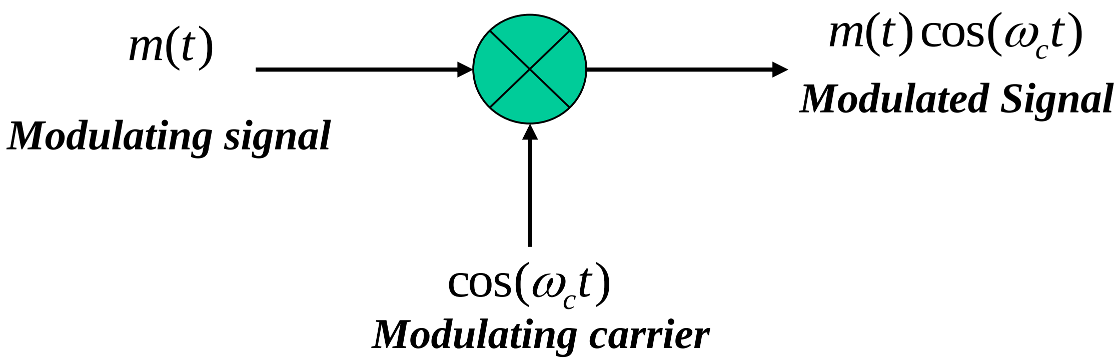
Amplitude Modulation – DSB-SC



Modulation: $m(t) \Leftrightarrow M(\omega)$

$$S_{\text{mod}}(t) = m(t)\cos(\omega_c t) \Leftrightarrow \frac{1}{2}[M(\omega+\omega_c) + M(\omega-\omega_c)]$$

Modulation: $m(t) \Leftrightarrow M(\omega)$

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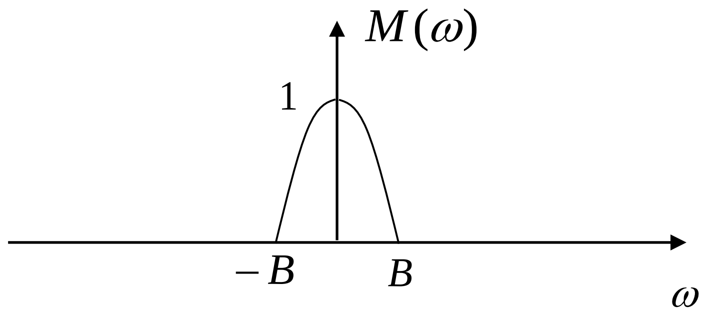
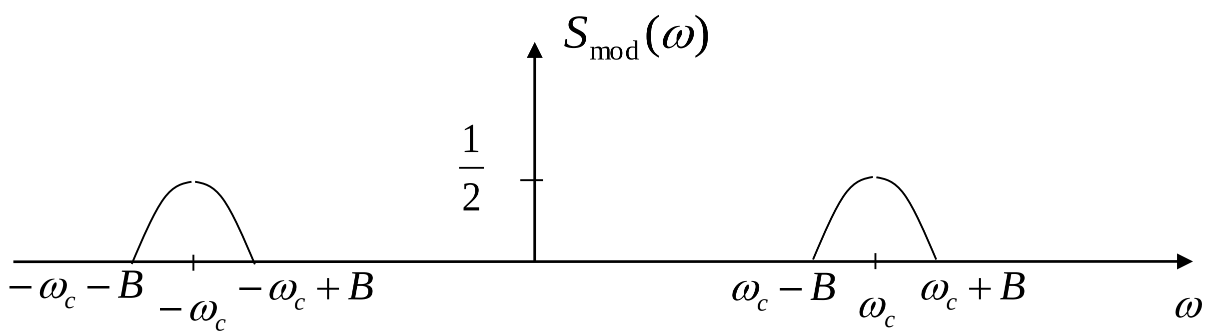
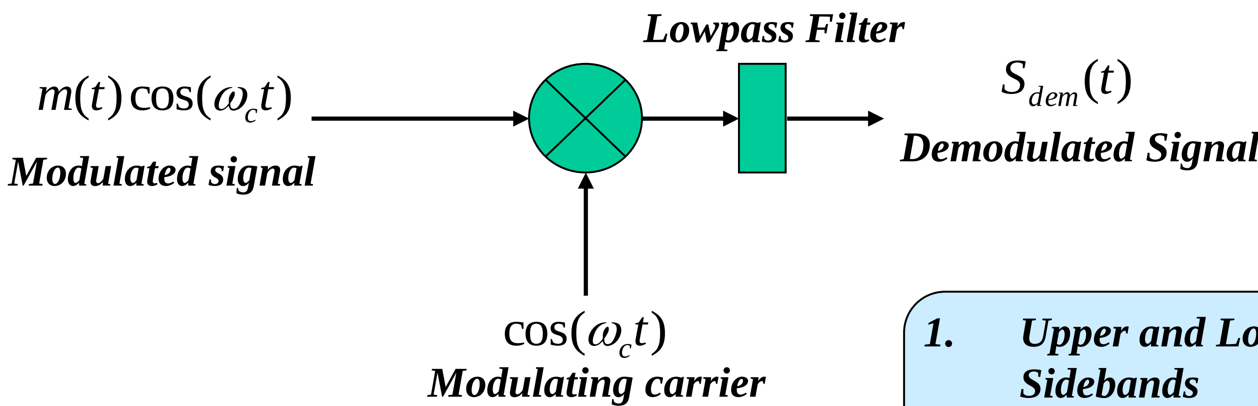




Amplitude Demodulation – DSB-SC



1. ***Upper and Lower Sidebands***
2. ***Relationship of carrier freq and signal bandwidth***

Demodulation:

$$S_{dem}(t) = m(t)\cos^2(\omega_c t)$$

$$= \frac{1}{2}[m(t) + m(t)\cos(2\omega_c t)]$$

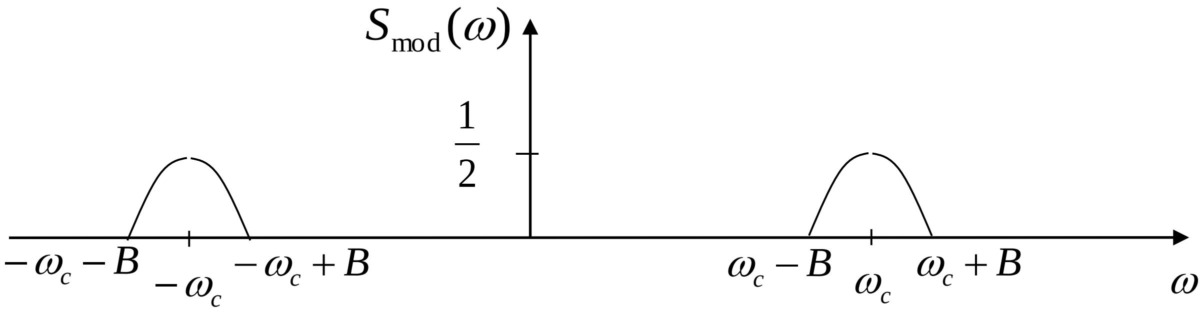
$$S_{dem}(\omega) \Leftrightarrow \frac{1}{2}M(\omega) + \frac{1}{4}[M(\omega + 2\omega_c) + M(\omega - 2\omega_c)]$$

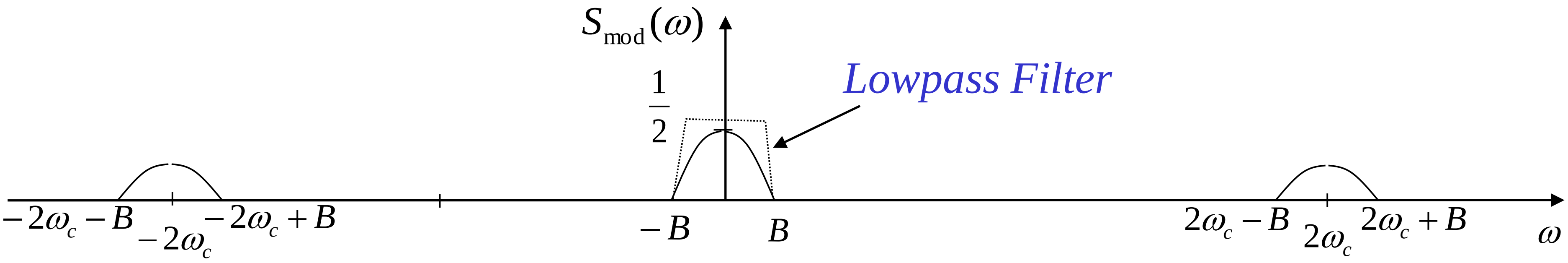
Demodulation:

$$S_{dem}(t) = m(t)\cos^2(\omega_c t)$$

$$= \frac{1}{2}[m(t) + m(t)\cos(2\omega_c t)]$$

$$S_{dem}(\omega) \Leftrightarrow \frac{1}{2}M(\omega) + \frac{1}{4}[M(\omega + 2\omega_c) + M(\omega - 2\omega_c)]$$





Example of DSB-SC Modulation

Let $$m(t) = \cos(\omega_m t)$$

then $$M(\omega) = \pi[\delta(\omega + \omega_m) + \delta(\omega - \omega_m)]$$

$$S_{\mathrm{mod}}(t) = m(t)\cos(\omega_c t)$$

$$= \cos(\omega_m t)\cos(\omega_c t)$$

$$= \frac{1}{2}[\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t]$$

Example of DSB-SC Demodulation

$$S_{dem}(t) = m(t)\cos^2(\omega_c t)$$

$$= \cos(\omega_m t)\cos^2(\omega_c t)$$

$$= \frac{1}{2}\cos(\omega_m t)(1 + \cos(2\omega_c t))$$

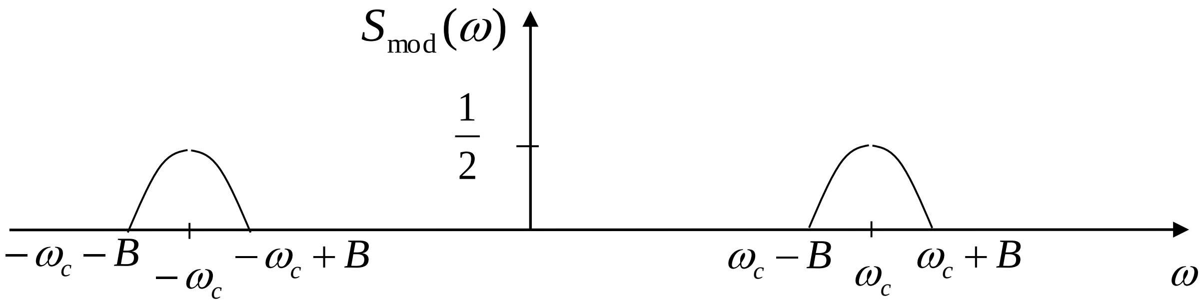
$$= \frac{1}{2}\cos(\omega_m t) + \frac{1}{2}\cos(\omega_m t)\cos(2\omega_c t)$$

Frequency Conversion or Mixing

How to change carrier frequency from one frequency to another?

$$m(t) \Leftrightarrow M(\omega)$$

$$S_{\text{mod}}(t) = m(t)\cos(\omega_c t) \Leftrightarrow \frac{1}{2}[M(\omega+\omega_c)+M(\omega-\omega_c)]$$



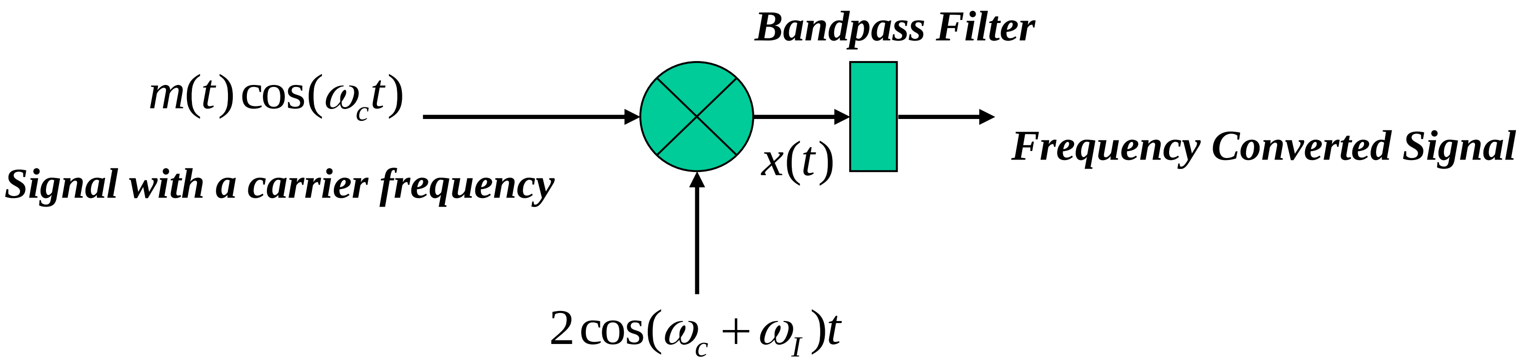
The Goal is to change ω_c *to* ω_I

$$m(t)\cos(\omega_c t) \quad \rightarrow \quad m(t)\cos(\omega_I t)$$

$$\text{if} \;\; \omega_I > \omega_c \;\; \text{up conversion}$$

$$\text{if} \;\; \omega_I < \omega_c \;\; \text{down conversion}$$

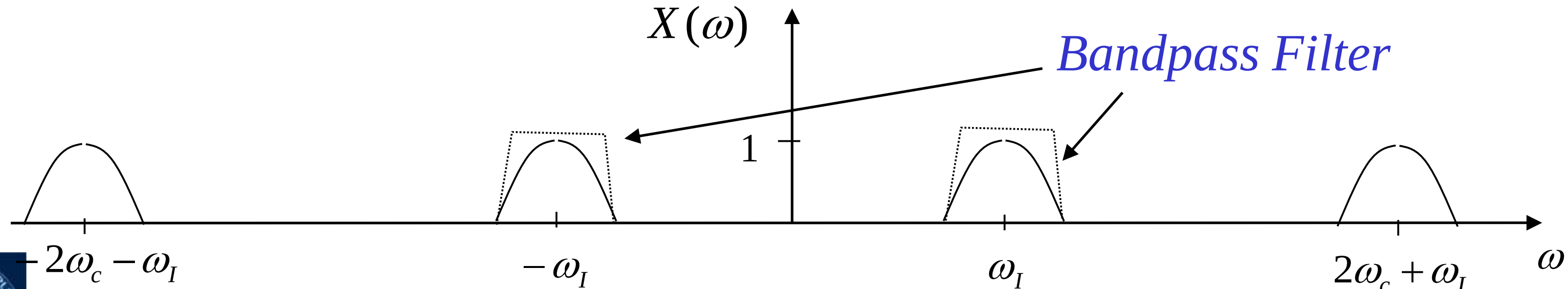
Option 1



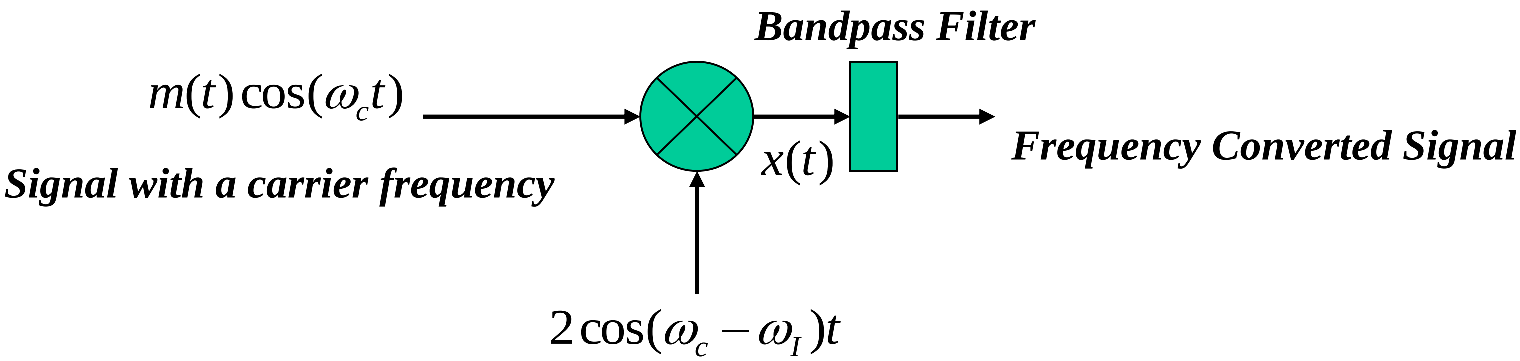
$$x(t) = m(t)\cos(\omega_c t)2\cos(\omega_c+\omega_I)t$$

$$= m(t)[2\cos(\omega_c t)\cos(\omega_c+\omega_I)t]$$

$$= m(t)[\cos(\omega_I t)+\cos(2\omega_c+\omega_I)t]$$



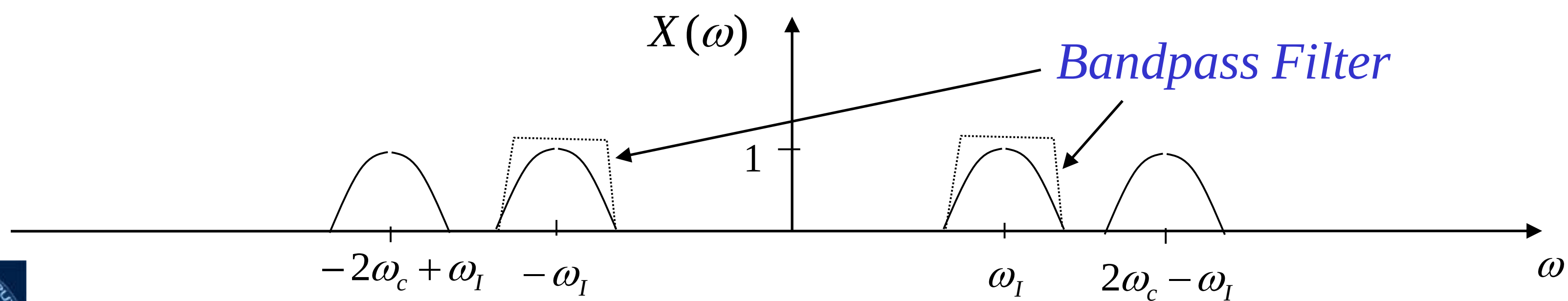
Option 2



$$x(t) = m(t)\cos(\omega_c t) 2\cos(\omega_c - \omega_I)t$$

$$= m(t)[2\cos(\omega_c t)\cos(\omega_c - \omega_I)t]$$

$$= m(t)[\cos(\omega_I t) + \cos(2\omega_c - \omega_I)t]$$



Switching Modulators

AM modulation can be obtained by not only multiplying with pure sinosoidal signal but by any periodic signal with fundamental frequency ω_c. Any periodic signal can be represented as:

$$\phi(t) = \sum_{n=0}^{\infty} C_n \cos(n\omega_c t + \theta_n)$$

$$\Rightarrow \quad m(t)\phi(t) = \sum_{n=0}^{\infty} C_n m(t) \cos(n\omega_c t + \theta_n)$$

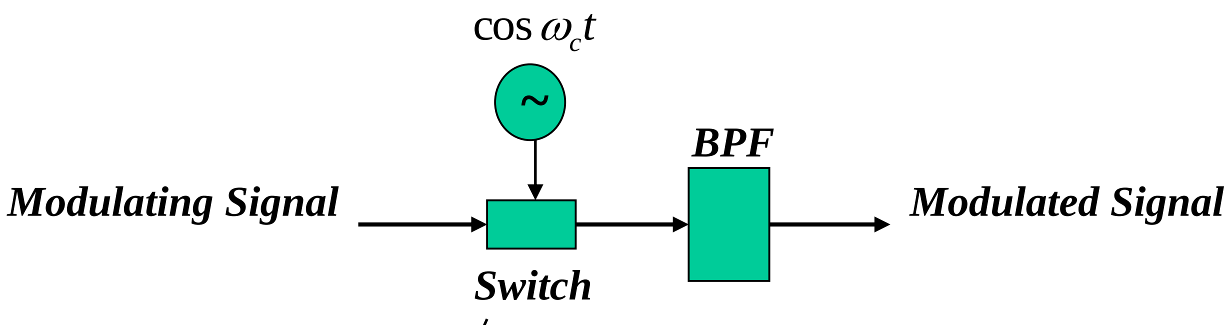
$$m(t)\phi(t) = C_0 m(t) + m(t) C_1(\omega_c t + \theta_n) + \sum_{n=2}^{\infty} C_n m(t) \cos(n\omega_c t + \theta_n)$$

Need a Bandpass Filter

$$w(t)=\frac{1}{2}+\frac{2}{\pi}[\cos\omega_c t-\frac{1}{3}\cos 3\omega_c t+\frac{1}{5}\cos 5\omega_c t-....]$$

$$m(t)w(t)=\frac{1}{2}m(t)+\frac{2}{\pi}[m(t)\cos\omega_c t-\frac{1}{3}m(t)\cos 3\omega_c t+\frac{1}{5}m(t)\cos 5\omega_c t-.....]$$

$$S_{\text{mod}}(\omega)\Leftrightarrow\frac{1}{2}M(\omega)+\frac{1}{\pi}[M(\omega+\omega_c)+M(\omega-\omega_c)]$$
$$-\frac{1}{3\pi}[M(\omega+3\omega_c)+M(\omega-3\omega_c)]$$
$$+\frac{1}{5\pi}[M(\omega+5\omega_c)+M(\omega-5\omega_c)].....$$



Can be realized with a diode (a basic switching element) or combination of diodes

Demodulation still needs to be synchronous

Regular Amplitude Modulation without carrier suppression

$$S_{\mathrm{mod}(AM)}(t) = A\cos(\omega_c t) + m(t)\cos(\omega_c t)$$

$$= [A + m(t)]\cos(\omega_c t)$$

$$S_{dem(AM)}(\omega) \Leftrightarrow \frac{1}{2}[M(\omega+\omega_c) + M(\omega-\omega_c)] + \pi A[\delta(\omega+\omega_c) + \delta(\omega-\omega_c)]$$

Important condition for envelop detection

A+m(t) $\geq$ *0 for all t*

Let m_p is the maximum peak value of m(t), then

$$A \geq m_p$$

Let's define modulation index $\mu = \frac{m_p}{A}$

$$0 \leq \mu \leq 1$$

Power of Carrier vs. Power in Sidebands

$$S_{\text{mod}(AM)}(t) = A\cos(\omega_c t) + m(t)\cos(\omega_c t)$$

$$P_c = \frac{A^2}{2} \qquad \textbf{and} \qquad P_s = \frac{1}{2}\overline{m^2(t)}$$

Power Efficiency

$$\eta = \frac{P_{useful}}{P_{total}} = \frac{P_s}{P_c + P_s} = \frac{\overline{m^2(t)}}{A^2 + \overline{m^2(t)}}$$

$$S_{\mathrm{mod}(AM)}(t) = A\cos(\omega_c t) + m(t)\cos(\omega_c t)$$

Let $m(t) = B\cos(\omega_m t)$

$$S_{\mathrm{mod}(AM)}(t) = A\cos(\omega_c t) + B\cos(\omega_m t)\cos(\omega_c t)$$

$$= [A + B\cos(\omega_m t)]\cos(\omega_c t)$$

$$\mu = \frac{m_p}{A} = \frac{B}{A} \Rightarrow B = \mu A$$

$$S_{\mathrm{mod}(AM)}(t) = [A + \mu A\cos(\omega_m t)]\cos(\omega_c t)$$

$$= A[1 + \mu\cos(\omega_m t)]\cos(\omega_c t)$$

$$P_c = \frac{A^2}{2} \qquad \textbf{and} \qquad P_s = \frac{1}{2}\overline{m^2(t)}$$

$$P_c = \frac{A^2}{2} \qquad\qquad P_s = \frac{1}{2}\frac{\mu^2 A^2}{2}$$

$$\eta = \frac{P_{useful}}{P_{total}} = \frac{P_s}{P_c + P_s} = \frac{\mu^2 A^2}{2A^2 + \mu^2 A^2} = \frac{\mu^2}{2+\mu^2}$$

$$= \frac{1}{2+1} = 0.33 = 33\% \qquad for \qquad \mu = 1$$

Example of Tone modulation => max efficiency = 33%

Generation of AM signals

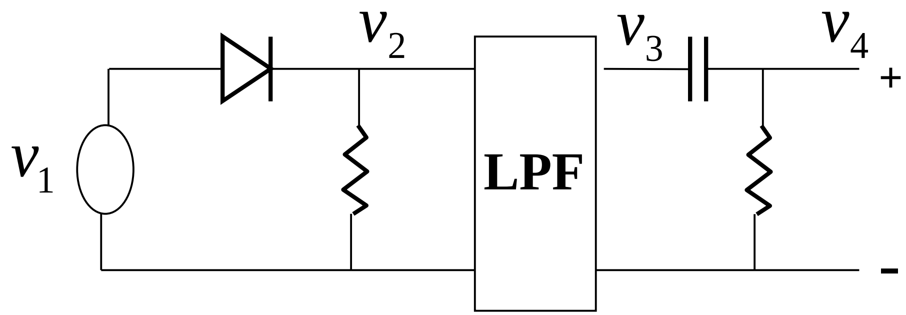
Generation of AM signals is the same as generation f AM-SC signals, the only difference is that m(t) is replaced by [A+m(t)]

Demodulation of AM signals

Two commonly used methods are:

1. ***Rectifier Detection – synchronous demodulation***
2. ***Envelop Detection – very simple detection***

Rectifier Detector



$$v_1 = [A + m(t)]\cos(\omega_c t)$$

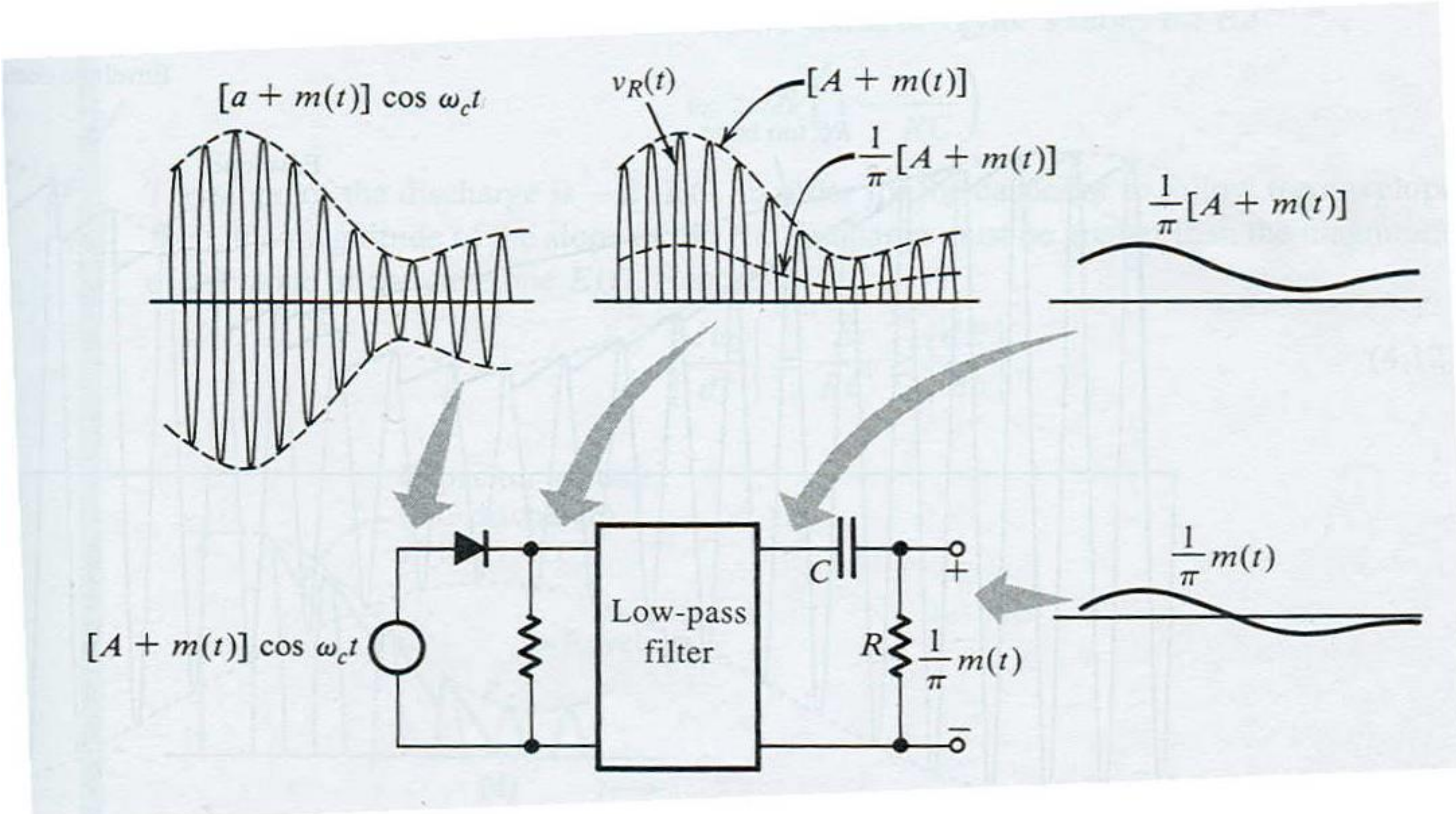
$$v_2 = \{[A + m(t)]\cos(\omega_c t)\}w(t)$$

$$= [A + m(t)]\cos(\omega_c t)[\frac{1}{2} + \frac{2}{\pi}\cos\omega_c t - \frac{2}{3\pi}\cos 3\omega_c t + \frac{2}{5\pi}\cos 5\omega_c t - ....]$$

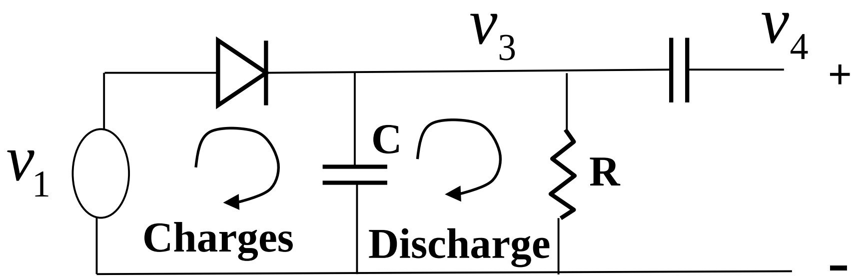
$$= [A + m(t)]\cos(\omega_c t) * \frac{2}{\pi}\cos\omega_c t + [A + m(t)]\cos(\omega_c t)[\frac{1}{2} - \frac{2}{3\pi}\cos 3\omega_c t + \frac{2}{5\pi}\cos 5\omega_c t - ....]$$

$$= \frac{1}{\pi}[A + m(t)] + higher \quad frequency \quad terms$$

$$v_3 = \frac{1}{\pi}[A + m(t)] \qquad\qquad v_4 = \frac{1}{\pi}m(t)$$


[a + m(t)] cos ω_c t
v_R(t)
[A + m(t)]
1/π [A + m(t)]
1/π [A + m(t)]
Low-pass filter
[A + m(t)] cos ω_c t
C
R
1/π m(t)
+
−
1/π m(t)

Envelop Detector



Time Constant = RC

$$RC > \frac{1}{\omega_c}$$

$$RC < \frac{1}{2\pi B}$$

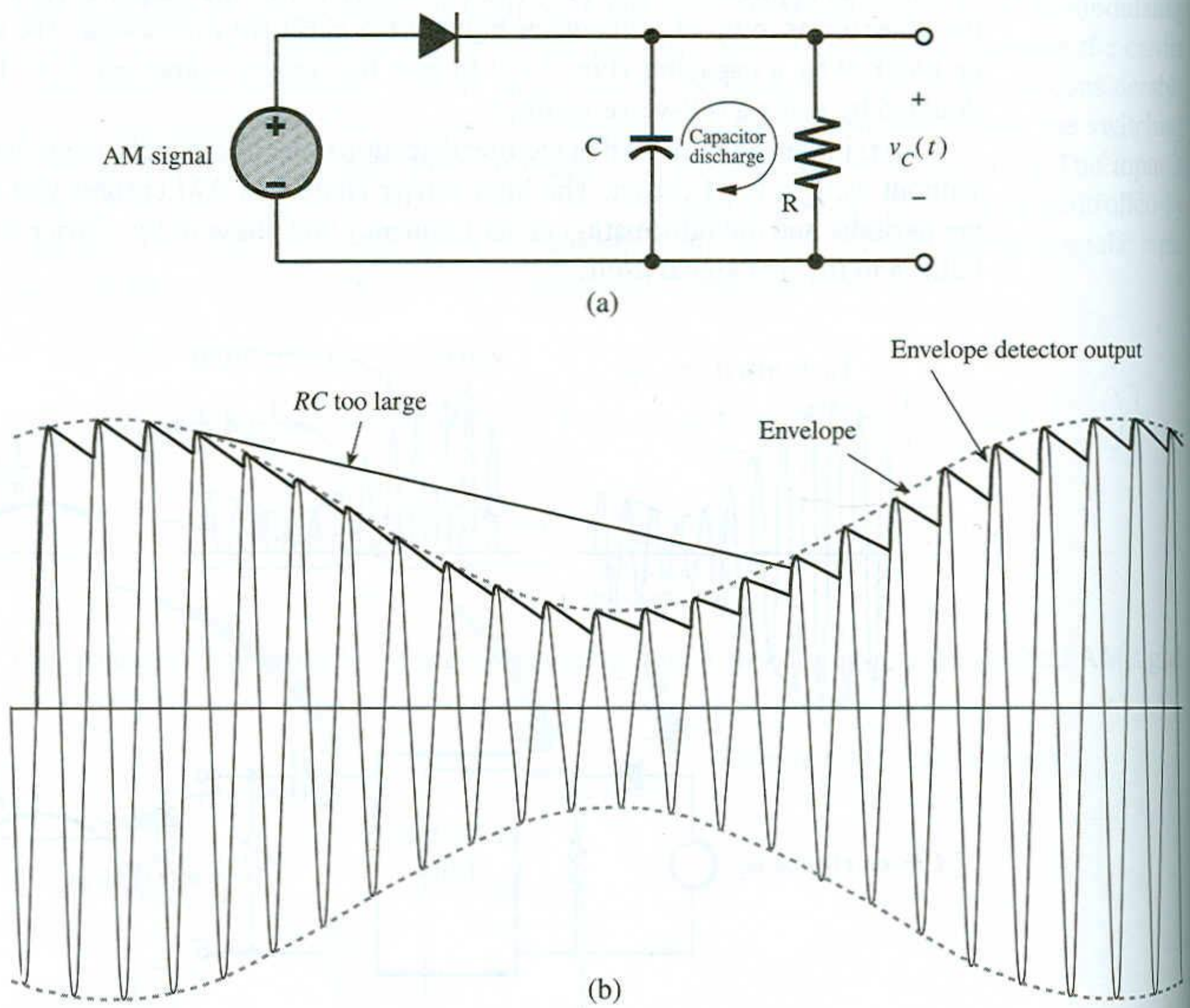
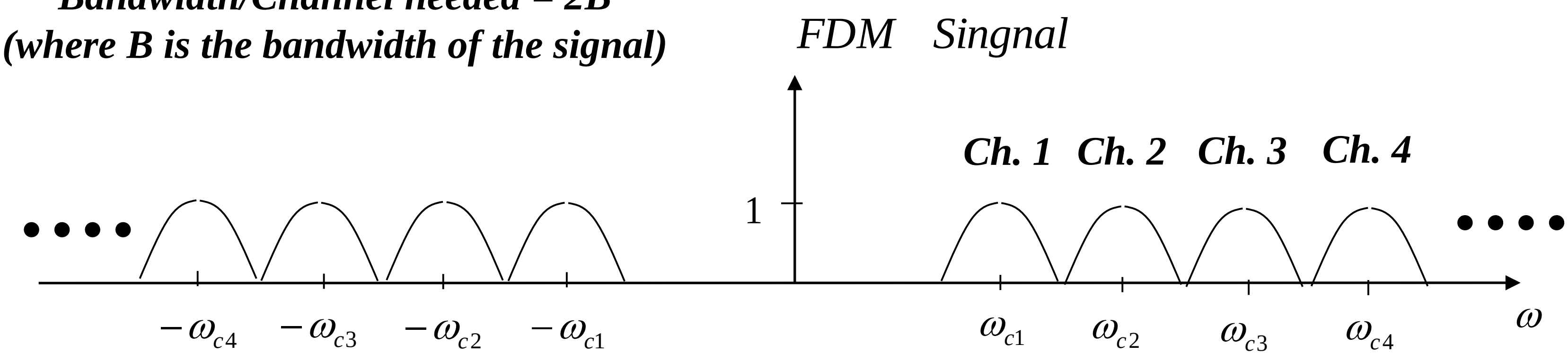


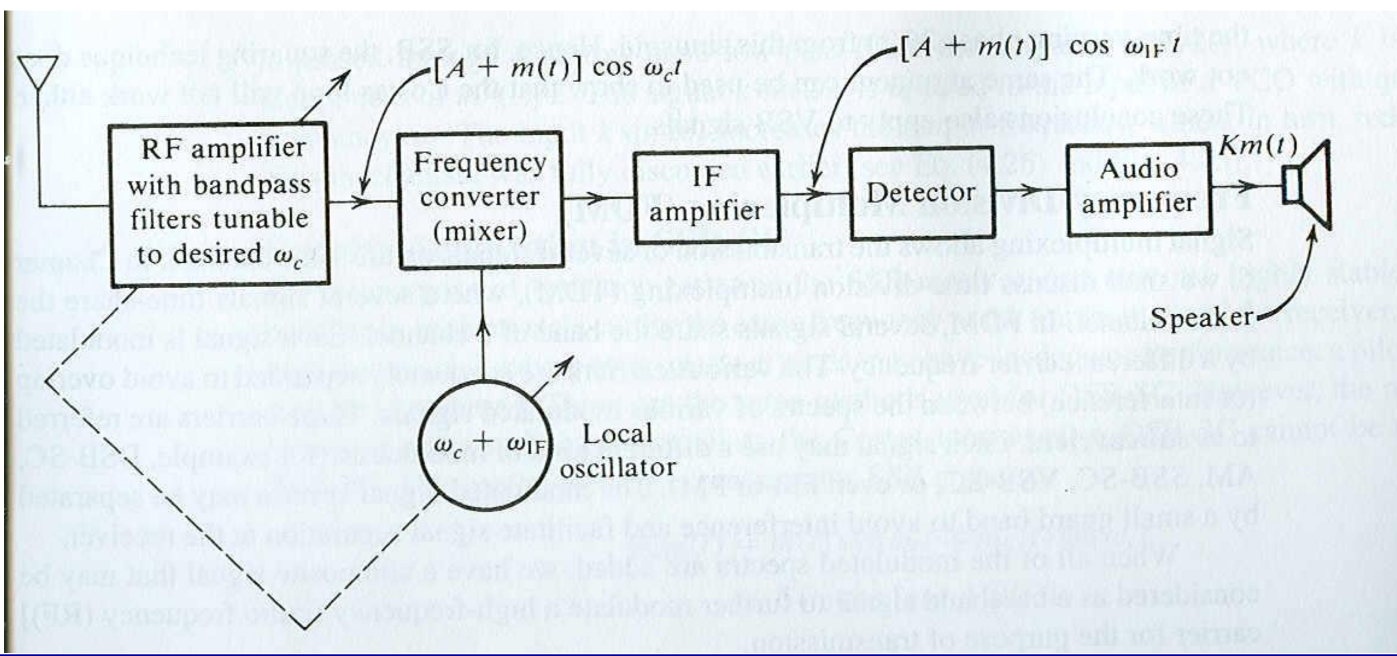
Figure 4.12 Envelope detector for AM.

Frequency Division Multiplexing

Bandwidth/Channel needed = 2B
(where B is the bandwidth of the signal)



Superheterodyne AM Receiver?

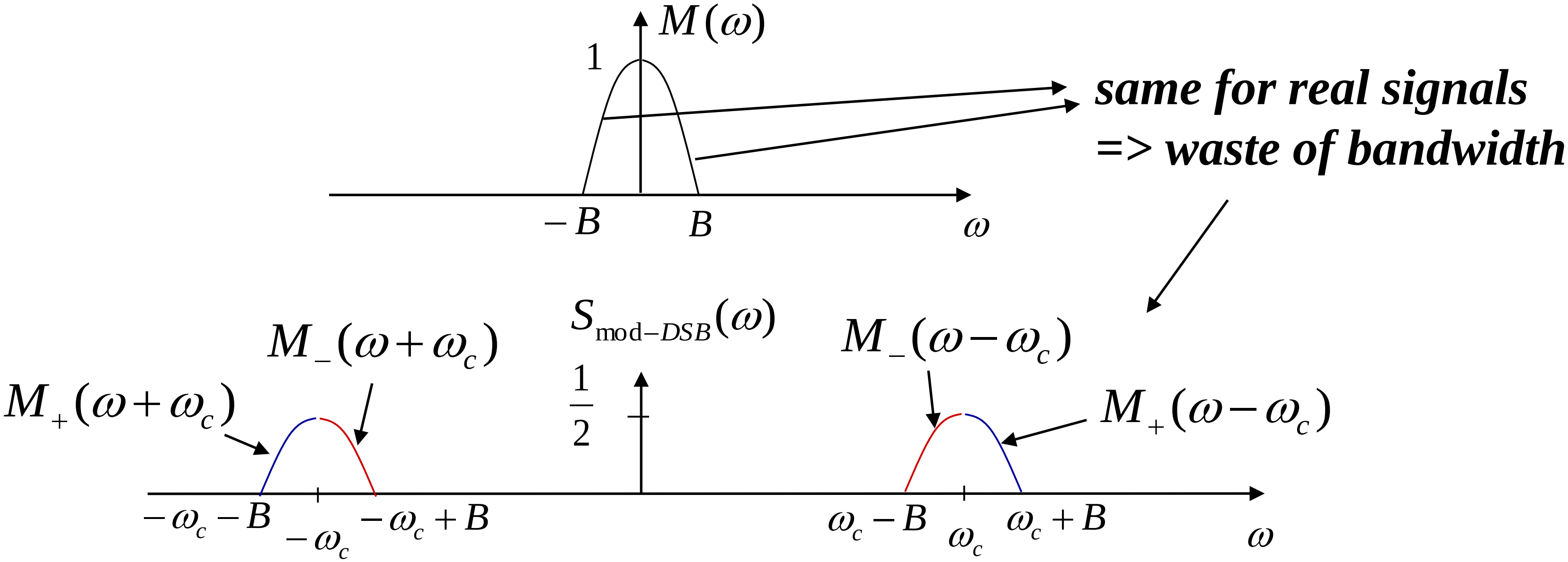


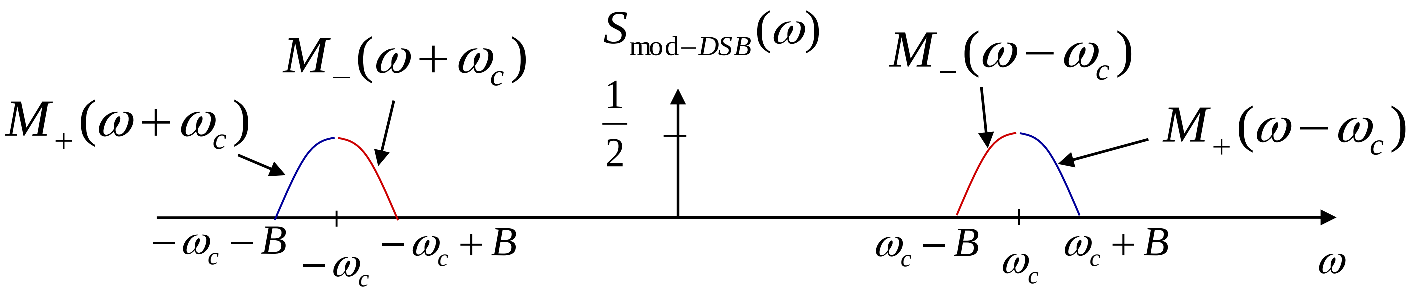
Amplitude Modulation – SSB-SC

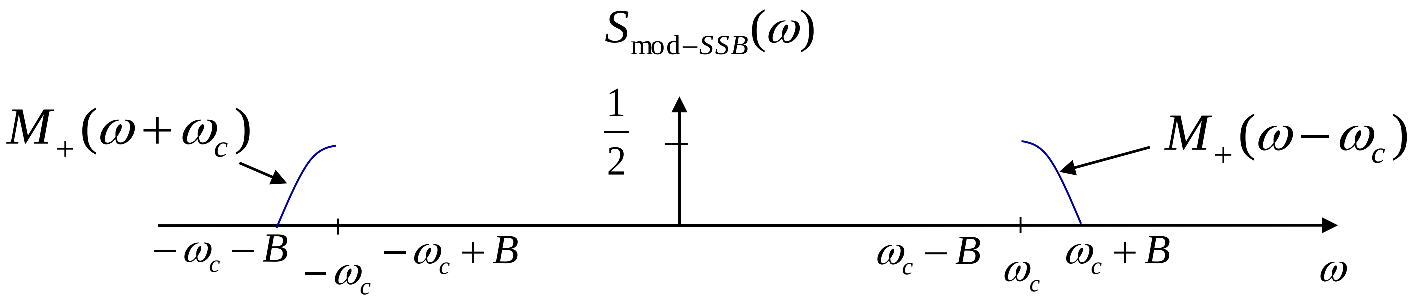
Modulation: $m(t) \Leftrightarrow M(\omega)$

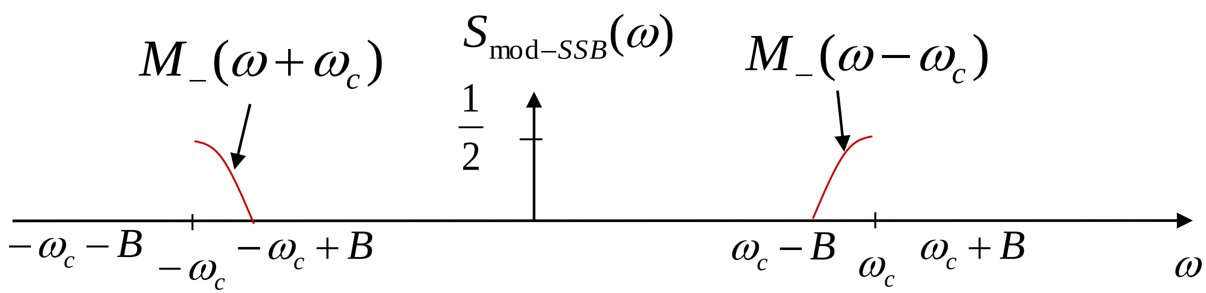
$$S_{\text{mod}-DSB}(t) = m(t)\cos(\omega_c t) \Leftrightarrow \frac{1}{2}[M(\omega+\omega_c)+M(\omega-\omega_c)]$$

$$S_{\text{mod}-SSB} \Leftrightarrow \frac{1}{2}[M_+(\omega+\omega_c)+M_-(\omega+\omega_c)+M_-(\omega-\omega_c)+M_+(\omega-\omega_c)]$$



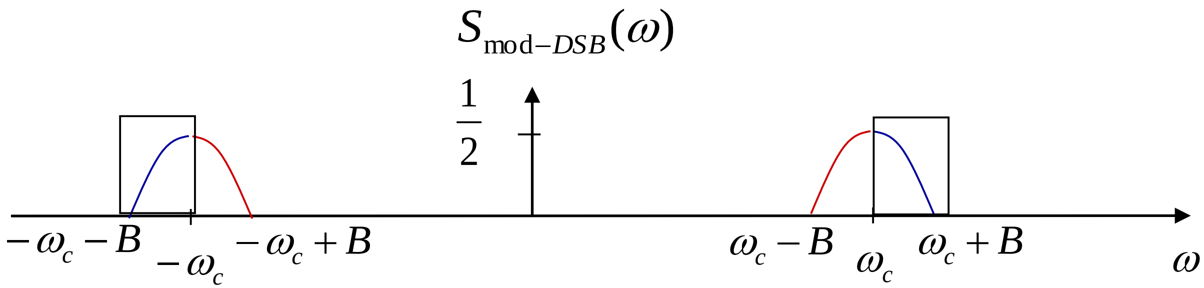




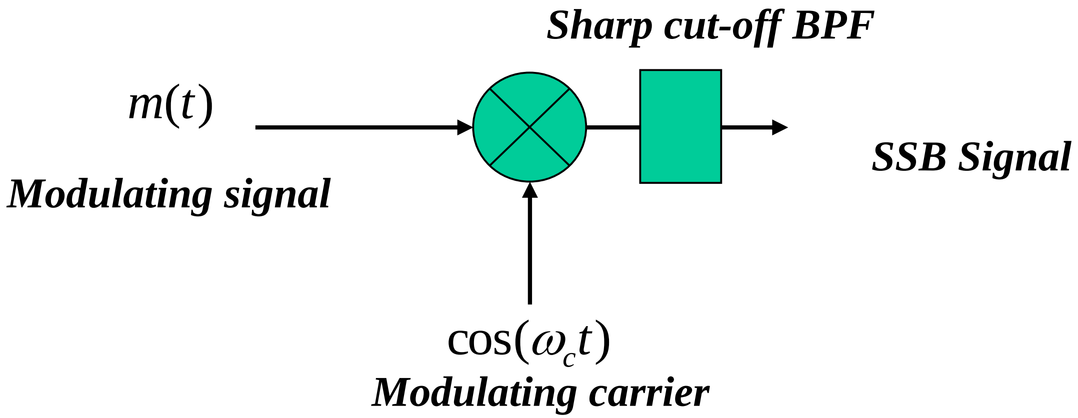


AM-SSB can potentially double the useable channel bandwidth

What we need to accomplish this?



A sharp cut-off bandpass filter!
Not practical to make!

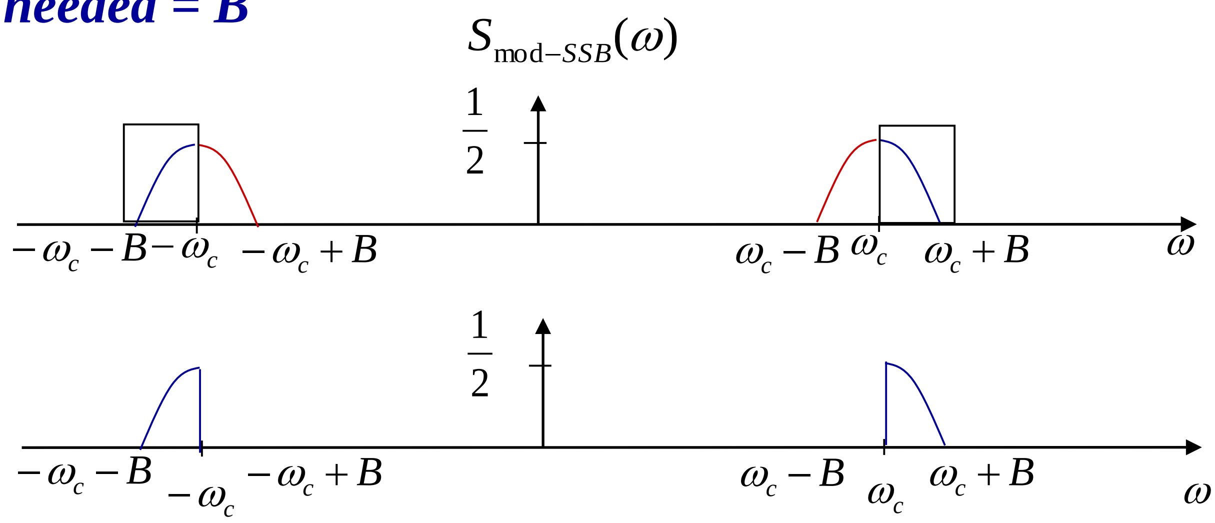


More Practical Bandwidth Saving Amplitude Modulation Formats

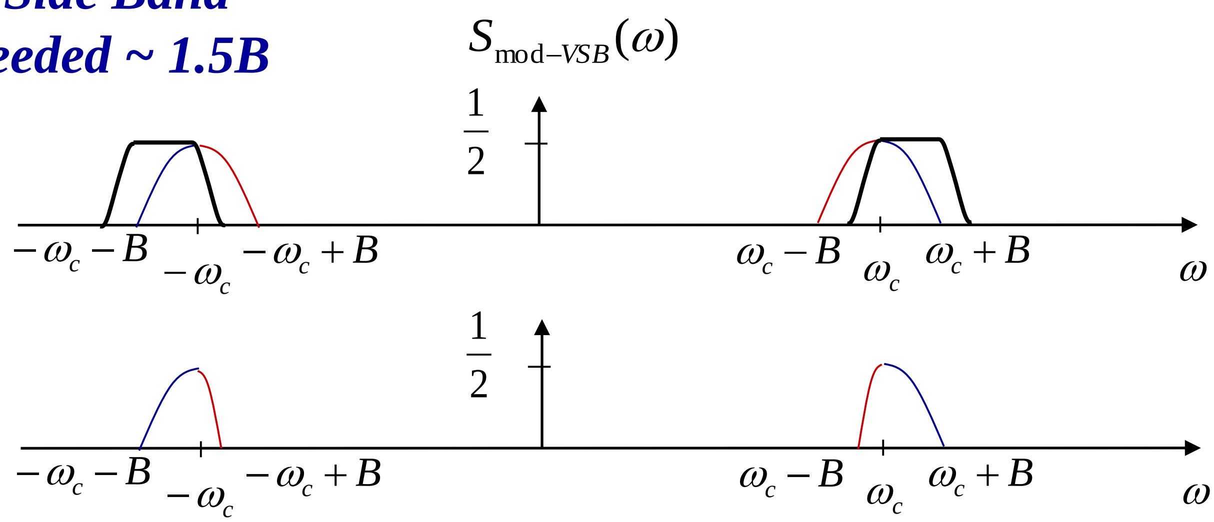
- ***Vestigial Side Band (VSB)***
  - ***increases almost 50% effective bandwidth***
  - ***used in TV***
  - ***Same as SSB but uses a practical bandpass filter***

- ***Quadrature Amplitude Modulation (QAM)***
  - ***Doubles the effective bandwidth***
  - ***Uses in limited bandwidth channel applications***

AM-Single Side Band
Bandwidth needed = B

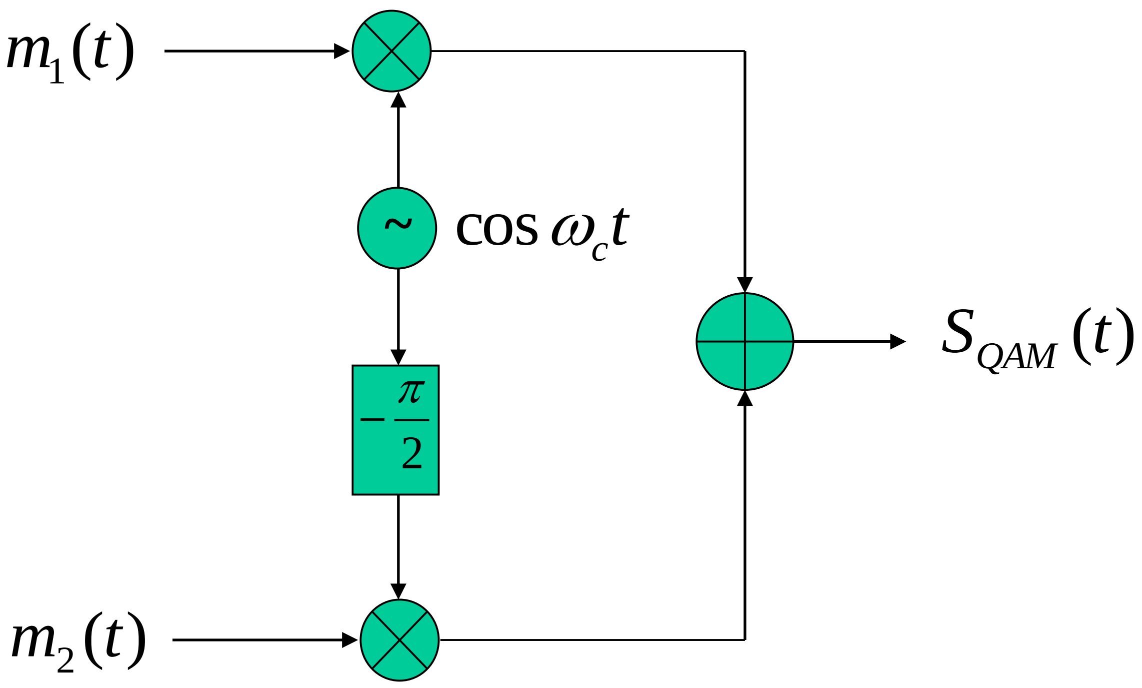


AM-Vestigial Side Band
Bandwidth needed ~ 1.5B



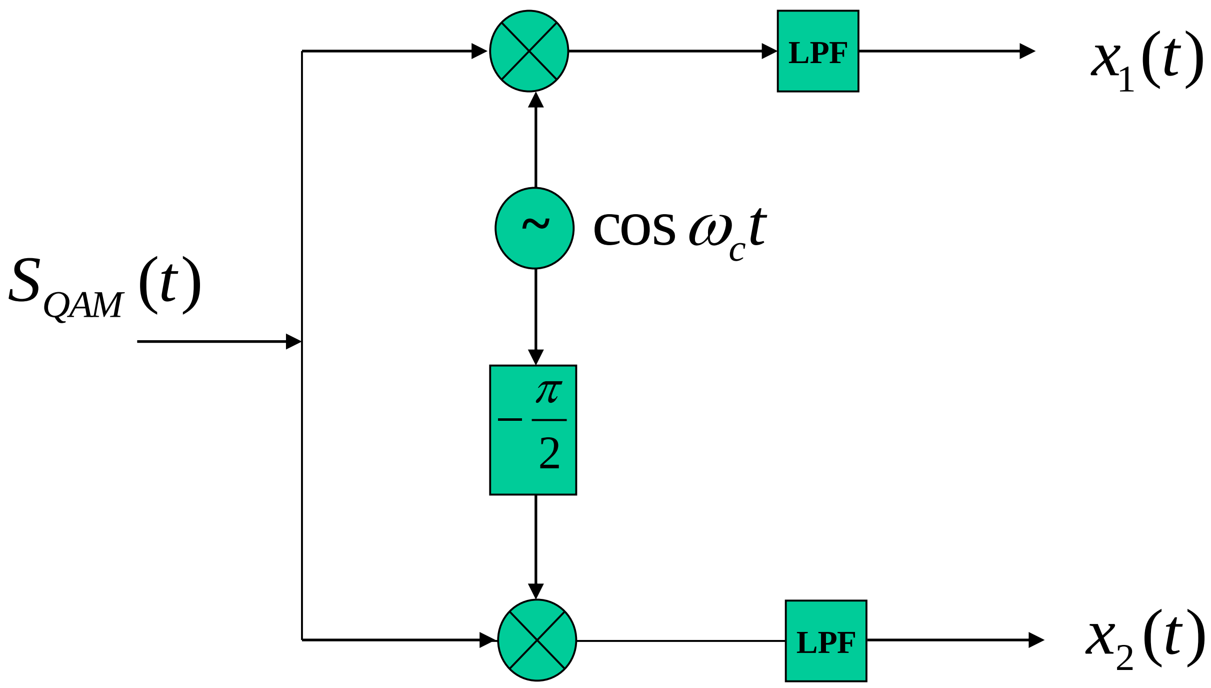
Quadrature Amplitude Modulation

$$S_{QAM}(t) = m_1(t)\cos(\omega_c t) + m_2(t)\sin(\omega_c t)$$



Quadrature Amplitude Demodulation

$$S_{QAM}(t) = m_1(t)\cos(\omega_c t) + m_2(t)\sin(\omega_c t)$$



$$S_{QAM}(t) = m_1(t)\cos(\omega_c t) + m_2(t)\sin(\omega_c t)$$

$$x_1(t) = [m_1(t)\cos(\omega_c t) + m_2(t)\sin(\omega_c t)]\cos(\omega_c t)$$

$$= m_1(t)\cos^2(\omega_c t) + m_2(t)\sin(\omega_c t)\cos(\omega_c t)$$

$$= \frac{1}{2}m_1(t) + \underbrace{\frac{1}{2}m_1(t)\cos(2\omega_c t) + m_2(t)\sin(\omega_c t)\cos(\omega_c t)}_{\text{will be filtered by lowpass filter}}$$

will be filtered by lowpass filter

$$S_{QAM}(t) = m_1(t)\cos(\omega_c t) + m_2(t)\sin(\omega_c t)$$

$$x_2(t) = [m_1(t)\cos(\omega_c t) + m_2(t)\sin(\omega_c t)]\sin(\omega_c t)$$

$$= m_1(t)\cos(\omega_c t)\sin(\omega_c t) + m_2(t)\sin^2(\omega_c t)$$

$$= m_1(t)\cos(\omega_c t)\sin(\omega_c t) + \frac{1}{2}m_2(t) - \frac{1}{2}m_2(t)\cos(2\omega_c t)$$

$$= \underbrace{m_1(t)\cos(\omega_c t)\sin(\omega_c t) - \frac{1}{2}m_2(t)\cos(2\omega_c t)}_{\text{will be filtered by lowpass filter}} + \frac{1}{2}m_2(t)$$

will be filtered by lowpass filter

Concept of Generalized Angle

$$\phi(t) = A\cos\theta(t)$$

$$\phi(t) = A\cos(\omega_c t + \theta_0)$$ —— ***Special Case***

Instantaneous Frequency

$$\omega_i(t) = \frac{d\theta}{dt} \quad \Rightarrow \quad \theta(t) = \int_{-\infty}^{t} \omega_i(x)dx$$

Phase Modulation

$$\phi(t) = A\cos(\omega_c t)$$

$$\theta(t) = \omega_c t + k_p m(t)$$

$$\phi_{PM}(t) = A\cos[\omega_c t + k_p m(t)]$$

$$\omega_i(t) = \frac{d\theta}{dt} = \omega_c + k_p \dot{m}(t)$$

Frequency Modulation

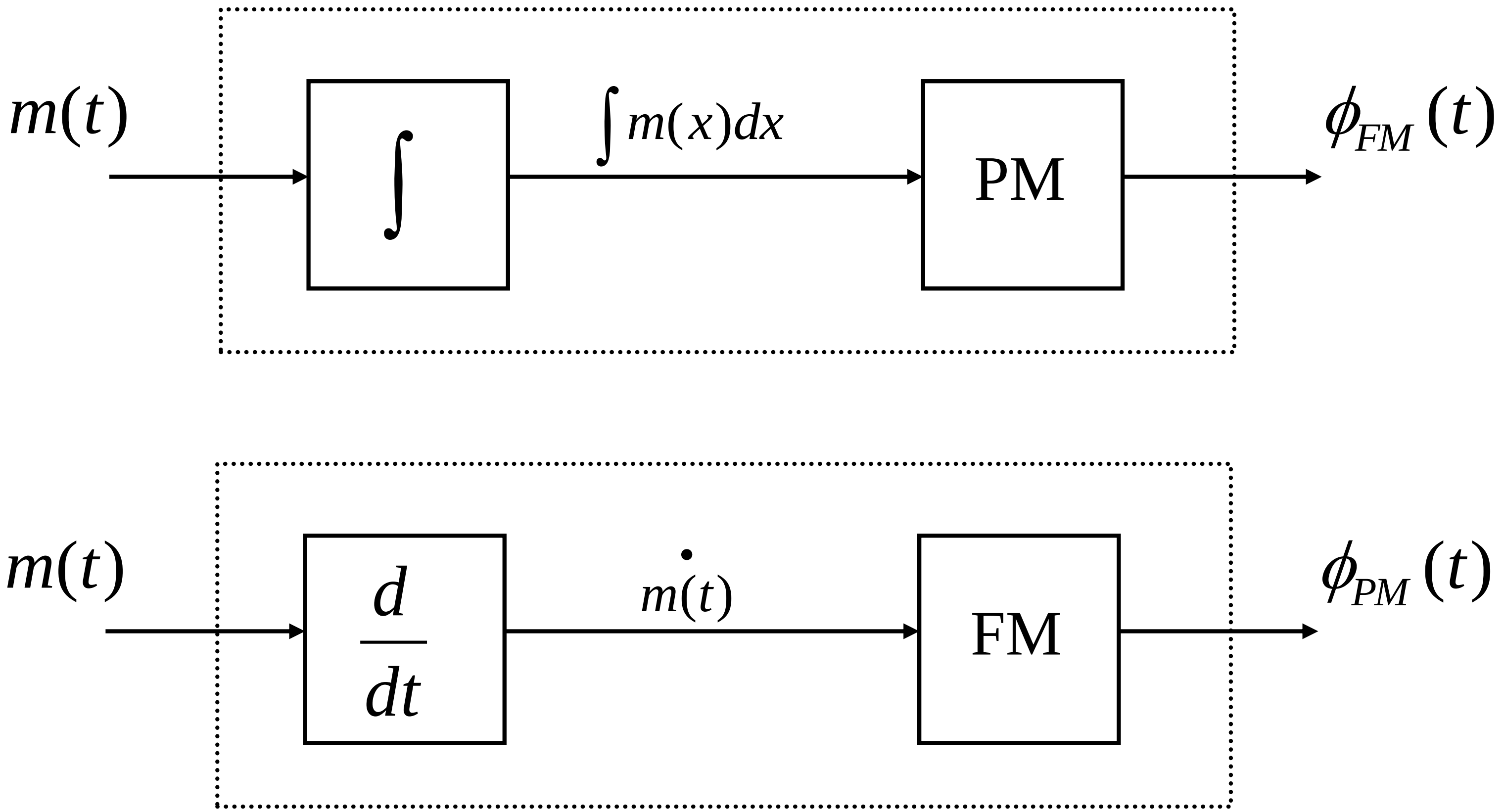
$$\phi(t) = A\cos(\omega_c t)$$

$$\omega_i(t) = \omega_c + k_f m(t)$$

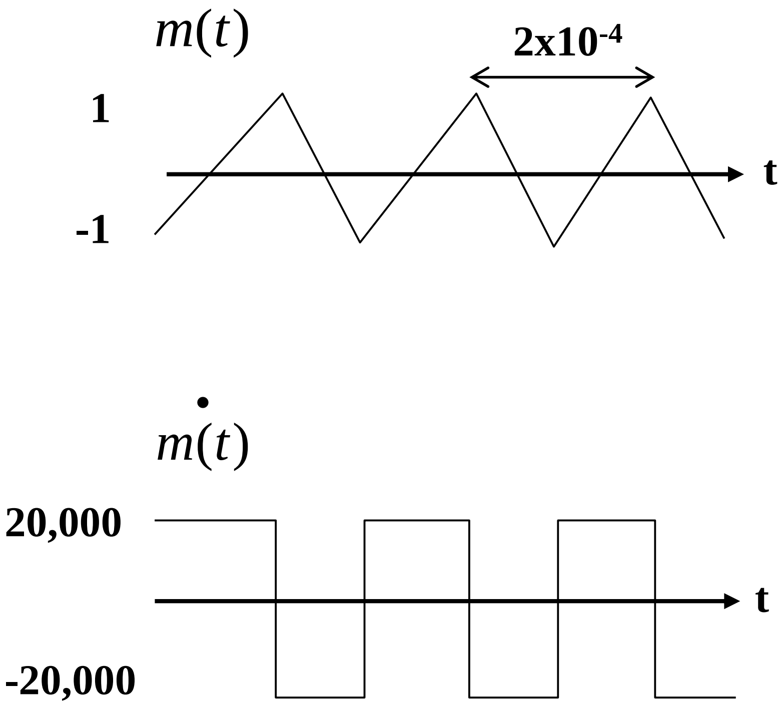
$$\theta(t) = \int_{-\infty}^{t} [\omega_c + k_f m(x)]dx \qquad = \omega_c t + k_f \int_{-\infty}^{t} m(x)dx$$

$$\phi_{FM}(t) = A\cos[\omega_c t + k_f \int_{-\infty}^{t} m(x)dx]$$

PM and FM - Inseparable



Example 1

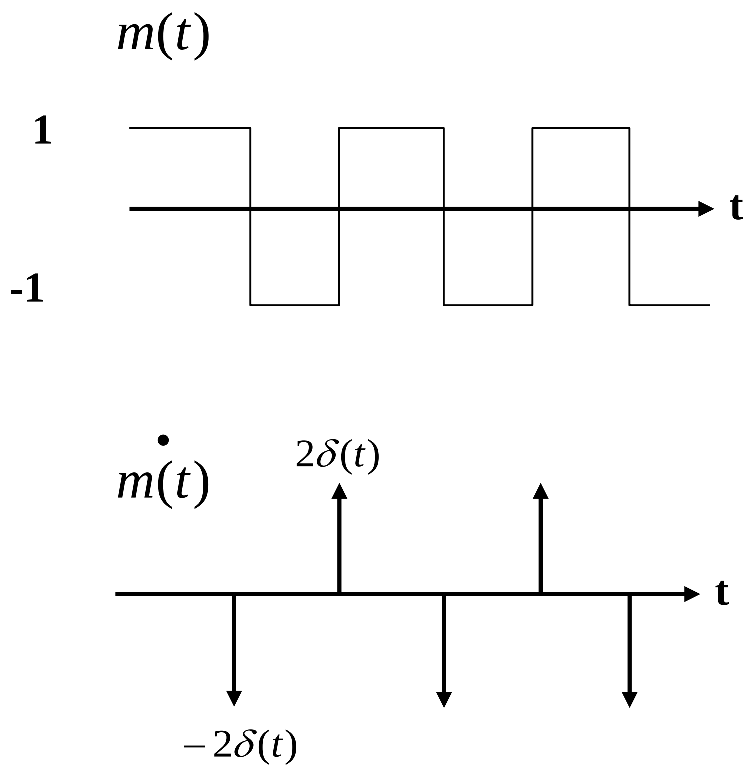


$$f_c = 100MHz$$

$$k_f = 2\pi \times 10^5$$

$$k_p = 10\pi$$

Example 2



$$f_c = 100MHz$$

$$k_f = 2\pi \times 10^5$$

$$k_p = \frac{\pi}{2}$$

Bandwidth of FM Signal

$$\phi_{FM}(t) = A\cos[\omega_c t + k_f \int_{-\infty}^{t} m(x)dx]$$

Let's define $$a(t) = \int_{-\infty}^{t} m(x)dx]$$

and $$\hat{\phi}_{FM}(t) = Ae^{j[\omega_c t + k_f a(t)]}$$

$$= Ae^{j\omega_c t} e^{jk_f a(t)}$$

then $$\phi_{FM}(t) = \text{Re}[\hat{\phi}_{FM}(t)]$$

$$\hat{\phi}_{FM}(t) = Ae^{j\omega_c t}e^{jk_f a(t)}$$

$$= Ae^{j\omega_c t}[1 + jk_f a(t) - \frac{k_f^2}{2!}a^2(t) + \ldots + j^n \frac{k_f^n}{n!}a^n(t)]$$

$$= A[1 + jk_f a(t) - \frac{k_f^2}{2!}a^2(t) + \ldots + j^n \frac{k_f^n}{n!}a^n(t)][\cos\omega_c t + j\sin\omega_c t]$$

$$\phi_{FM}(t) = \mathrm{Re}[\hat{\phi}_{FM}(t)]$$

$$= A[\cos\omega_c t - k_f a(t)\sin\omega_c t - \frac{k_f^2}{2!}a^2(t)\cos\omega_c t + \frac{k_f^3}{3!}a^3(t)\sin\omega_c t + \ldots]$$

If m(t) is band limited to B, then Bandwidth of $a^n(t)$ is nB

Narrow Band FM and PM (NBFM or NWPM)

$$\phi_{FM}(t) = A[\cos\omega_c t - k_f a(t)\sin\omega_c t - \frac{k_f^2}{2!}a^2(t)\cos\omega_c t + \frac{k_f^3}{3!}a^3(t)\sin\omega_c t + \ldots]$$

if $\quad k_f a(t) << 1.0$

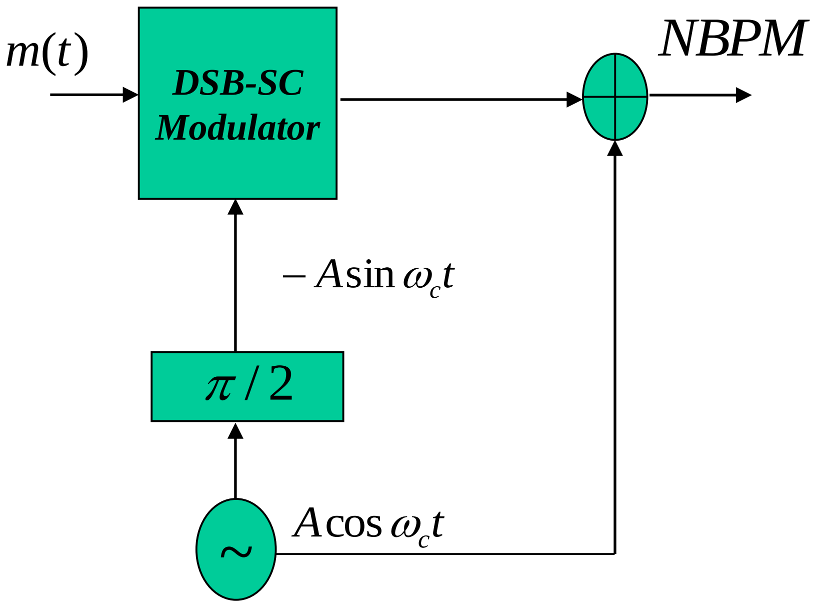
then $\quad \phi_{FM}(t) \approx A[\cos\omega_c t - k_f a(t)\sin\omega_c t]$

Similarly $\quad \phi_{PM}(t) \approx A[\cos\omega_c t - k_p m(t)\sin\omega_c t]$

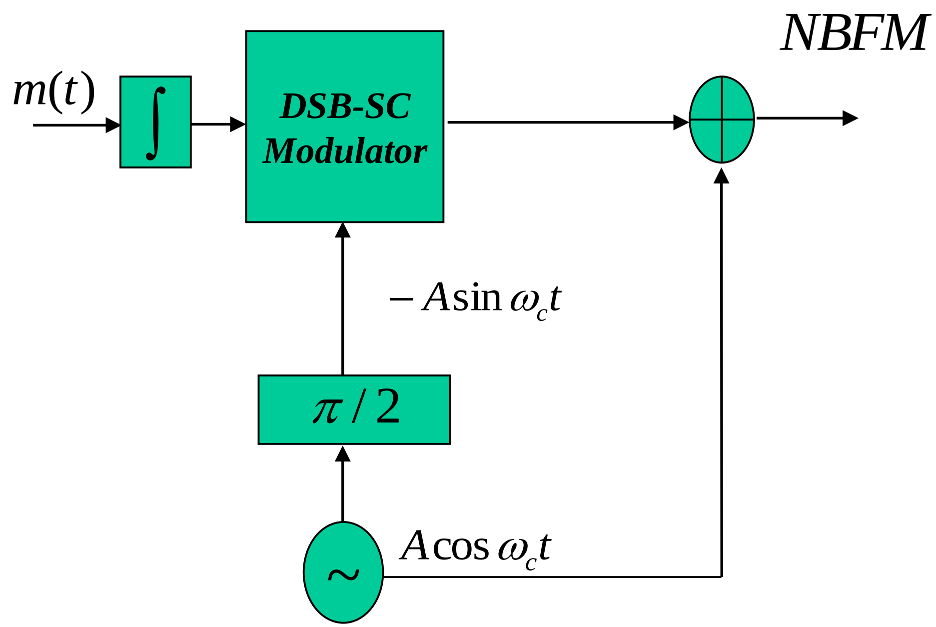
Bandwidth of narrowband FM or PM is 2B

Generation of NBFM or NBPM

$$\phi_{PM}(t) \approx A[\cos\omega_c t - k_p m(t)\sin\omega_c t]$$



$$\phi_{FM}(t) \approx A[\cos\omega_c t - k_f a(t)\sin\omega_c t]$$



Distortion Estimate in NBFM Signals

$$\phi_{FM}(t) \approx A[\cos\omega_c t - k_f a(t)\sin\omega_c t]$$

$$= AE(t)\cos[\omega_c t + \theta(t)]$$

where

$$E(t) = \sqrt{1 + k_f^2 a^2(t)}$$

Amplitude distortion can be minimized by bandbass limiter

$$\theta(t) = \tan^{-1}[k_f a(t)]$$

Phase Distortion

$$\theta(t) = \tan^{-1}[k_f a(t)]$$

$$\omega_i(t) = \dot{\theta}(t) = \frac{k_f \dot{a}(t)}{1 + k_f^2 a^2(t)}$$

$$= \frac{k_f m(t)}{1 + k_f^2 a^2(t)}$$

$$= k_f m(t)[1 - k_f^2 a^2(t) + k_f^4 a^4(t) - \ldots]$$

Example of Tone Modulation

$$m(t) = \alpha \cos \omega_m t \qquad a(t) = \frac{\alpha \sin \omega_m t}{\omega_m}$$

Let's define $\beta = \dfrac{\Delta F}{B} \Rightarrow \qquad \beta = \dfrac{\alpha k_f}{\omega_m}$

$$\omega_i(t) = k_f m(t)[1 - k_f^2 a^2(t) + k_f^4 a^4(t) - \ldots]$$

$$= \beta\omega_m \cos \omega_m t(1 - \beta^2 \sin^2 \omega_m t + \beta^4 \sin^4 \omega_m t - \ldots]$$

$$\omega_i(t) \approx \beta\omega_m \cos \omega_m t(1 - \beta^2 \sin^2 \omega_m t)$$

$$= \beta\omega_m \cos \omega_m t(1 - \beta^2 \frac{[1 - \cos 2\omega_m t]}{2})$$

$$= \beta\omega_m \cos\omega_m t(1-\beta^2 \frac{[1-\cos 2\omega_m t]}{2})$$

$$= \beta\omega_m \cos\omega_m t(1-\frac{\beta^2}{2}+\frac{\beta^2\cos 2\omega_m t}{2})$$

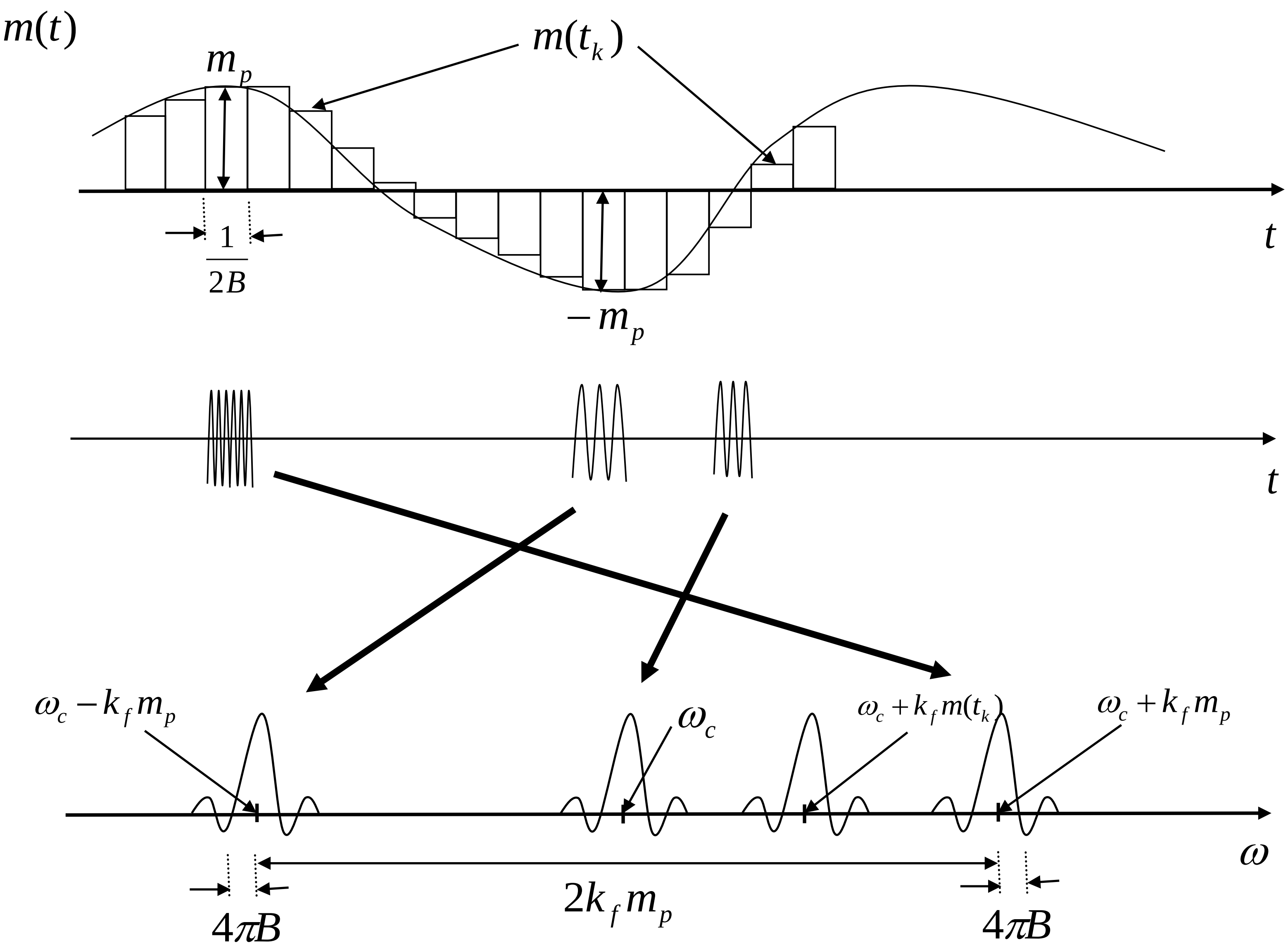
$$= \beta\omega_m \cos\omega_m t - \frac{\beta^3\omega_m}{2}\cos\omega_m t + \frac{\beta^3\omega_m \cos\omega_m t \cos 2\omega_m t}{2}$$

$$= \beta\omega_m \cos\omega_m t - \frac{\beta^3\omega_m}{2}\cos\omega_m t + \frac{\beta^3\omega_m (\cos\omega_m t + \cos 3\omega_m t)}{4}$$

$$= \beta\omega_m (1-\frac{\beta^2}{4})\cos\omega_m t + \frac{\beta^3\omega_m}{4}\cos 3\omega_m t$$

$$\approx \beta\omega_m \cos\omega_m t + \frac{\beta^3\omega_m}{4}\cos 3\omega_m t \quad \mathit{for} \quad \beta << 1$$

Wide Band FM (WBFM)



Wide Band FM (WBFM)

$$\Delta f = \frac{k_f m_p}{2\pi}$$

$$B_{FM} = 2\times(\Delta f + 2B)$$

This bandwidth is slightly more than the actual because this corresponds to the staircase approximation of the signal

Let's make the adjustment, we know that

if $\Delta f << B$ **then** $B_{FM} = 4B$

Which is not right from our NBFW analysis, we know that

$$B_{FM} = 2B$$

Wide Band FM (WBFM)

Therefore,

$$B_{FM} = 2\times(\Delta f + 2B)$$

$$\Big\Downarrow \quad \textit{Should change to}$$

$$B_{FM} = 2\times(\Delta f + B)$$

Now, if $\Delta f >> B$ **then** $B_{FM} = 2\Delta f$ **WBFM**

And if $\Delta f << B$ **then** $B_{FM} = 2B$ **NBFM**

if we define $\beta = \dfrac{\Delta f}{B}$ **then** $B_{FM} = 2B(\beta + 1)$

Wide Band PM (WBPM)

$$\omega_i(t) = \omega_c + k_p \dot{m}(t)$$

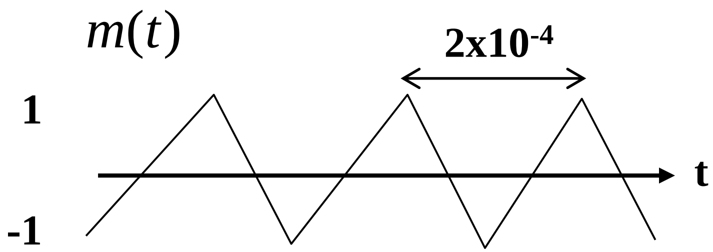
$$\Delta f = \frac{k_p m'_p}{2\pi}$$

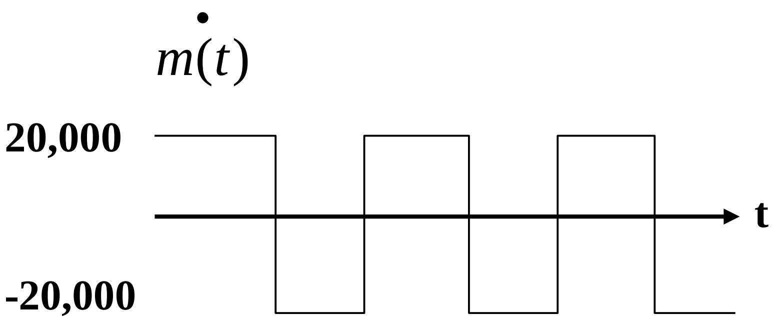
$$B_{PM} = 2 \times (\Delta f + B)$$

$$B_{PM} = 2B(\beta + 1)$$

$$\beta = \frac{\Delta f}{B}$$

Example 1





$$k_f = 2\pi \times 10^5$$

$$k_p = 10\pi$$

1. ***What is the bandwidth of corresponding FM or PM signals ?***
2. ***What if m(t) is doubled in magnitude?***
3. ***What if the period is doubled?***

Understanding the Spectrum of FM

Let's use the example of tone modulation

$$m(t) = \alpha \cos \omega_m t \qquad a(t) = \frac{\alpha \sin \omega_m t}{\omega_m}$$

We previously defined $\hat{\phi}_{FM}(t) = Ae^{j[\omega_c t + k_f a(t)]}$

$$\Rightarrow \hat{\phi}_{FM}(t) = Ae^{j[\omega_c t + \frac{\alpha k_f}{\omega_m}\sin(\omega_m t)]}$$

Also, we know $\Delta\omega = k_f m_p = \alpha k_f$

Now $\beta = \frac{\Delta f}{B} \Rightarrow \qquad \beta = \frac{\alpha k_f}{\omega_m}$

$$\Rightarrow \hat{\phi}_{FM}(t) = Ae^{j[\omega_c t + \beta \sin(\omega_m t)]}$$

$$= Ae^{j\omega_c t}[e^{j\beta \sin(\omega_m t)}]$$

Now

$$e^{j\beta \sin \omega_m t} = \sum_{n=-\infty}^{\infty} C_n e^{j\omega_m t}$$

Where

$$C_n = \frac{\omega_m}{2\pi} \int_{-\pi/\omega_m}^{\pi/\omega_m} e^{\beta \sin \omega_m t} e^{-jn\omega_m t} dt = J_n(\beta)$$

Bessel Function

$$\Rightarrow \hat{\phi}_{FM}(t) = Ae^{j\omega_c t} \sum_{n=-\infty}^{\infty} J_n(\beta) e^{jn\omega_m t}$$

$$\Rightarrow \hat{\phi}_{FM}(t) = Ae^{j\omega_c t} \sum_{n=-\infty}^{\infty} J_n(\beta) e^{jn\omega_m t}$$

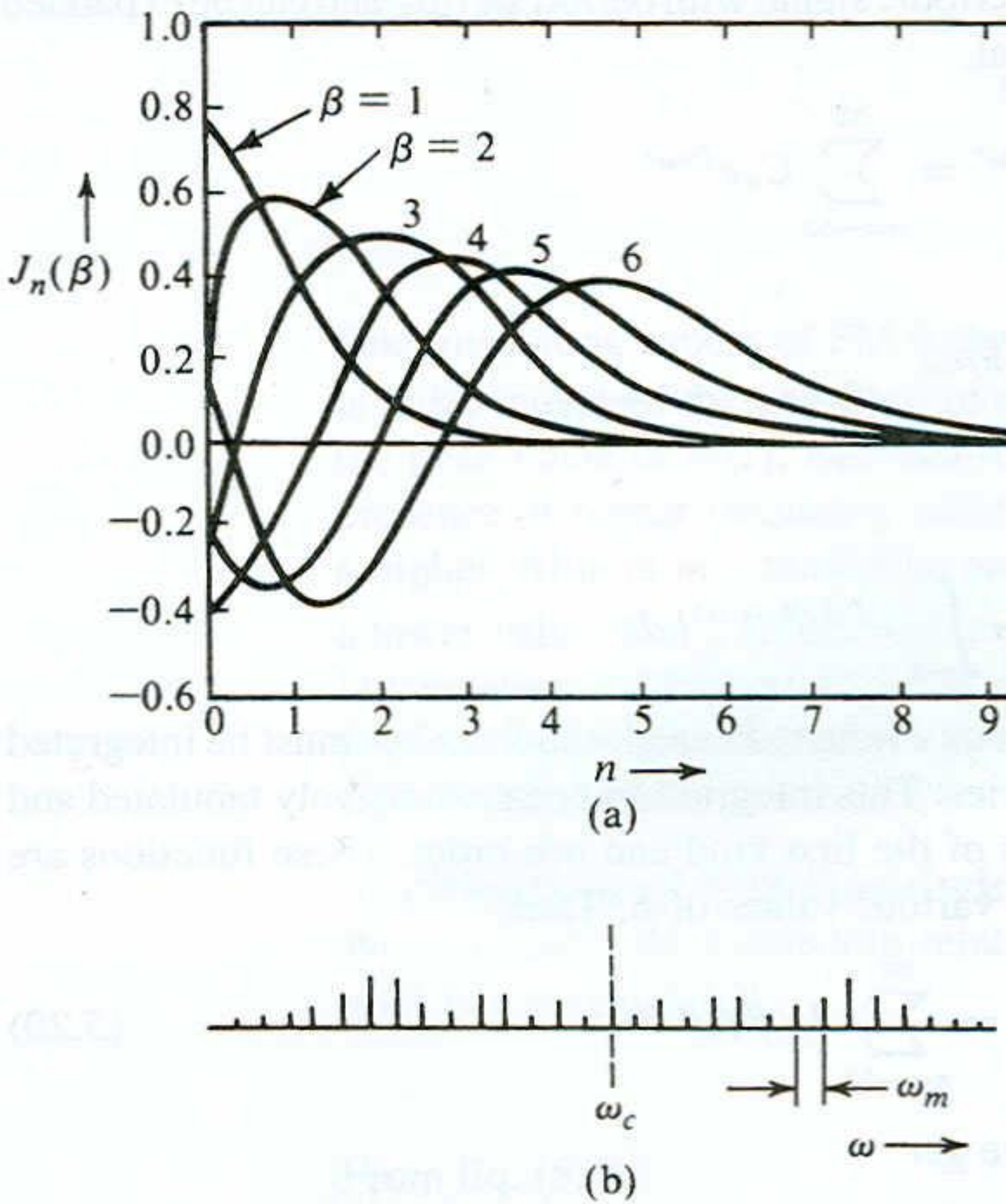
$$= A \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j(\omega_c t + n\omega_m t)}$$

Remember we were interested in

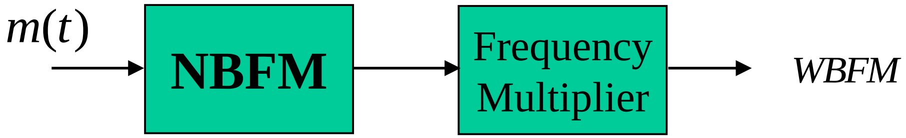
$$\phi_{FM}(t) = \mathrm{Re}[\hat{\phi}_{FM}(t)]$$

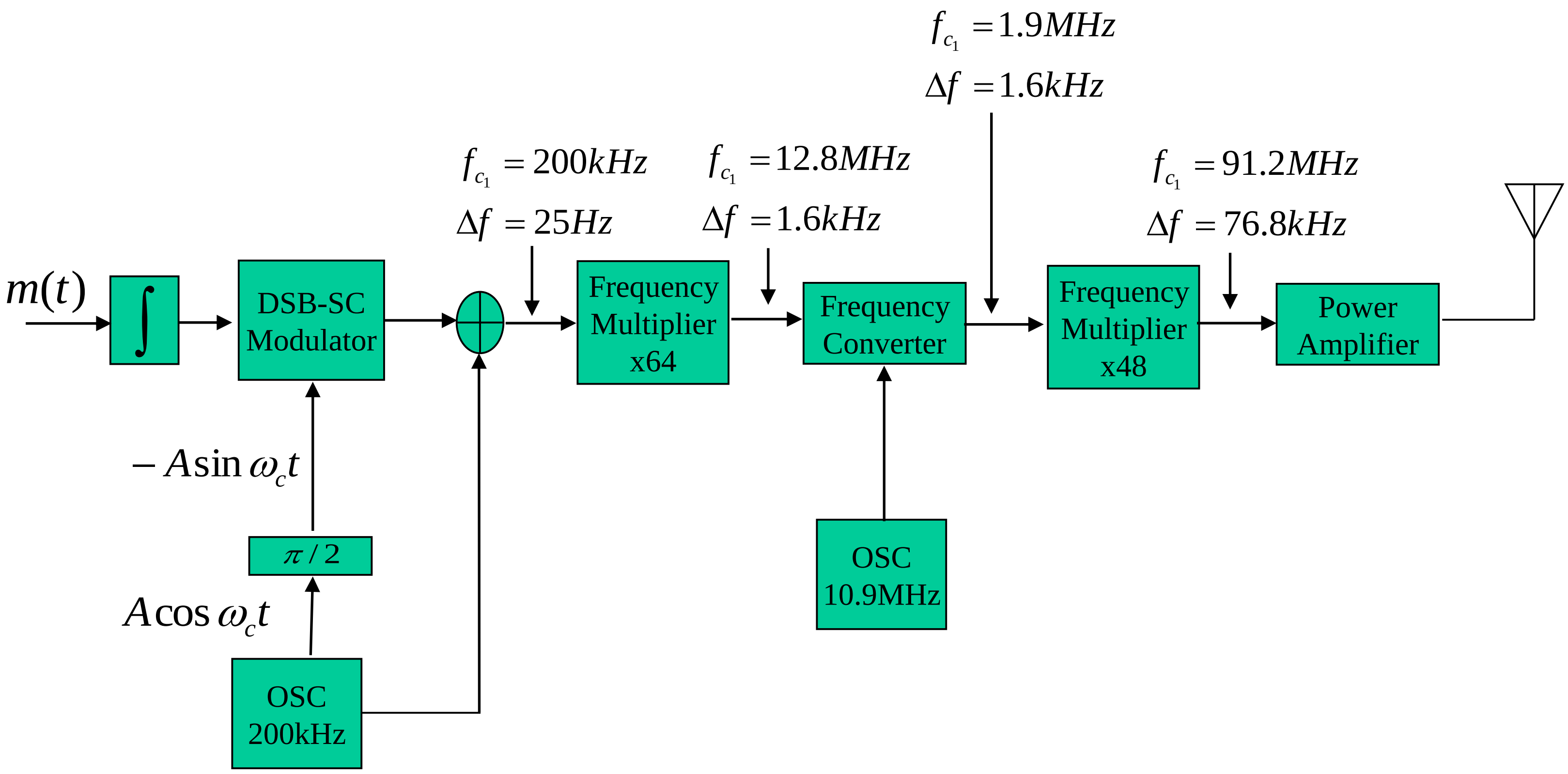
$$\Rightarrow \phi_{FM}(t) = A \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c t + n\omega_m t)$$

Bessel Function



Wideband FM Generation - Indirect





Wideband FM Generation - Direct

Voltage Controlled Oscillator

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

can be varied with bias voltage (C)

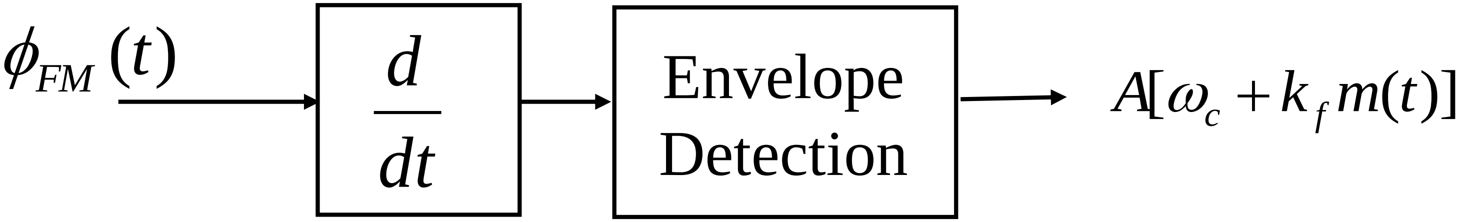
can be varied with current through coil (L)

Demodulation of FM

$$\phi_{FM}(t) = A\cos[\omega_c t + k_f \int_{-\infty}^{t} m(x)dx]$$

$$\dot{\phi}_{FM}(t) = \frac{d}{dt}\{A\cos[\omega_c t + k_f \int_{-\infty}^{t} m(x)dx]\}$$

$$= A[\omega_c + k_f m(t)]\sin[\omega_c t + k_f \int_{-\infty}^{t} m(x)dx]$$

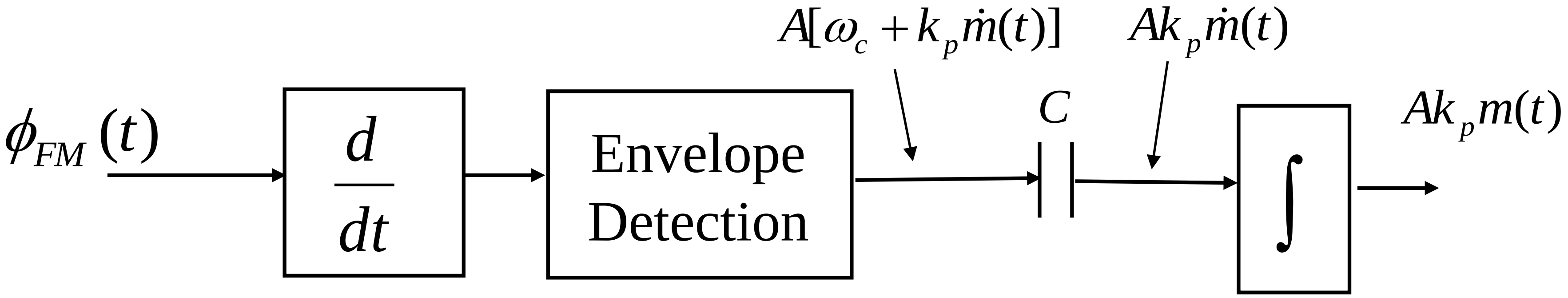


Demodulation of PM

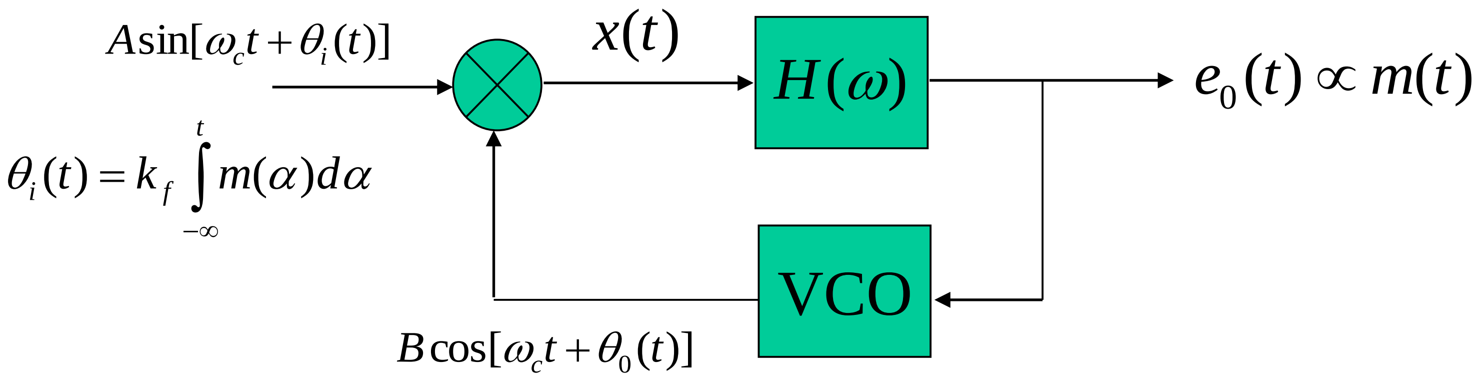
$$\phi_{PM}(t) = A\cos[\omega_c t + k_p m(t)]$$

$$\dot{\phi}_{PM}(t) = \frac{d}{dt}\{A\cos[\omega_c t + k_p m(t)]\}$$

$$= A[\omega_c + k_p \dot{m}(t)]\sin[\omega_c t + k_p m(t)]$$



Demodulation of FM – Phase Lock Loop



$$\omega_{VCO} = \omega_c + ce_0(t)$$

$$\Rightarrow \dot{\theta}_0(t) = ce_o(t)$$

$$x(t) = AB\sin[\omega_c t + \theta_i(t)]\cos[\omega_c t + \theta_0(t)]$$

$$= \frac{AB}{2}\{\sin[\theta_i(t) - \theta_0(t)] + \sin[2\omega_c t + \theta_i(t) + \theta_0(t)]\}$$

Suppressed by loop filter, $H(\omega)$

So the effective input to the loop filter will be

$$x(t) = \frac{AB}{2}\sin[\theta_i(t) - \theta_0(t))]$$

$$= \frac{AB}{2}\sin[\theta_e(t)]$$

where $\theta_e(t) = \theta_i(t) - \theta_0(t)$

$$\Rightarrow \theta_0(t) = \theta_i(t) - \theta_e(t)$$

$$\theta_0(t) = \theta_i(t) - \theta_e(t)$$

For FM Signal

Now

$$\theta_o(t) = k_f \int_{-\infty}^{t} m(\alpha)d\alpha - \theta_e(t)$$

$$[\theta_i(t) = k_f \int_{-\infty}^{t} m(\alpha)d\alpha]$$

$$\theta_o(t) = k_f \int_{-\infty}^{t} m(\alpha)d\alpha$$

For small error

$$\Rightarrow \dot{\theta}_o(t) = k_f m(t)$$

Taking derivative on both sides

$$\Rightarrow e_o(t) = \frac{k_f}{c} m(t)$$

$$[\dot{\theta}_0(t) = ce_o(t)]$$

For PM Signal

Now $$\theta_o(t) = k_p m(t) - \theta_e(t)$$ $[\theta_i(t) = k_p m(t)]$

$$\theta_o(t) = k_p m(t)$$ ***For small error***

$$\Rightarrow \dot{\theta}_o(t) = k_p \dot{m}(t)$$

$$\Rightarrow e_o(t) = \frac{k_p}{c}\dot{m}(t)$$ $[\dot{\theta}_0(t) = c e_o(t)]$

You need an integrator after PLL for PM demodulation

Interference in Angle Modulated Signals

Desired signal **Interfering signal**

$$r(t) = A\cos\omega_c t + I\cos(\omega_c + \omega)t$$

$$= A\cos\omega_c t + I\cos\omega_c t\cos\omega t - I\sin\omega_c t\sin\omega t$$

$$= (A + I\cos\omega t)\cos\omega_c t - I\sin\omega t\sin\omega_c t$$

$$= E_r(t)\cos[\omega_c t + \psi_d(t)]$$

where

$$E_r(t) = \sqrt{(A + I\cos\omega t)^2 + (I\sin\omega t)^2}$$

$$\psi_d(t) = \tan^{-1}\frac{I\sin\omega t}{A + I\cos\omega t}$$

$$\psi_d(t) = \tan^{-1}\frac{I\sin\omega t}{A + I\cos\omega t}$$

When I<<A

$$\psi_d(t) \approx \tan^{-1}\frac{I}{A}\sin\omega t \approx \frac{I}{A}\sin\omega t$$

Remember original signal

$$r(t) = E_r(t)\cos[\omega_c t + \psi_d(t)]$$

After passing through PM or FM demodulator

$$y_d(t) = \frac{I}{A}\sin\omega t$$

For PM

$$y_d(t) = \frac{I\omega}{A}\cos\omega t$$

For FM

Pre-emphasis and De-emphasis in FM

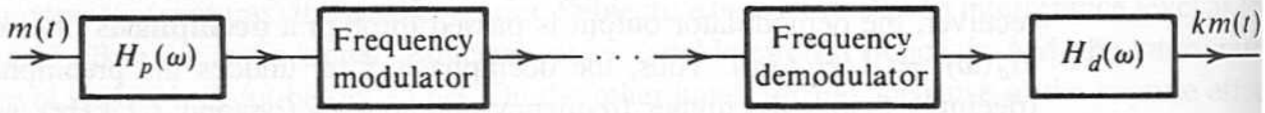


Figure 5.17 Preemphasis-deemphasis in an FM system.

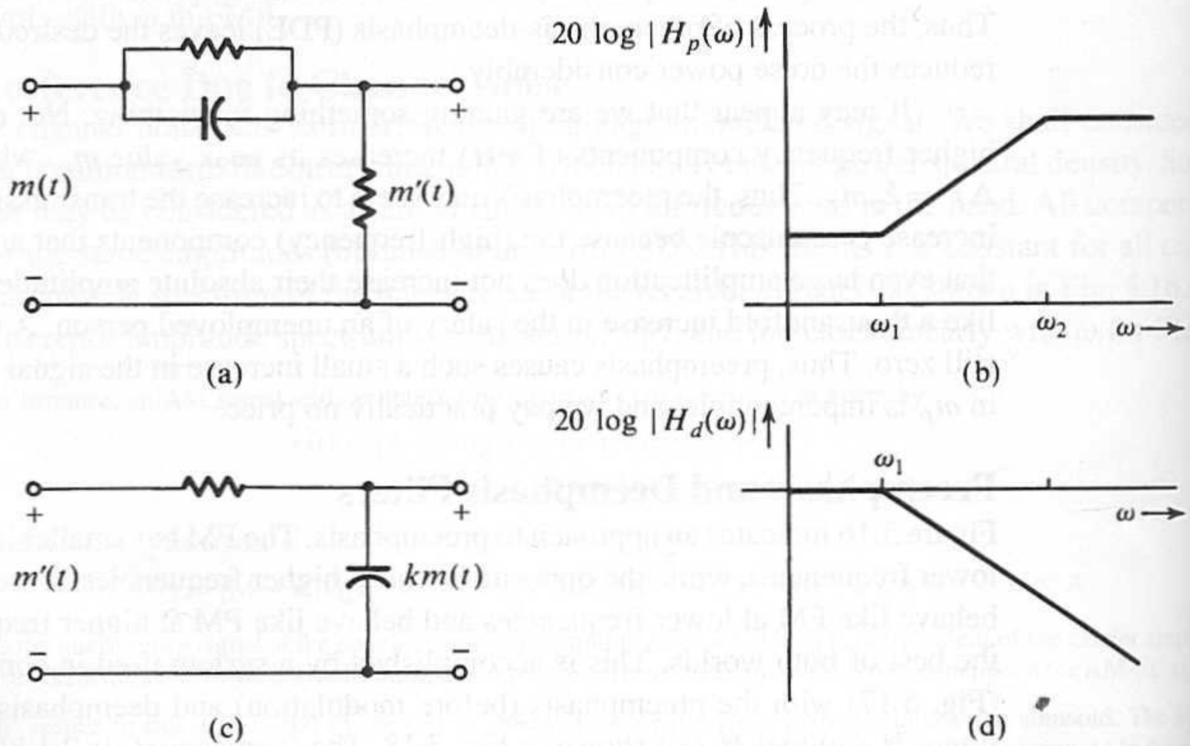


Figure 5.18 (a) Preemphasis filter. (b) Its frequency response. (c) Deemphasis filter. (d) Its frequency respons

Stereo FM Receiver