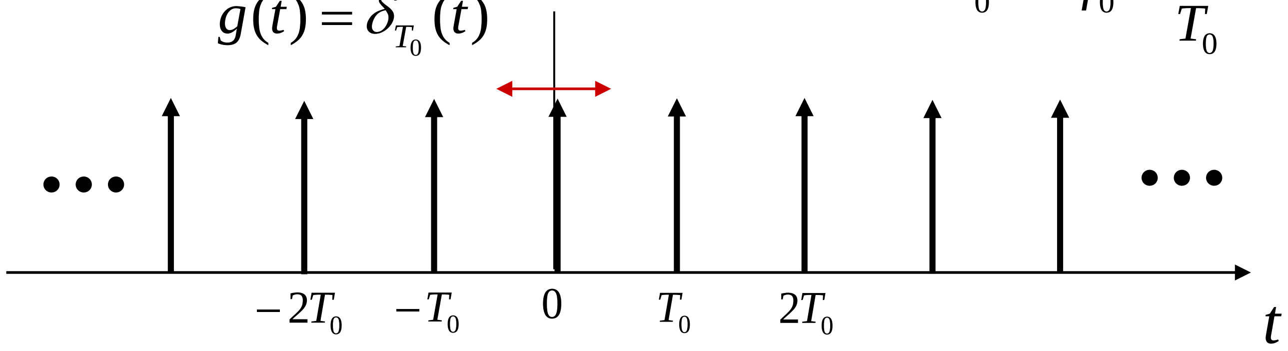
Therefore, with time period of T_0

$$y(t) = 1 + \frac{4}{\pi}\cos(\omega_0 t) - \frac{4}{3\pi}\cos(3\omega_0 t) + \frac{4}{5\pi}\cos(5\omega_0 t) + \ldots$$

Example

$$T_0 \Rightarrow f_0 = \frac{1}{T_0} \Rightarrow \omega_0 = \frac{2\pi}{T_0}$$

$$g(t) = \delta_{T_0}(t)$$



$$g(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$$

Remember definition of impulse function

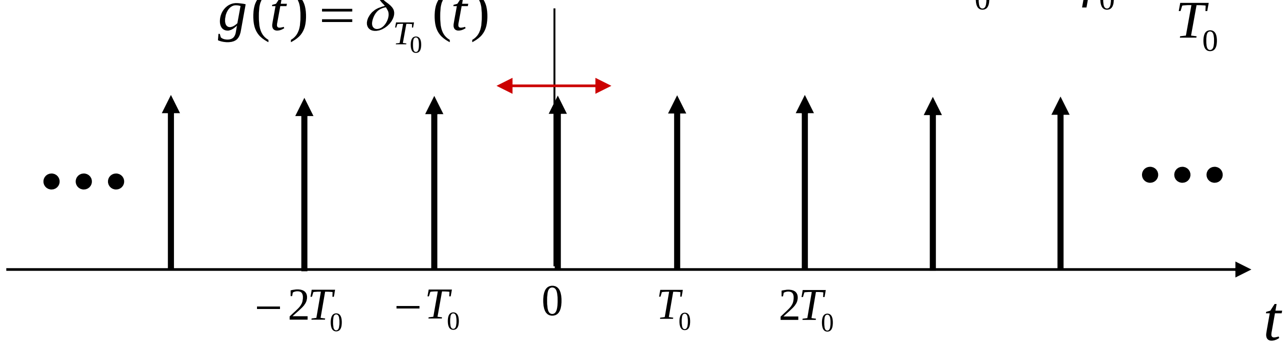
$$a_0 = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} \delta(t)dt = \frac{1}{T_0}$$

$$\int_{-\infty}^{\infty} \delta(t) = 1$$

Example … cont.

$$T_0 \Rightarrow f_0 = \frac{1}{T_0} \Rightarrow \omega_0 = \frac{2\pi}{T_0}$$

$$g(t) = \delta_{T_0}(t)$$



$$a_n = \frac{2}{T_0}\int_{t_0}^{t_0+T_0} \delta(t)\cos(n\omega_0 t)dt = \frac{2}{T_0}$$

$$b_n = \frac{2}{T_0}\int_{t_0}^{t_0+T_0} \delta(t)\sin(n\omega_0 t)dt = 0$$

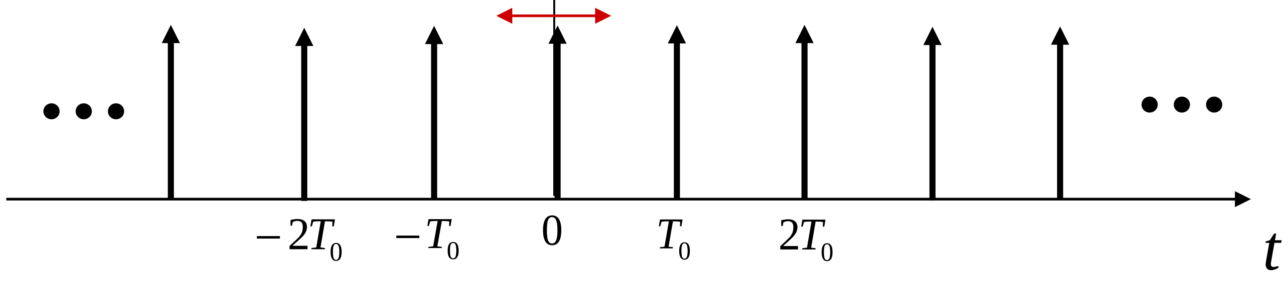
Remember extended sampling properties of impulse function

$$\int_{-\infty}^{\infty} g(t)\delta(t) = g(0)$$

Example … cont.

$$T_0 \Rightarrow f_0 = \frac{1}{T_0} \Rightarrow \omega_0 = \frac{2\pi}{T_0}$$

$$g(t) = \delta_{T_0}(t)$$



$$a_0 = \frac{1}{T_0}$$

$$a_n = \frac{2}{T_0}$$

$$b_n = 0$$

$$\Rightarrow$$

$$C_0 = a_0 = \frac{1}{T_0}$$

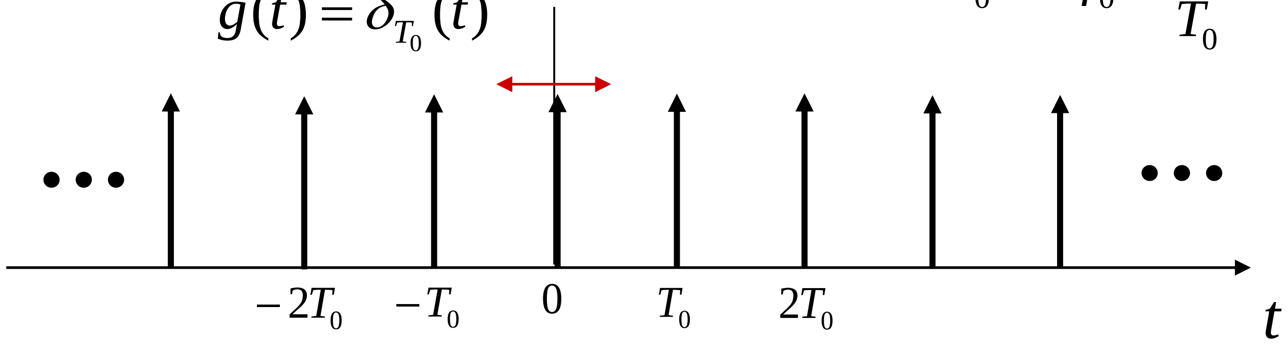
$$C_n = \sqrt{a_n^2 + b_n^2} = a_n = \frac{2}{T_0}$$

$$\theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right) = 0$$

Example … cont.

$$T_0 \Rightarrow f_0 = \frac{1}{T_0} \Rightarrow \omega_0 = \frac{2\pi}{T_0}$$

$$g(t) = \delta_{T_0}(t)$$



$$g(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$$

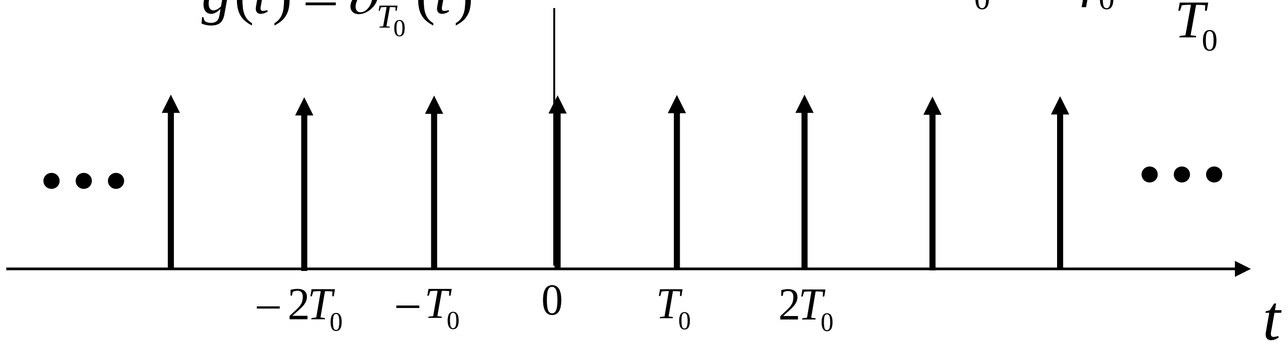
$$= \frac{1}{T_0} + \sum_{n=1}^{\infty} \frac{2}{T_0} \cos(n\omega_0 t)$$

Example … cont.

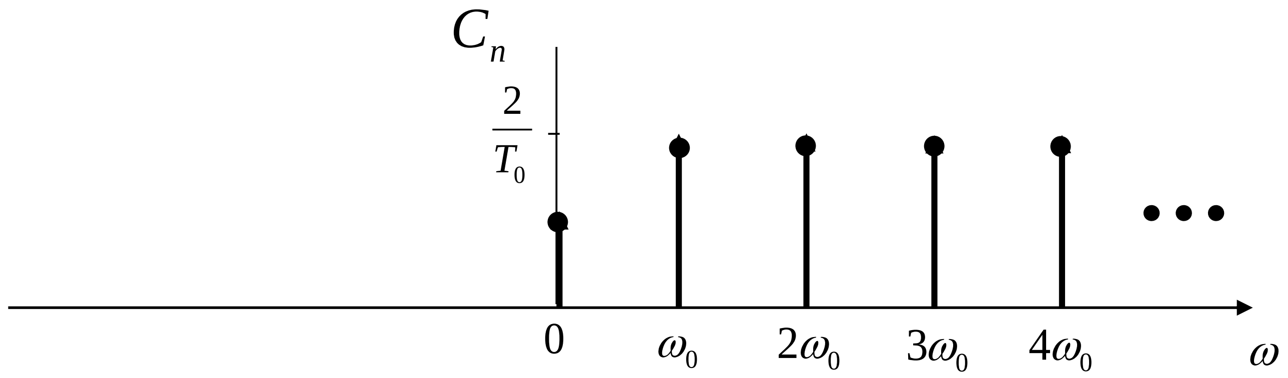
Time Domain Signal

$$T_0 \Rightarrow f_0 = \frac{1}{T_0} \Rightarrow \omega_0 = \frac{2\pi}{T_0}$$

$$g(t) = \delta_{T_0}(t)$$



Fourier Spectrum



Energy of Signal Revisited

Energy of Signal is defined

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt \quad \textbf{and not} \quad E_g = \int_{-\infty}^{\infty} g^2(t) dt$$

To accommodate complex signals

Because

$$|g(t)|^2 = g(t)g(t) = g^2(t) \quad \textbf{for real signals}$$

$$|g(t)|^2 = g(t)g^*(t) \quad \textbf{for complex signals}$$

Similarly,

$$g(t)x(t) \quad becomes = g(t)x^*(t) \quad complex \quad signals$$

Another Example of orthogonal Signal Set

(again periodic)

$$e^{jn\omega_0 t} \qquad (n = 0, \pm 1, \pm 2, \ldots)$$

$$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$$

$$\int_{T_0} e^{jm\omega_0 t}\left(e^{jn\omega_0 t}\right)^* dt = \int_{T_0} e^{j(m-n)\omega_0 t} dt$$

$$= \int_{T_0} \left(\cos(m-n)\omega_0 t + j\sin(m-n)\omega_0 t\right) dt$$

$$= \int_{T_0} \cos(m-n)\omega_0 t dt + j\int_{T_0} \sin(m-n)\omega_0 t dt$$

$$= \begin{cases} 0 & m \neq n \\ T_0 & m = n \neq 0 \end{cases}$$

Exponential Fourier Series

$$g(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$

By orthogonality

$$D_n = \frac{1}{T_0} \int_{T_0} g(t) \left(e^{jn\omega_0 t}\right)^{*} dt$$

or

$$D_n = \frac{1}{T_0} \int_{T_0} g(t) e^{-jn\omega_0 t} dt$$

Compact to Exponential Fourier Series

$$g(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$$

$$= C_0 + \sum_{n=1}^{\infty} \frac{C_n}{2}\left[e^{j(n\omega_0 t+\theta_n)} + e^{-j(n\omega_0 t+\theta_n)}\right]$$

$$= C_0 + \sum_{n=1}^{\infty}\left[\left(\frac{C_n}{2}e^{j\theta_n}\right)e^{jn\omega_0 t} + \left(\frac{C_n}{2}e^{-j\theta_n}\right)e^{-jn\omega_0 t}\right]$$

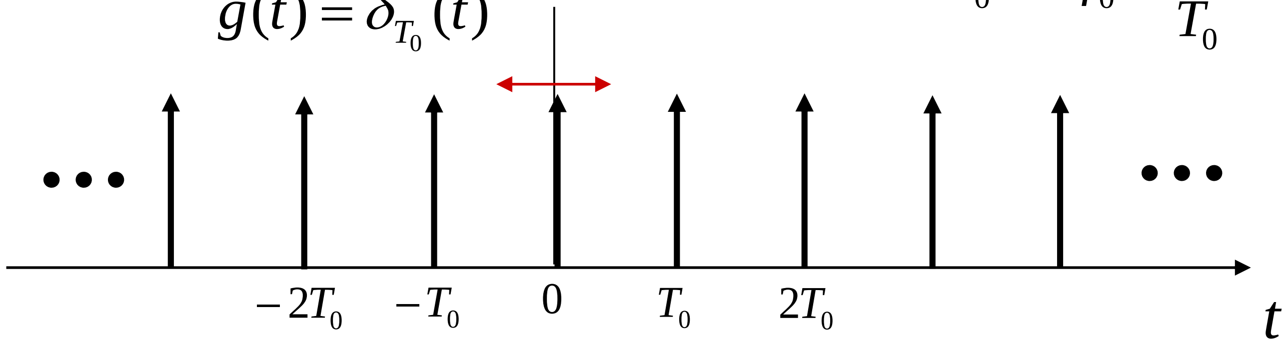
$$= D_0 + \sum_{n=1}^{\infty}\left[D_n e^{jn\omega_0 t} + D_{-n}e^{-jn\omega_0 t}\right]$$

$$= D_0 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} D_n e^{jn\omega_0 t} \qquad = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$

Let's look back on the Example of Impulse Train

$$T_0 \Rightarrow f_0 = \frac{1}{T_0} \Rightarrow \omega_0 = \frac{2\pi}{T_0}$$

$$g(t) = \delta_{T_0}(t)$$



$$g(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n) = \frac{1}{T_0} + \sum_{n=1}^{\infty} \frac{2}{T_0} \cos(n\omega_0 t)$$

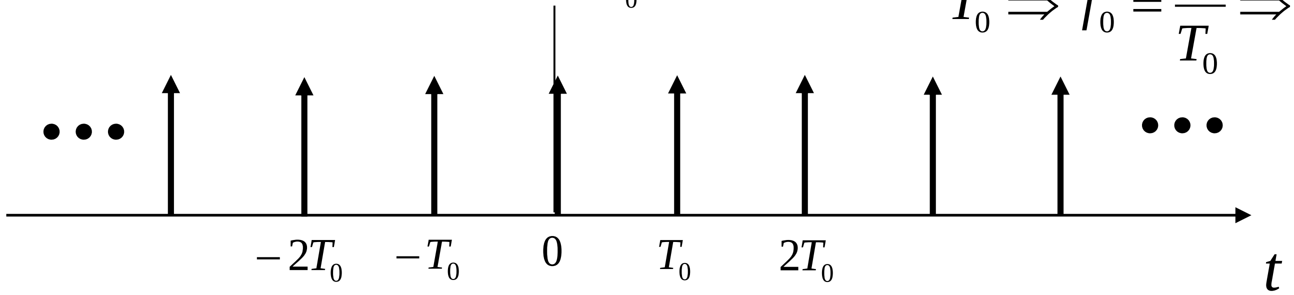
$$D_0 = C_0 = \frac{1}{T_0}$$

$$D_n = \frac{C_n}{2} e^{j\theta_n} = \frac{1}{T_0} \qquad D_{-n} = \frac{C_n}{2} e^{-j\theta_n} = \frac{1}{T_0}$$

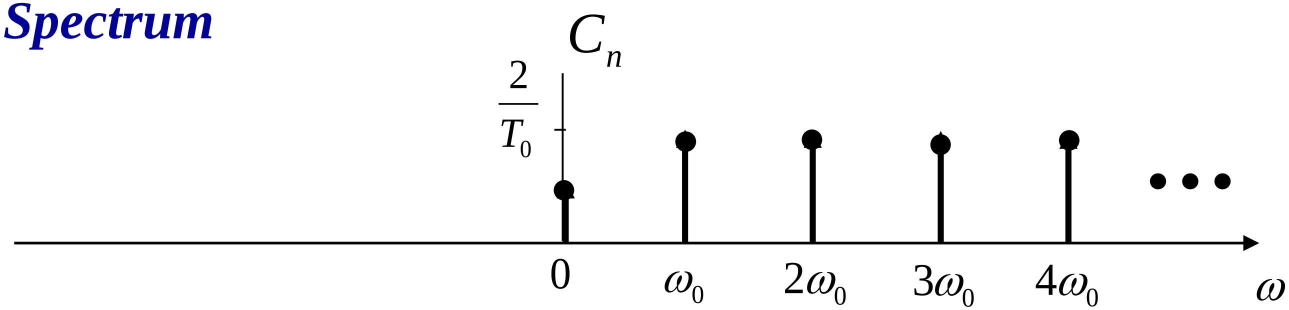
Time Domain Signal

$$g(t) = \delta_{T_0}(t)$$

$$T_0 \Rightarrow f_0 = \frac{1}{T_0} \Rightarrow \omega_0 = \frac{2\pi}{T_0}$$



Fourier Spectrum



Exponential Fourier Spectrum