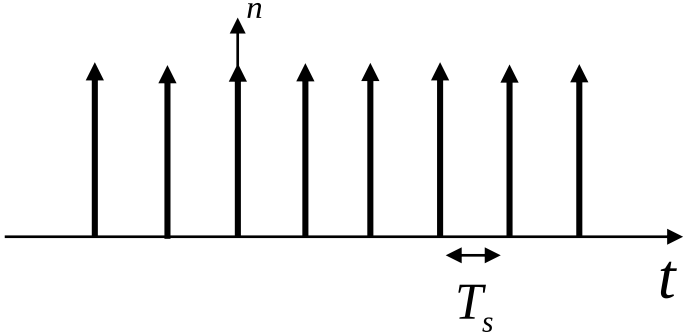
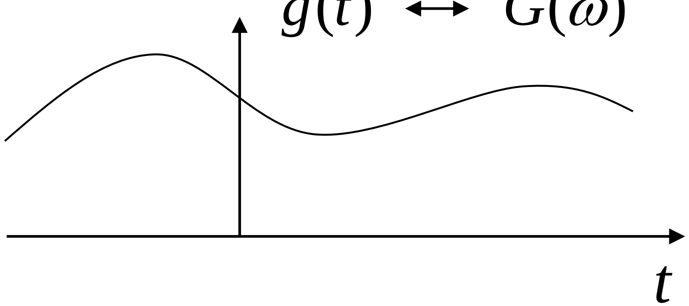
Sampling Theorem

$$\delta_{T_s}(t) = \sum_n \delta(t - nT_s) \leftrightarrow G[n\omega_s]$$

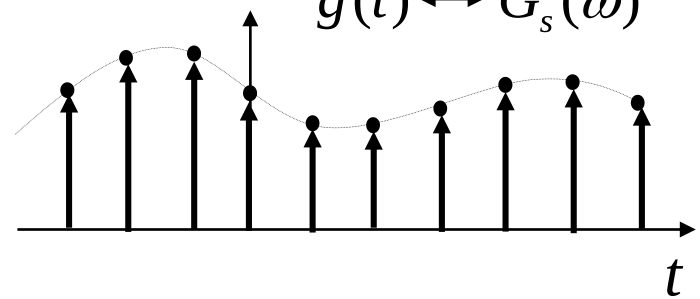


$\otimes$

$$g(t) \leftrightarrow G(\omega)$$



$$\bar{g}(t) \leftrightarrow G_s(\omega)$$



Sampling Frequency

$$f_s = \frac{1}{T_s}$$

Sampling Theorem

Let's g(t) is band limited to B Hz, then samples of g(t) could be written as

$$\bar{g}(t) = g(t)\delta_{T_s}(t) = \sum_n g(nT_s)\delta(t - nT_s)$$

We know that

$$\delta_{T_s}(t) = \frac{1}{T_s}[1 + 2\cos\omega_s t + 2\cos 2\omega_s t + 2\cos 3\omega_s t + \ldots]$$

where $$\omega_s = \frac{2\pi}{T_s} = 2\pi f_s$$

$$\bar{g}(t) = g(t)\delta_{T_s}(t)$$

$$= \frac{1}{T_s}[g(t) + 2g(t)\cos\omega_s t + 2g(t)\cos 2\omega_s t + 2g(t)\cos 3\omega_s t + \ldots]$$

$$= \frac{g(t)}{T_s} + \frac{2}{T_s}\sum_{n=1}^{\infty} g(t)\cos n\omega_s t$$

By taking Fourier Transform on both sides

$$\overline{G}(\omega) = \frac{G(\omega)}{T_s} + \frac{1}{T_s}\sum_{n=1}^{\infty}[G(\omega - n\omega_s) + G(\omega + n\omega_s)]$$

$$= \frac{1}{T_s}\sum_{n=-\infty}^{\infty} G(\omega - n\omega_s)$$

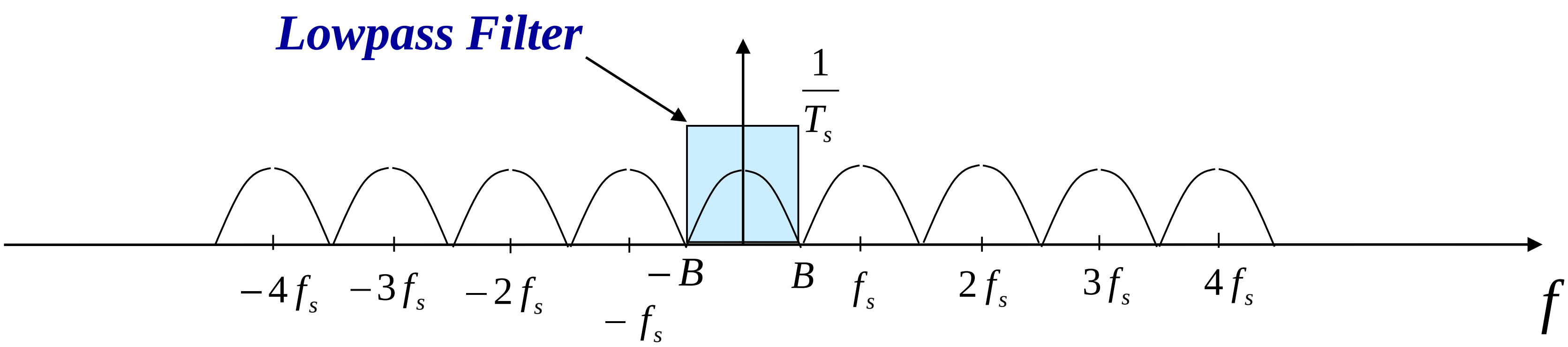
Nyquist Criterion

$f_s > 2B$ ***or*** $T_s < \frac{1}{2B}$

Sampling Theorem

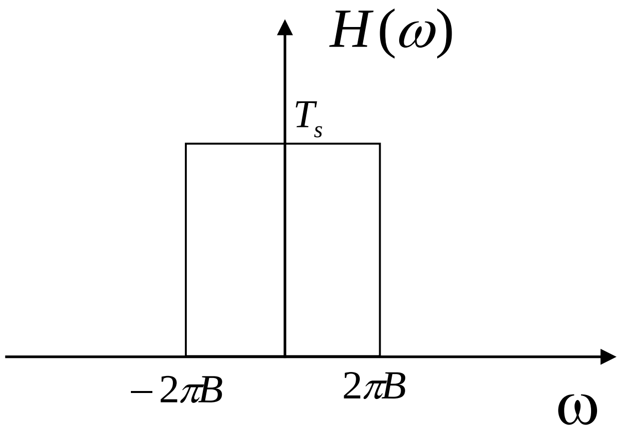
Sampling frequency should be at least equal to or greater than twice the bandwidth of the message signal for successful recovery of the signal from it's samples

$$f_s \geq 2B$$

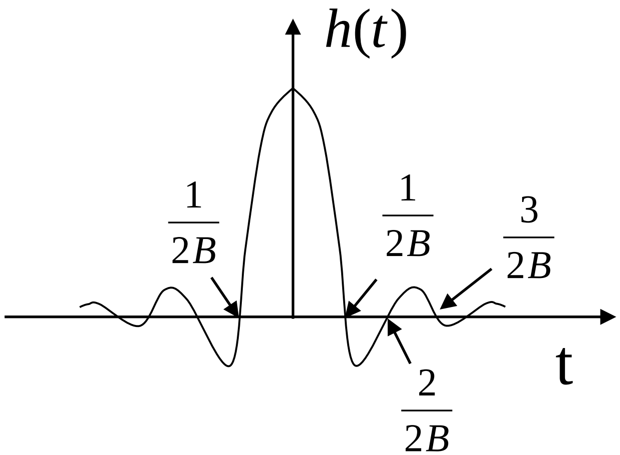


bandwidth of lowpass filter = B Hz

Ideal Reconstruction from Samples



$$\Leftrightarrow$$



$$H(\omega) = T_s rect(\frac{\omega}{4\pi B})$$

$$h(t) = 2BT_s \sin c(2\pi Bt)$$

$$h(t) = \sin c(2\pi Bt) \qquad (2BT_s = 1)$$

$$\delta(t) \longrightarrow \boxed{H(\omega)} \longrightarrow h(t)$$

$$\bar{g}(t) \longrightarrow \boxed{H(\omega)} \longrightarrow g(t)$$

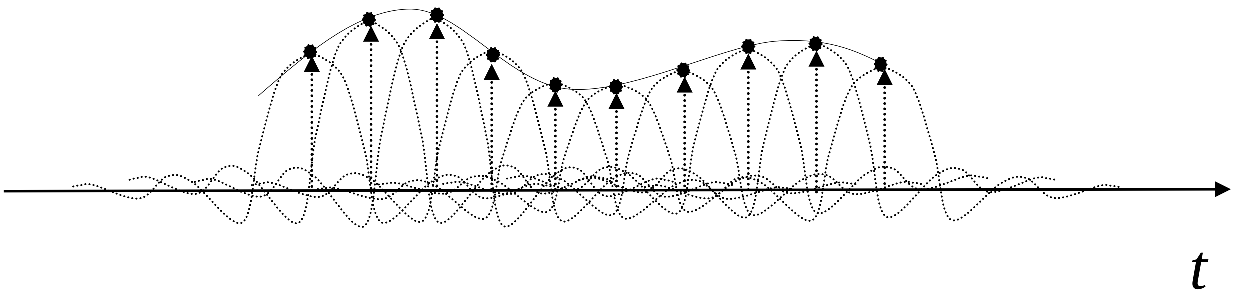
Recall $$\bar{g}(t) = g(t)\delta_{T_s}(t) = \sum_n g(nT_s)\delta(t - nT_s)$$

$$\Rightarrow \quad g(t) = \sum g(nT_s)h(t - nT_s)$$

$$= \sum g(nT_s)\sin c[2\pi B(t - nT_s)]$$

$$= \sum g(nT_s)\sin c(2\pi Bt - n\pi)$$

Recovered $g(t)$



Practical Considerations in Nyquist Sampling

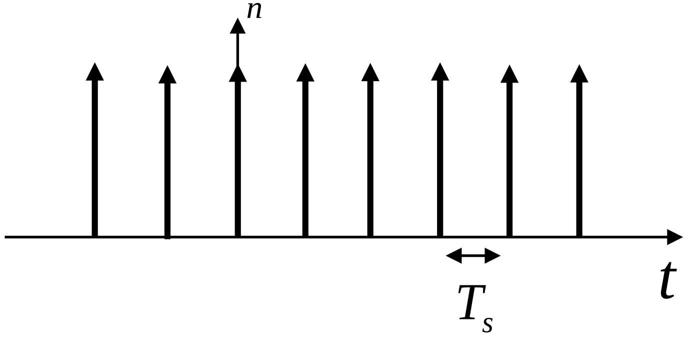
- ***Gradual roll-off lowpass filter***

- ***Aliasing***

- ***Practical Pulses vs. Ideal Impulses***

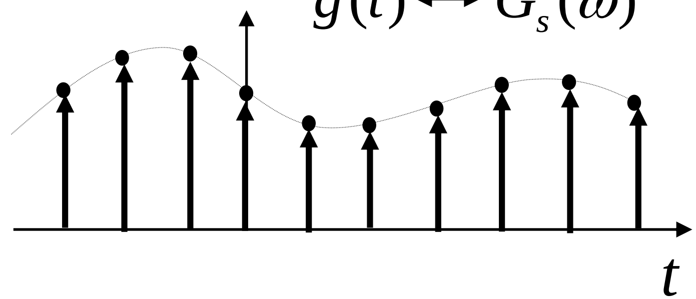
Sampling Theorem

$$\delta_{T_s}(t) = \sum_n \delta(t - nT_s) \leftrightarrow G[n\omega_s]$$

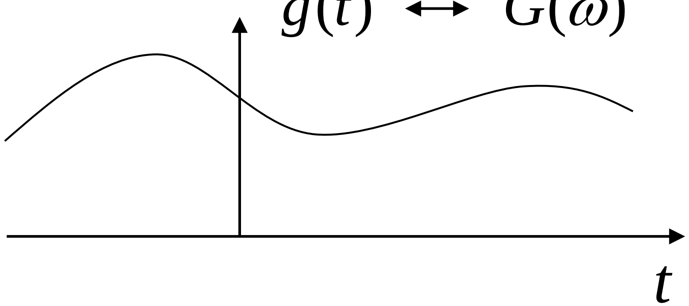


Not practical to generate!

$$\bar{g}(t) \leftrightarrow G_s(\omega)$$



$$g(t) \leftrightarrow G(\omega)$$

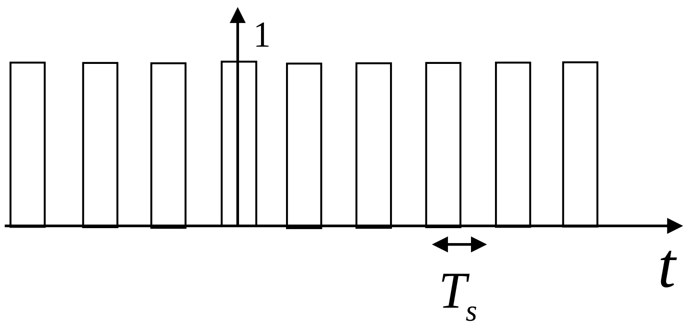


Sampling Frequency

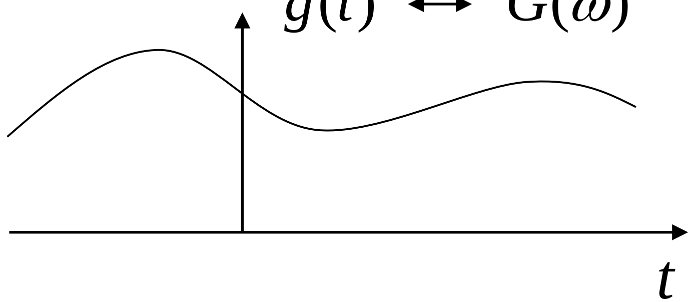
$$f_s = \frac{1}{T_s}$$

Let's Try Square Wave

$$x(t) \leftrightarrow X[n\omega_s]$$

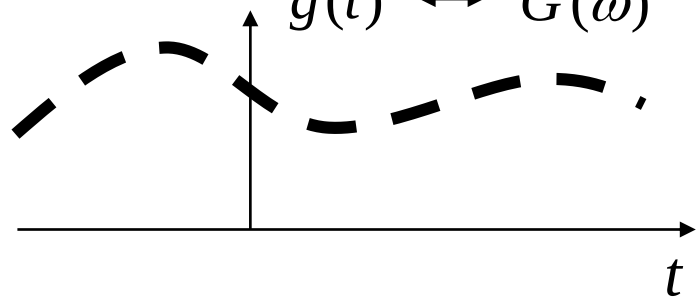


$$g(t) \leftrightarrow G(\omega)$$



Problem Resolved

$$\bar{g}(t) \leftrightarrow \bar{G}(\omega)$$



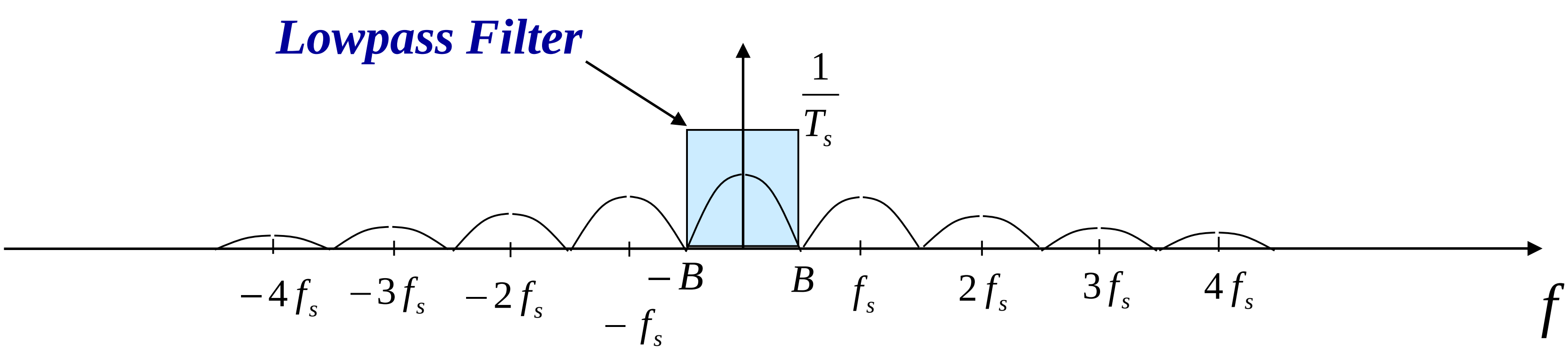
Sampling Frequency

$$f_s = \frac{1}{T_s}$$

Sampling Theorem

Sampling frequency should be at least equal to or greater than twice the bandwidth of the message signal for successful recovery of the signal from it's samples

$$f_s \geq 2B$$



bandwidth of lowpass filter = B Hz

Maximum Information Rate

Signal of bandwidth B

$\Downarrow$

2B samples per second

$\Downarrow$

2B Independent pieces of information per second

$\Downarrow$

Two independent pieces of information per second per hertz of bandwidth

Sampling Theorem and Digital Communication

- ***Pulse Amplitude Modulation*** ***(M-ary)***
- ***Pulse Width Modulation*** ***(M-ary)***
- ***Pulse Position Modulation*** ***(M-ary)***
- ***Pulse Code Modulation*** ***(Binary)***

Pulse Code Modulation (PCM)

Sampling → ***Quantization*** → ***Binary Coding***

Speech Signal from 15Hz to 15kHz

	Maximum Frequency	*Sampling Rate*	*Quantization Levels*	*Number of Bits/code*	*Bit Rate*
Telephone	3.4 kHz	8 kHz	256	8	64 kb/s
Compact Disk	15 kHz	44.1 kHz	65,536	16	705.6 kb/s

Quantization

Reconstruction of actual samples after sampling

$$g(t) = \sum g(nT_s) \sin c(2\pi Bt - n\pi)$$

Reconstruction of quantized samples after sampling

$$\hat{g}(t) = \sum \hat{g}(nT_s) \sin c(2\pi Bt - n\pi)$$

Distortion in reconstructed signal

$$q(t) = \hat{g}(t) - g(t)$$

$$\Rightarrow q(t) = \sum [\hat{g}(nT_s) - g(nT_s)] \sin c(2\pi Bt - n\pi)$$

$$\Rightarrow q(t) = \sum q(nT_s) \sin c(2\pi Bt - n\pi)$$

where $q(nT_s)$ is the quantization error in nth sample

Power or mean square value of q(t) (quantization noise) is given by

$$\overline{q(t)}^2 = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} q^2(t)dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} [\sum_n q(nT_s) \sin c(2\pi Bt - n\pi)]^2 dt$$

Now by using the fact that sinc() are orthogonal functions i.e.,

$$\int_{-\infty}^{\infty} \sin c(2\pi Bt - n\pi) \sin c(2\pi Bt - m\pi)dt = \begin{cases} 0 & \boldsymbol{m \neq n} \\ \frac{1}{2B} & \boldsymbol{m = n} \end{cases}$$

$$\Rightarrow \overline{q(t)}^2 = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \sum_n q^2(nT_s) \sin c^2(2\pi Bt - n\pi)dt$$

$$\Rightarrow \overline{q(t)}^2 = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \sum_n q^2(nT_s) \sin c^2(2\pi Bt - n\pi) dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \sum_n q^2(nT_s) \int_{-T/2}^{T/2} \sin c^2(2\pi Bt - n\pi) dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \sum_n q^2(nT_s)(\frac{1}{2B})$$

$$= \lim_{T\to\infty} \frac{1}{2BT} \sum_n q^2(nT_s)$$

=> quantization noise is average of square of quantization error

Let's calculate this

quantization error lies in the range of -Δv/2 and Δv/2

where $$\Delta v = \frac{2m_p}{L}$$

Assuming quantization error is equally likely in the range of -Δv/2 and Δv/2

$$\overline{q^2} = \frac{1}{\Delta v} \int_{-\Delta v/2}^{\Delta v/2} q^2 dq$$

$$= \frac{1}{\Delta v} \left| \frac{q^3}{3} \right|_{-\Delta v/2}^{\Delta v/2}$$

$$= \frac{1}{\Delta v} \frac{(\Delta v)^3}{12}$$

$$= \frac{(\Delta v)^2}{12} = \frac{m_p^2}{3L^2}$$

$$\Rightarrow N_q = \overline{q^2(t)} = \frac{m_p^2}{3L^2}$$

$$\hat{g}(t) = \underbrace{g(t)}_{\textit{Desired signal}} + \underbrace{q(t)}_{\textit{Unwanted noise}}$$

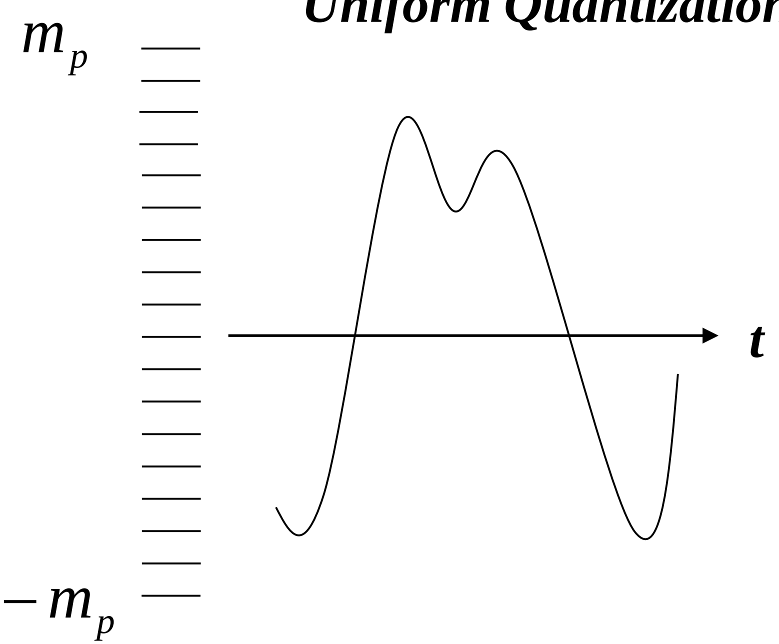
$$\Rightarrow S_0 = \overline{g^2(t)}$$

$$and \qquad N_0 = N_q = \frac{m_p^2}{3L^2}$$

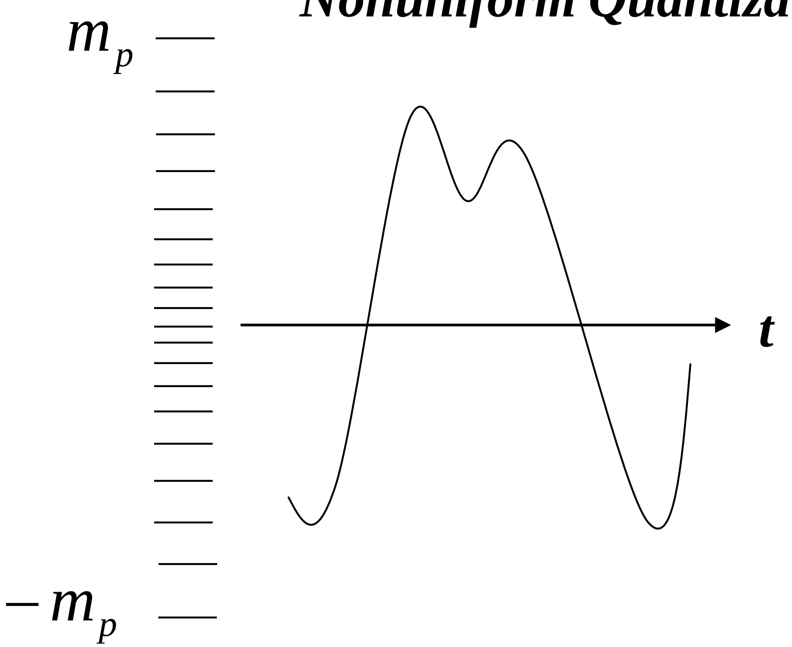
Therefore $$SNR = \frac{S_0}{N_0} = 3L^2 \frac{\overline{m^2(t)}}{m_p^2}$$

Nonuniform Quantization

Uniform Quantization

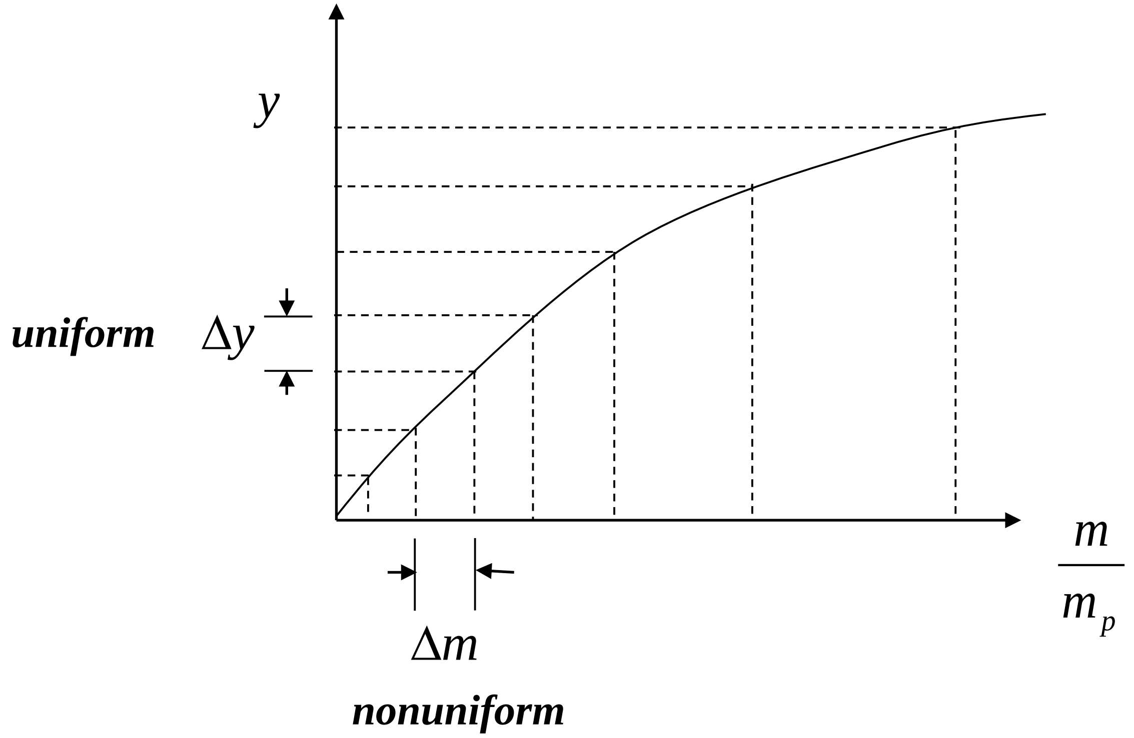


Nonuniform Quantization



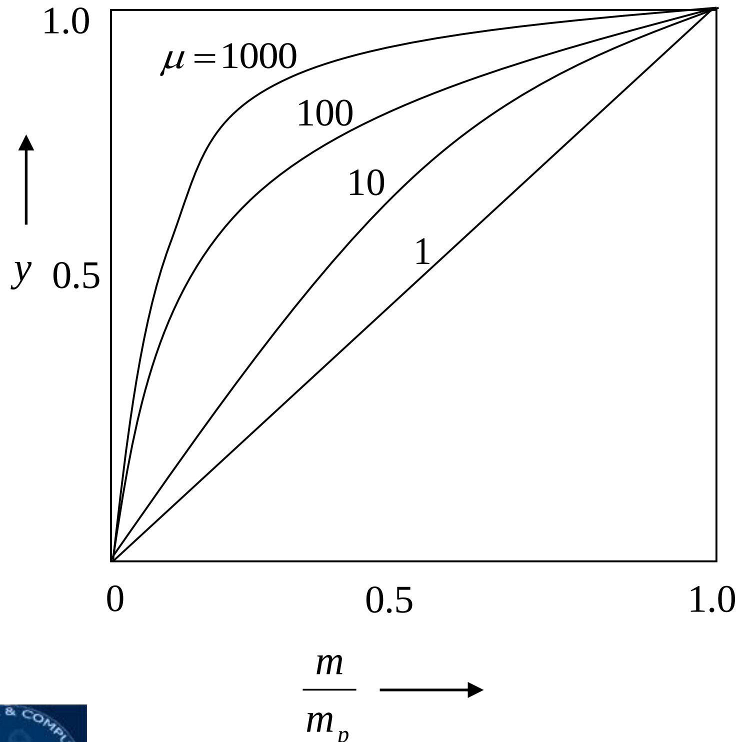
Nonuniform Quantization = Compression + Uniform Quantization

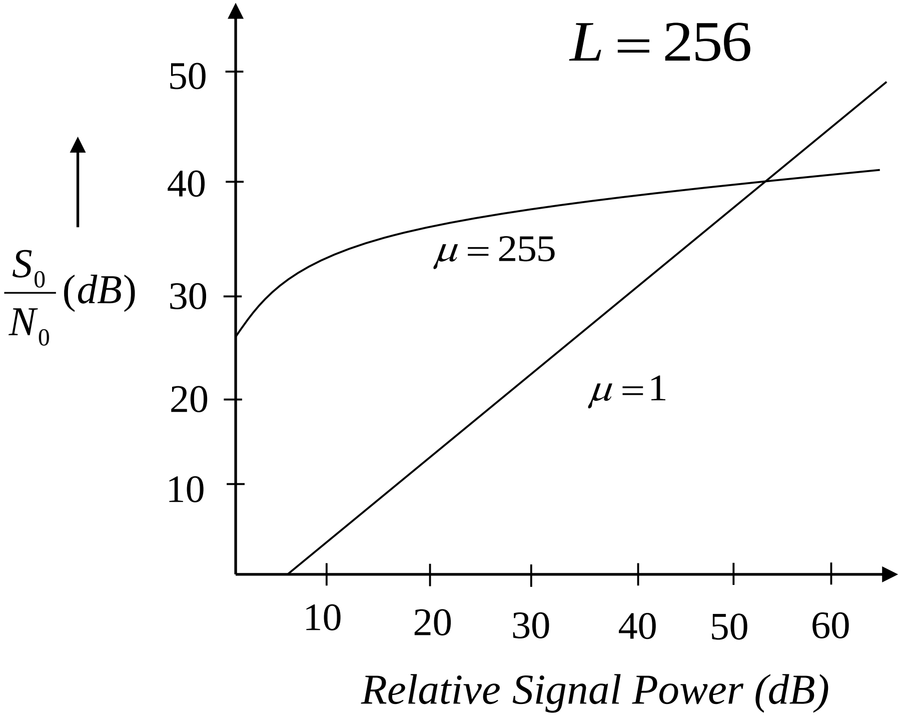
(Usually compression increases the bandwidth but not applicable here)



Example

μ-law $$y = \frac{1}{\ln(1+\mu)} \ln\left(1+\frac{\mu m}{m_p}\right)$$





Transmission Bandwidth for PCM

$$Signal \quad Bandwidth = B$$

$$Sampling \quad Rate = 2B$$

$$Quantized \quad levels = L = 2^n$$

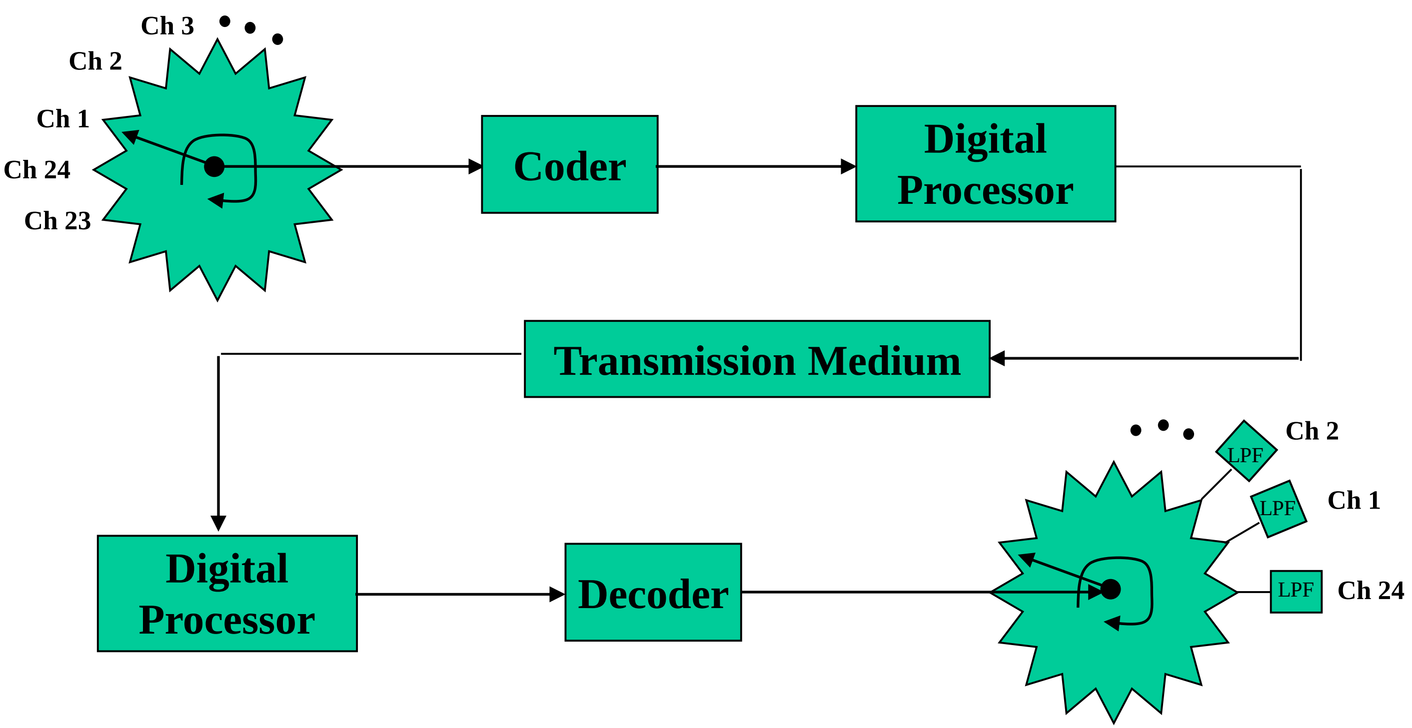
$$\Rightarrow Number \quad of \quad encoded \quad bits = n$$

$$Bit \quad Rate = 2nB \quad bits/\sec$$

$$\Rightarrow \mathrm{Re}quired \quad Transmission \quad Bandwidth = nB \quad Hz$$

Digital Communication and Time Division Multiplexing

- ***T1 Carrier System (DS1) is an example***
- ***1.544Mbits/sec total data rate for 24 channels***
- ***Each channel is encoded with 8 bits***



What does Digital Processor do?

1. ***Synchronization (Framing)***
2. ***Signaling***
3. ***Parity Insertion and Check***
4. ***Scrambling***

Energy of a Signal and Parseval's Theorem

$$E_g = \int_{-\infty}^{\infty} g(t)g^*(t)dt$$

$$E_g = \int_{-\infty}^{\infty} g(t)\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} G^*(\omega)e^{-j\omega t}d\omega\right]dt$$

$$E_g = \frac{1}{2\pi}\int_{-\infty}^{\infty} G^*(\omega)\left[\int_{-\infty}^{\infty} g(t)e^{-j\omega t}dt\right]d\omega$$

$$E_g = \frac{1}{2\pi}\int_{-\infty}^{\infty} G^*(\omega)G(\omega)d\omega$$

$$E_g = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left|G(\omega)\right|^2 d\omega$$

Energy Spectral Density

$$E_g = \frac{1}{2\pi}\int_{-\infty}^{\infty} |G(\omega)|^2 d\omega \qquad \Rightarrow \qquad \Psi_g(\omega) = |G(\omega)|^2$$

energy per unit bandwidth

ESD of the Input and the Output

$$g(t) \longrightarrow \boxed{\quad h(t) \quad} \longrightarrow y(t)$$

$$Y(\omega) = H(\omega)G(\omega)$$

$$|Y(\omega)|^2 = |H(\omega)|^2 |G(\omega)|^2$$

$$\Rightarrow \Psi_y(\omega) = |H(\omega)|^2 \Psi_g(\omega)$$

Autocorrelation function of a signal

$$\psi_g(\tau) = \int_{-\infty}^{\infty} g(t)g(t+\tau)dt$$

$$let \quad x = t + \tau$$

$$\psi_g(\tau) = \int_{-\infty}^{\infty} g(x-\tau)g(x)dx = \int_{-\infty}^{\infty} g(x)g(x-\tau)dx$$

$$= \int_{-\infty}^{\infty} g(t)g(t-\tau)dt$$

$$\Rightarrow \psi_g(\tau) = \psi_g(-\tau)$$

$$F[\psi_g(\tau)] = \int_{-\infty}^{\infty} e^{-j\omega\tau}[\int_{-\infty}^{\infty} g(t)g(t+\tau)dt]d\tau$$

$$= \int_{-\infty}^{\infty} g(t)[\int_{-\infty}^{\infty} g(t+\tau)e^{-j\omega\tau}d\tau]dt$$

$$= \int_{-\infty}^{\infty} g(t)[\int_{-\infty}^{\infty} g(\tau+t)e^{-j\omega\tau}d\tau]dt$$

$$= \int_{-\infty}^{\infty} g(t)[G(\omega)e^{j\omega t}]dt$$

$$= G(\omega)\int_{-\infty}^{\infty} g(t)e^{j\omega t}dt$$

$$= G(\omega)G(-\omega) = \left|G(\omega)\right|^2$$

$$\Rightarrow \psi_g(\tau) \Leftrightarrow \Psi_g(\omega)$$

Signal Power and Power Spectral Density

$$P_g = \lim_{T \to \infty} \frac{1}{T} \int_{-\infty}^{\infty} g^2(t)\,dt$$

$$g_T(t) = \begin{cases} g(t) & |t| \le T/2 \\ 0 & |t| \ge T/2 \end{cases}$$

$$P_g = \lim_{T \to \infty} \frac{\int_{-\infty}^{\infty} g_T{}^2(t)\,dt}{T}$$

$$= \lim_{T \to \infty} \frac{E_{g_T}}{T}$$

$$E_{g_T} = \int_{-\infty}^{\infty} g_T^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |G_T(\omega)|^2 d\omega$$

$$\Rightarrow P_g = \lim_{T\to\infty}\frac{E_{g_T}}{T} = \lim_{T\to\infty}\frac{1}{T}[\frac{1}{2\pi}\int_{-\infty}^{\infty}|G_T(\omega)|^2 d\omega]$$

$$\Rightarrow P_g = \frac{1}{2\pi}\int_{-\infty}^{\infty}\lim_{T\to\infty}\frac{|G_T(\omega)|^2}{T}d\omega$$

We define PSD as

$$S_g(\omega) = \lim_{T\to\infty}\frac{|G_T(\omega)|^2}{T}$$

So that $$P_g = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_g(\omega)d\omega = \frac{1}{\pi}\int_{0}^{\infty} S_g(\omega)d\omega$$

Autocorrelation function of a power signal

$$\Re_g(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-\infty}^{\infty} g(t)g(t+\tau)dt$$

Same argument as in energy signals will prove that

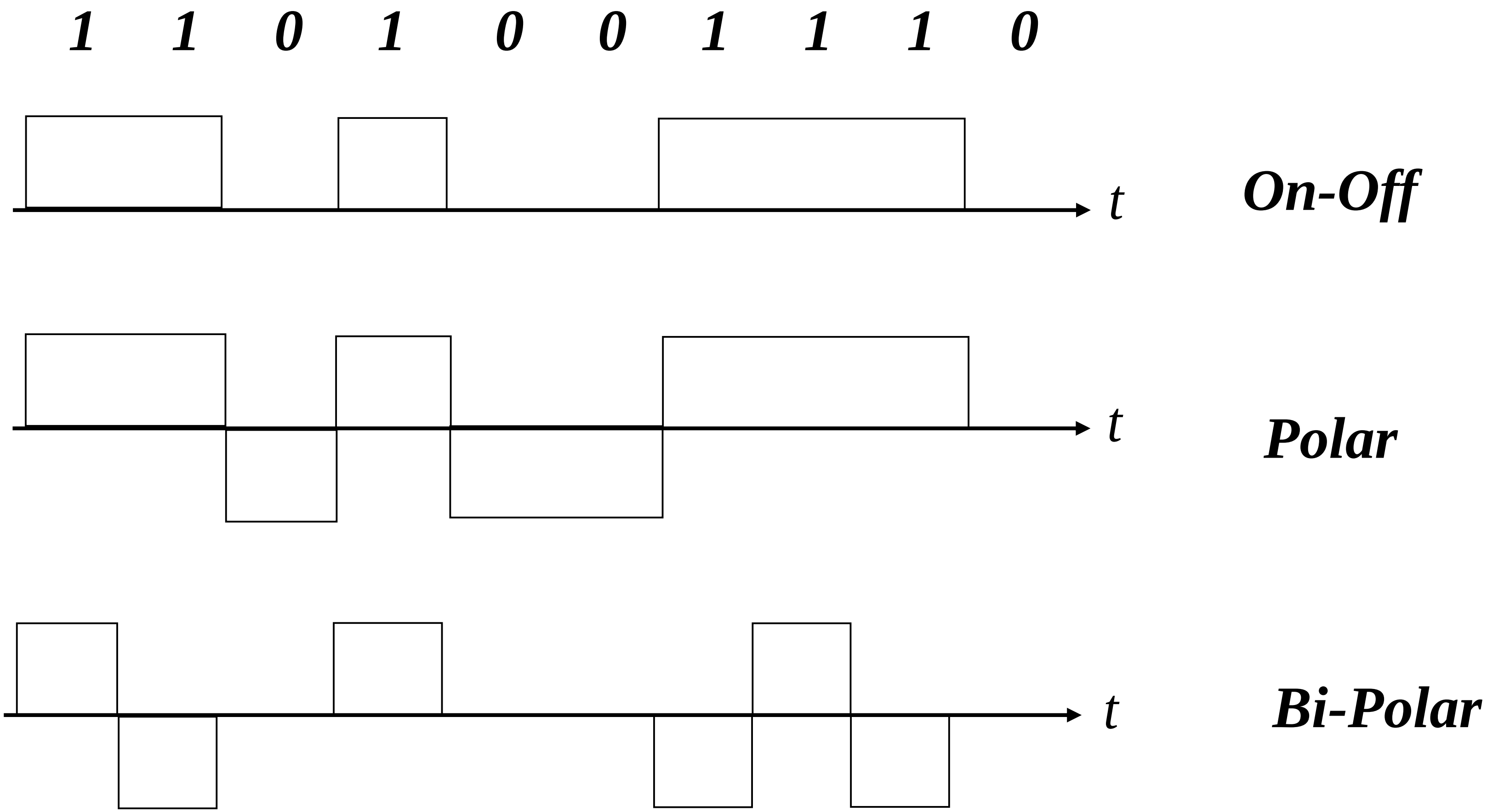
$$\Re_g(\tau) = \Re_g(-\tau)$$

and $$\Re_g(\tau) \Leftrightarrow S_g(\omega)$$

also $$S_y(\omega) = |H(\omega)|^2 S_g(\omega)$$

if $$Y(\omega) = H(\omega)G(\omega)$$

Various Digital Line Codes

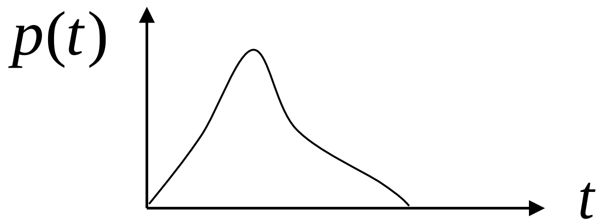


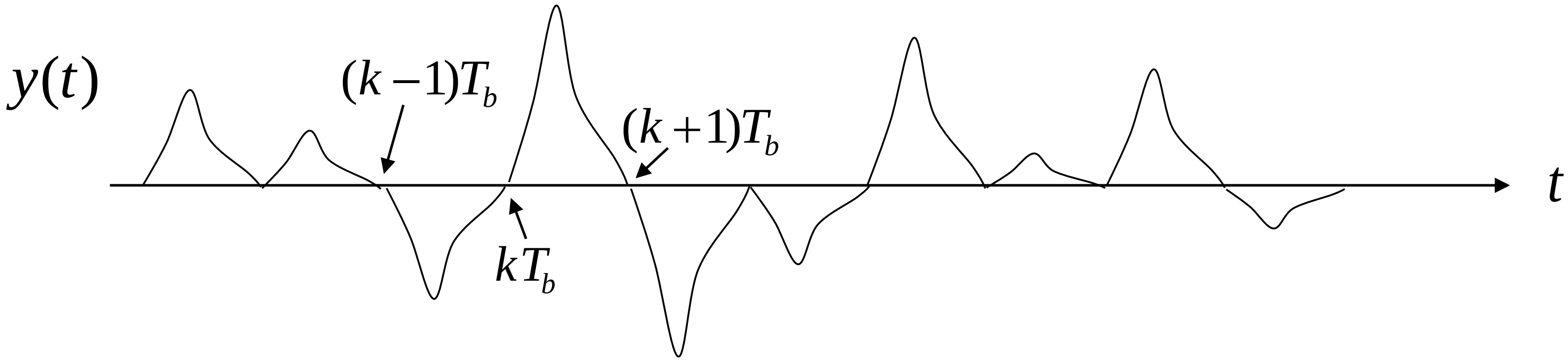
Pulse shape p(t) could be arbitrary

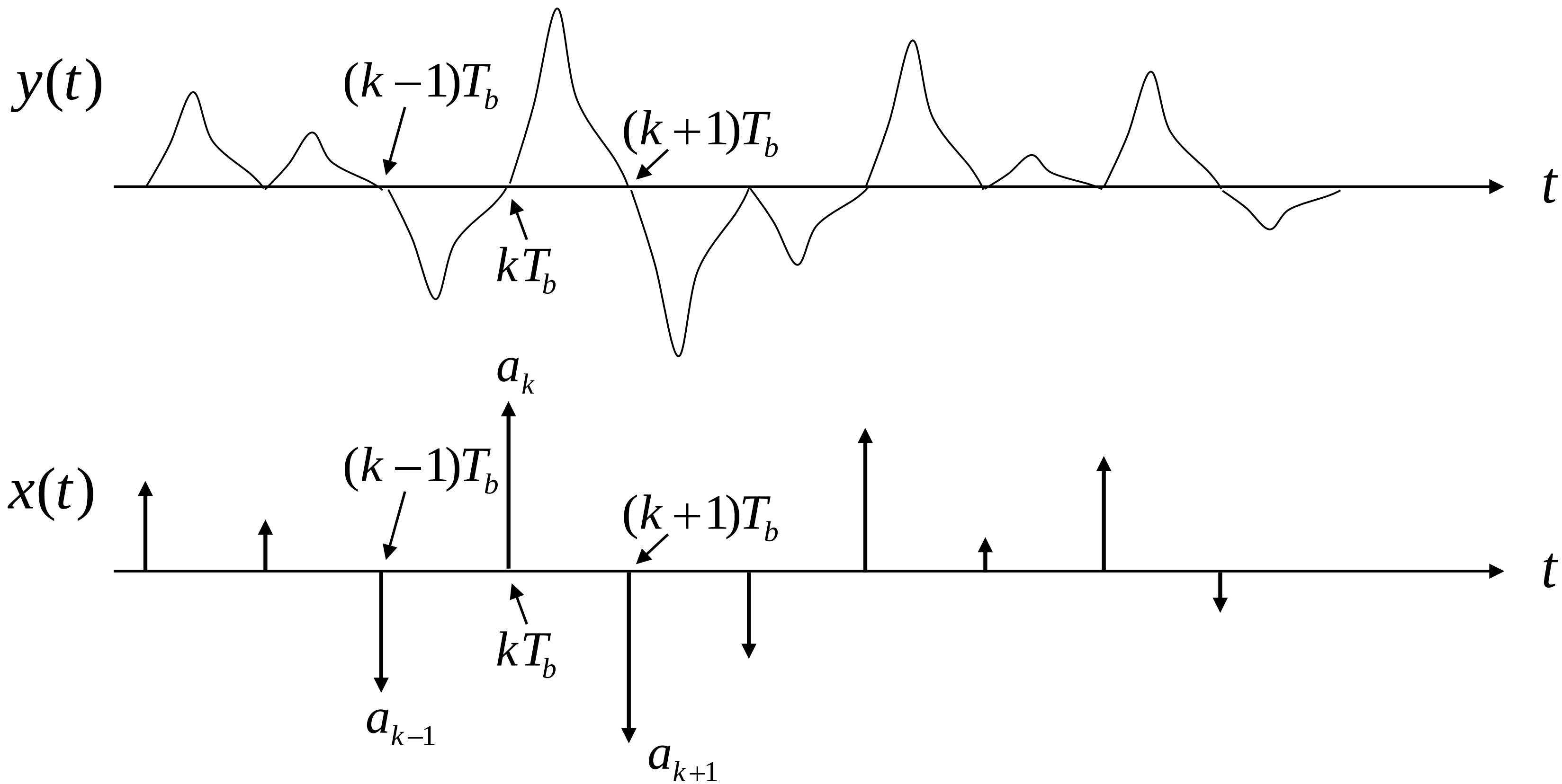
Some Characteristics of Good Line Codes

1. ***Bandwidth Efficient***
2. ***Power Efficient***
3. ***Error Detection or Correction Capability***
4. ***Favorable PSD***
5. ***Adequate Timing Content***
6. ***Transparency***

PSD of various Line Codes



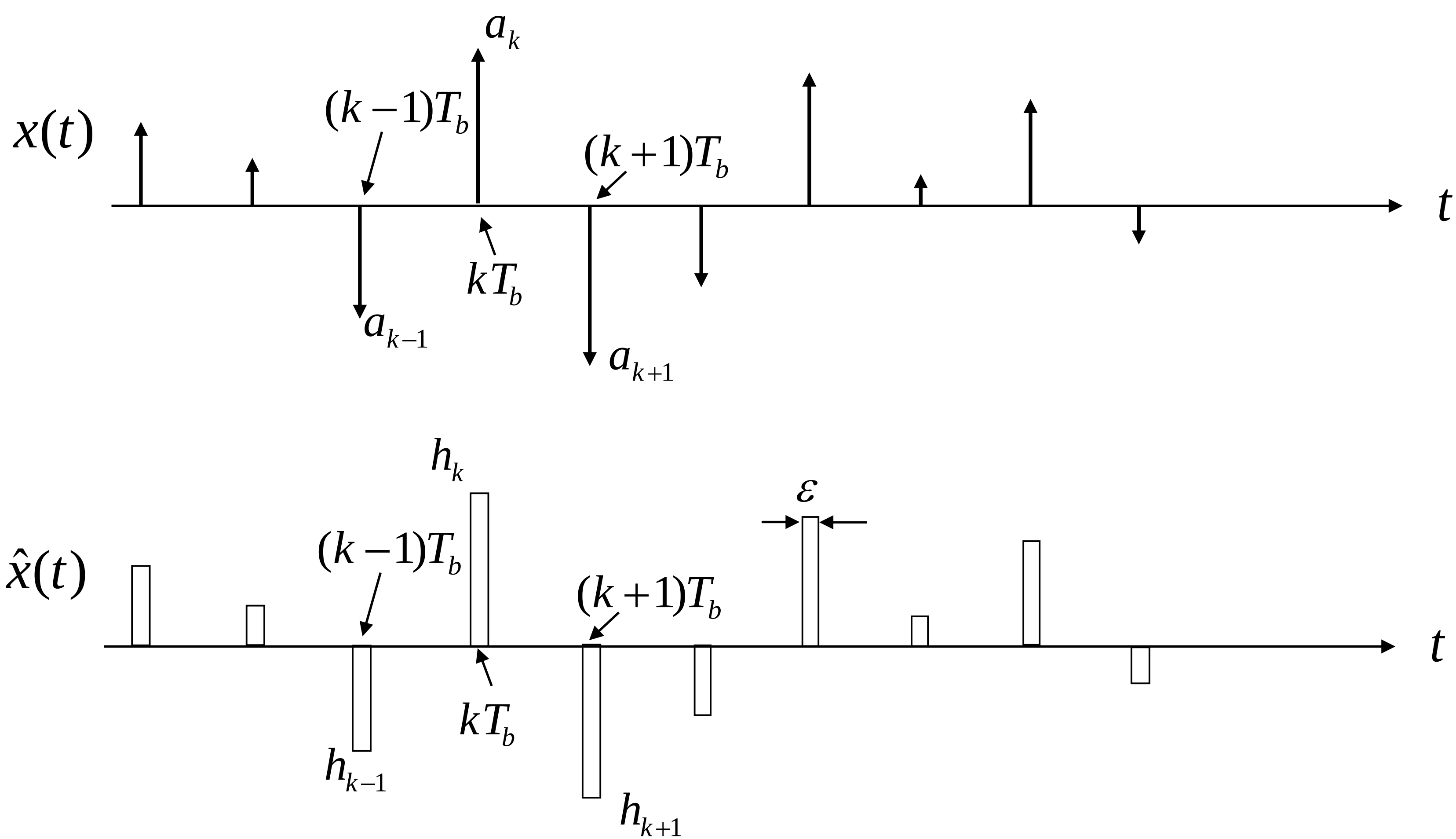




$$x(t) \longrightarrow \boxed{h(t) = p(t)} \longrightarrow y(t)$$

$$\Rightarrow S_y(\omega) = \left|P(\omega)\right|^2 S_x(\omega)$$

PSD of x(t)



$$\varepsilon h_k = a_k$$

$$\Re_{\hat{x}}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \hat{x}(t)\hat{x}(t+\tau)dt$$

$$when \qquad \tau < \varepsilon$$

$$\Re_{\hat{x}}(\tau) = \lim_{T\to\infty} \frac{1}{T} \sum_k h_k^2(\varepsilon - \tau)$$

$$= \lim_{T\to\infty} \frac{1}{T} \sum_k a_k^2 (\frac{\varepsilon - \tau}{\varepsilon^2}) \qquad (\varepsilon h_k = a_k)$$

$$= \lim_{T\to\infty} \frac{1}{\varepsilon T} \sum_k a_k^2 (1 - \frac{\tau}{\varepsilon})$$

$$= \lim_{T\to\infty} \frac{1}{\varepsilon T} (1 - \frac{\tau}{\varepsilon}) \frac{T_b}{T_b} \sum_k a_k^2$$

$$= \frac{R_0}{\varepsilon T_b} (1 - \frac{\tau}{\varepsilon}) \quad where \quad R_0 = \lim_{T\to\infty} \frac{T_b}{T} \sum_k a_k^2$$

$$\Re_{\hat{x}}(\tau) = \frac{R_0}{\varepsilon T_b}(1 - \frac{\tau}{\varepsilon}) \qquad where \qquad R_0 = \lim_{T\to\infty} \frac{T_b}{T} \sum_k a_k^2$$

$$notice \quad N = \frac{T}{T_b} \qquad\qquad N \to \infty \quad when \quad T \to \infty$$

$$\Rightarrow \quad R_0 = \lim_{N\to\infty} \frac{1}{N} \sum_k a_k^2$$

$$= \overline{a_k^2}$$

as $\Re_{\hat{x}}(\tau)$ ***is even function of time***

$$\Re_{\hat{x}}(\tau) = \frac{R_0}{\varepsilon T_b}(1 - \frac{|\tau|}{\varepsilon}) \qquad\qquad |\tau| < \varepsilon$$

$$at \quad \tau = T_b$$

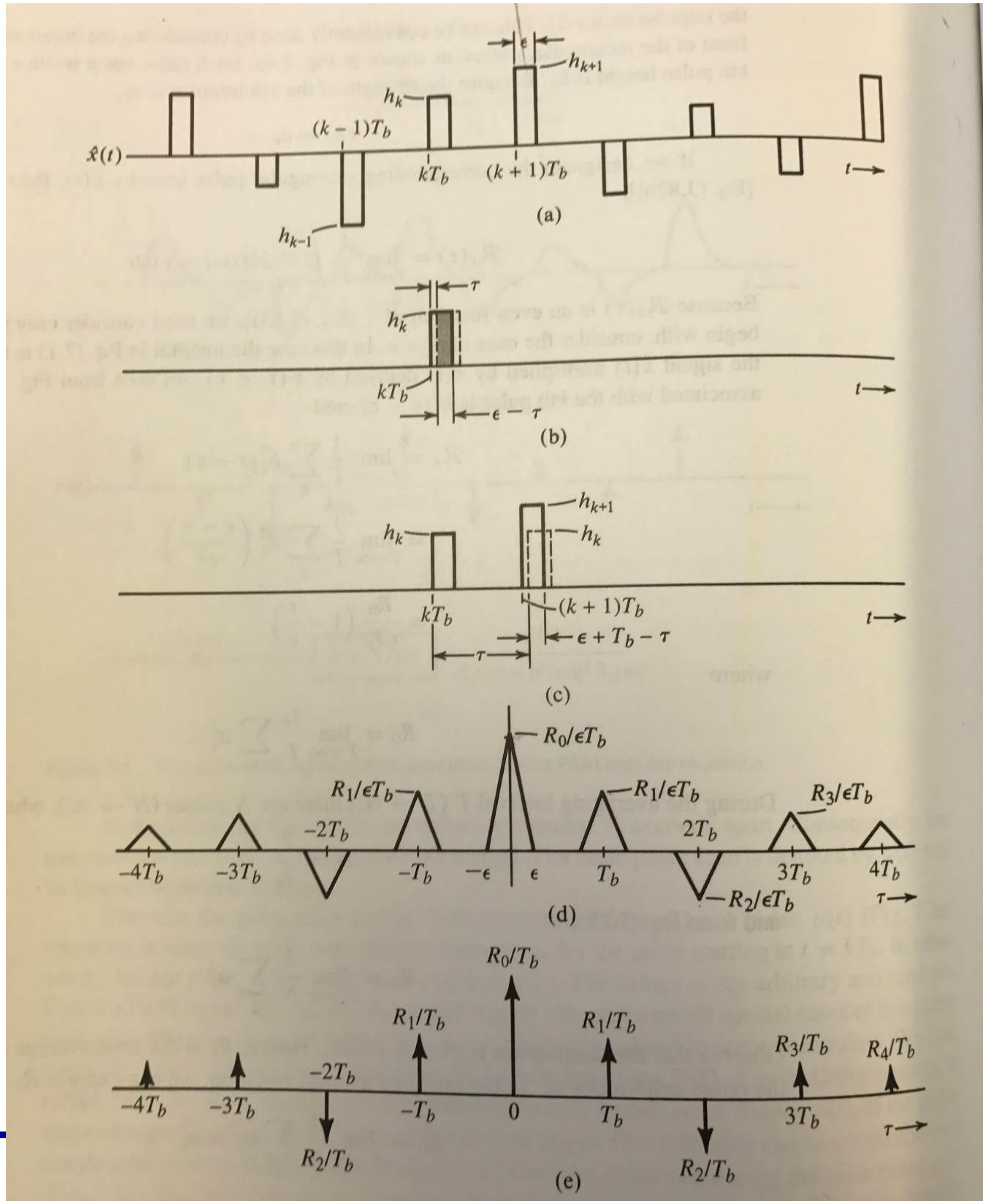
$$\Re_{\hat{x}}(\tau) = \frac{R_1}{\varepsilon T_b}\left(1 - \frac{|\tau|}{\varepsilon}\right)$$

$$where \quad R_1 = \lim_{N\to\infty} \frac{1}{N} \sum_k a_k a_{k+1} = \overline{a_k a_{k+1}}$$

Similarly, at $\tau = nT_b$

$$\Re_{\hat{x}}(\tau) = \frac{R_n}{\varepsilon T_b}\left(1 - \frac{|\tau|}{\varepsilon}\right)$$

$$Where \quad R_n = \lim_{N\to\infty} \frac{1}{N} \sum_k a_k a_{k+n} = \overline{a_k a_{k+n}}$$



ϵ
h_{k+1}
h_k
$(k-1)T_b$
$\hat{x}(t)$
kT_b
$(k+1)T_b$
t
h_{k-1}
(a)
τ
h_k
kT_b
$\epsilon - \tau$
t
(b)
h_{k+1}
h_k
h_k
kT_b
$(k+1)T_b$
$\epsilon + T_b - \tau$
τ
t
(c)
$R_0/\epsilon T_b$
$R_1/\epsilon T_b$
$R_1/\epsilon T_b$
$R_3/\epsilon T_b$
$-2T_b$
$2T_b$
$-4T_b$
$-3T_b$
$-T_b$
$-\epsilon$
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$3T_b$
$4T_b$
$R_2/\epsilon T_b$
τ
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R_0/T_b
R_1/T_b
R_1/T_b
R_3/T_b
R_4/T_b
$-2T_b$
$-4T_b$
$-3T_b$
$-T_b$
0
T_b
$3T_b$
τ
R_2/T_b
R_2/T_b
(e)

$$at \quad \tau = nT_b, \qquad \Re_{\hat{x}}(\tau) = \frac{R_n}{\varepsilon T_b}\left(1 - \frac{|\tau|}{\varepsilon}\right)$$

$$\Re_x(\tau) = \lim_{\varepsilon \to 0} \Re_{\hat{x}}(\tau)$$

$$\Rightarrow \Re_x(\tau) = \frac{R_n}{T_b}\delta(\tau - nT_b)$$

For all τ

$$\Re_x(\tau) = \sum_{n=-\infty}^{\infty} \frac{R_n}{T_b}\delta(\tau - nT_b)$$

$$= \frac{1}{T_b}\sum_{n=-\infty}^{\infty} R_n \delta(\tau - nT_b)$$

$$\Rightarrow S_x(\omega) = \frac{1}{T_b}\sum_{n=-\infty}^{\infty} R_n e^{-jn\omega T_b}$$

$$\Rightarrow S_x(\omega) = \frac{1}{T_b}\sum_{n=-\infty}^{\infty} R_n e^{-jn\omega T_b}$$

$$= \frac{1}{T_b}(R_0 + 2\sum_{n=1}^{\infty} R_n \cos n\omega T_b)$$

We know that

$$S_y(\omega) = |P(\omega)|^2 S_x(\omega)$$

$$\Rightarrow S_y(\omega) = \frac{|P(\omega)|^2}{T_b}(R_0 + 2\sum_{n=1}^{\infty} R_n \cos n\omega T_b)$$

PSD of Polar Signaling

transmission of 1 with p(t) and 0 with –p(t)

$$S_y(\omega) = \frac{|P(\omega)|^2}{T_b}(R_0 + 2\sum_{n=1}^{\infty} R_n \cos n\omega T_b)$$

$$R_0 = \lim_{N\to\infty} \frac{1}{N}\sum_k a_k^2$$

$$= \lim_{N\to\infty} \frac{1}{N}(N) = 1$$

$$R_1 = \lim_{N\to\infty} \frac{1}{N}\sum_k a_k a_{k+1}$$

$$= \lim_{N\to\infty} \frac{1}{N}[\frac{N}{2}(1) + \frac{N}{2}(-1)] = 0$$

$$\Rightarrow R_n = 0 \quad \mathit{if} \quad n > 0$$

$$\Rightarrow S_y(\omega) = \frac{|P(\omega)|^2}{T_b}$$

$$\Rightarrow S_y(\omega) = \frac{|P(\omega)|^2}{T_b}$$

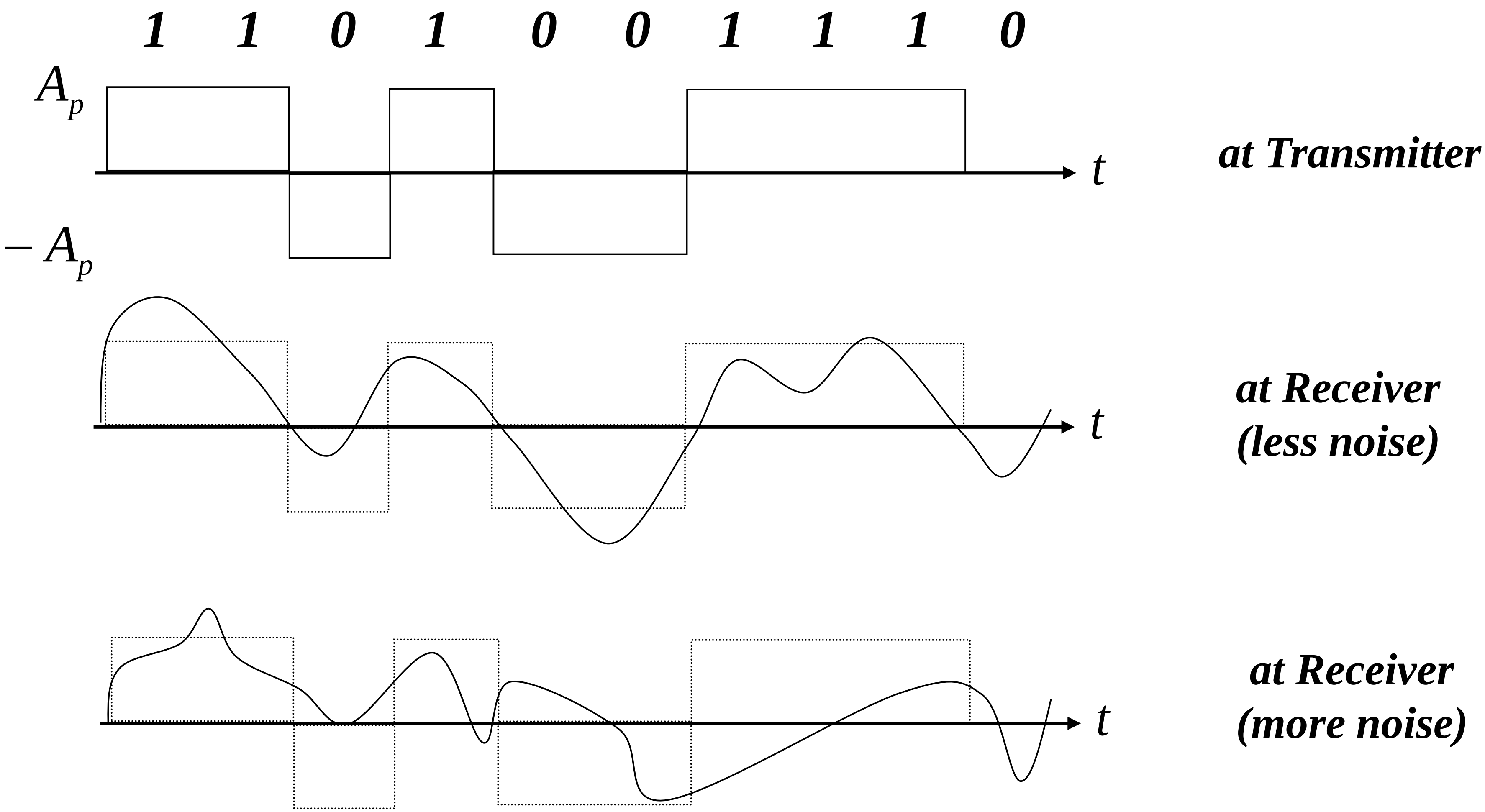
$$if \qquad p(t) = rect(\frac{t}{T_b})$$

$$then \qquad P(\omega) = T_b \sin c(\frac{\omega T_b}{2})$$

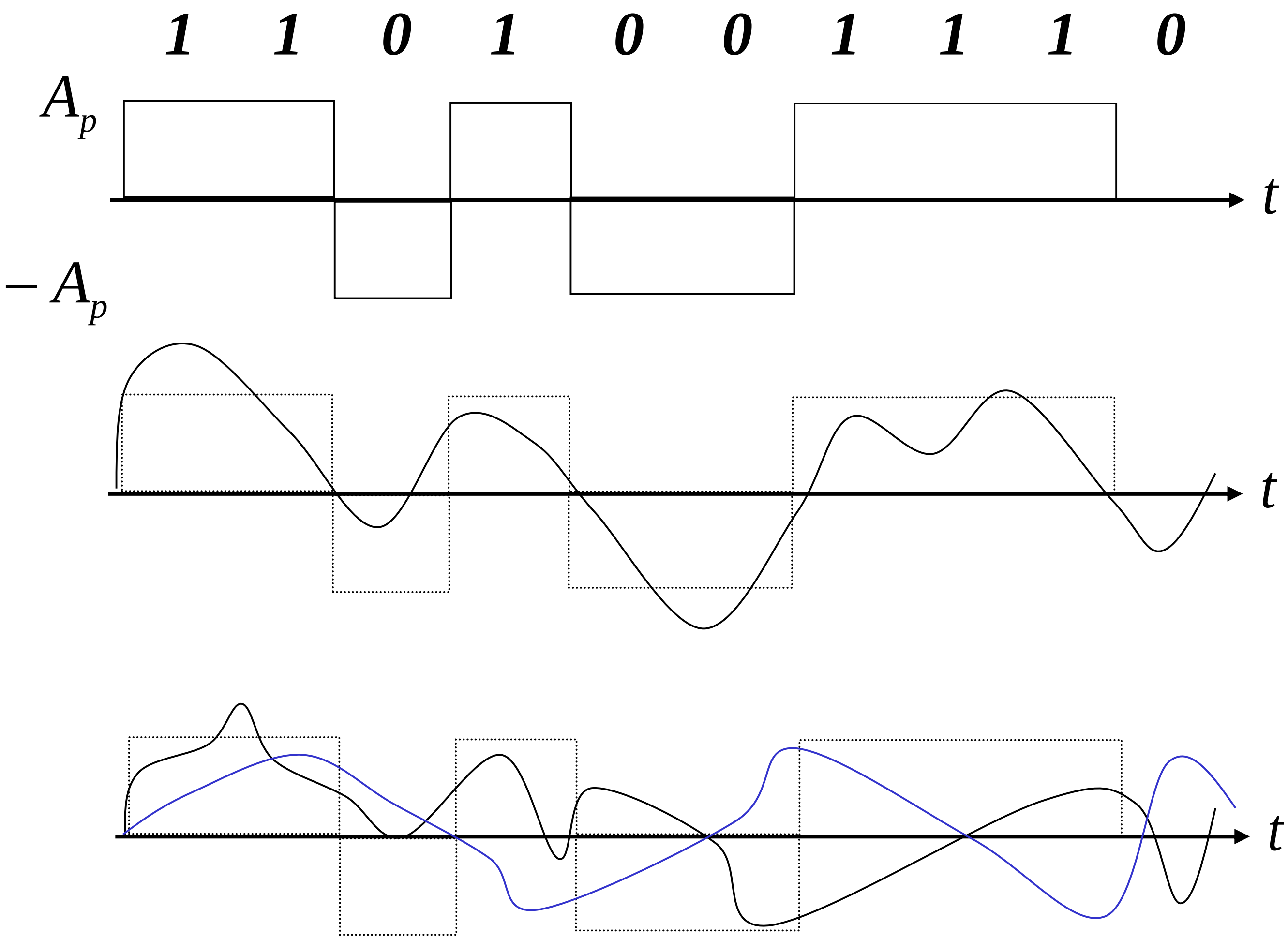
$$\Rightarrow S_y(\omega) = T_b \sin c^2(\frac{\omega T_b}{2})$$

Tweaking PSD with Pulse Shaping having null at zero frequency

Noise and Digital Communication



Noise and Digital Communication

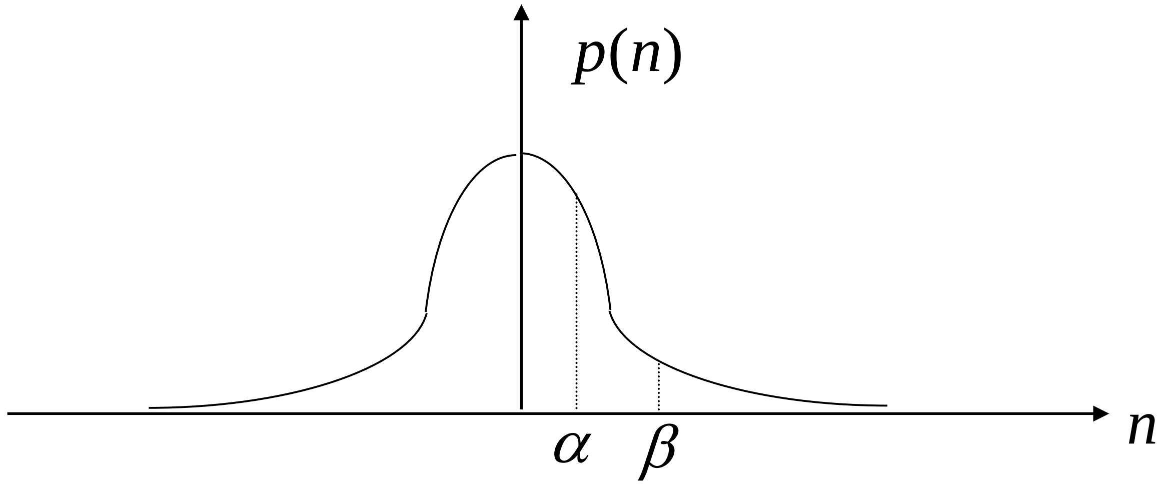


at Transmitter

at Receiver (less noise)

at Receiver (more noise)

Gaussian Noise and Probability Density Function



$$p(n) = \frac{1}{\sigma_n \sqrt{2\pi}} e^{\frac{-n^2}{2\sigma_n^2}}$$

$$\Pr ob(\alpha < n < \beta) = \int_{\alpha}^{\beta} p(n)dn = \int_{\alpha}^{\beta} \frac{1}{\sigma_n \sqrt{2\pi}} e^{\frac{-n^2}{2\sigma_n^2}} dn$$

Error Probability for Polar Signaling

$$\Pr ob(E) = \frac{1}{2}[\Pr ob(E|0) + \Pr ob(E|1)]$$

$$\Pr ob(E|0) = probability \quad that \quad n > A_p$$

$$\Pr ob(E|1) = probability \quad that \quad n < -A_p$$

$$\Pr ob(E|0) = \int_{A_p}^{\infty} p(n)dn = \int_{A_p}^{\infty} \frac{1}{\sigma_n \sqrt{2\pi}} e^{\frac{-n^2}{2\sigma_n^2}} dn$$

$$= \frac{1}{\sigma_n \sqrt{2\pi}} \int_{A_p}^{\infty} e^{\frac{-n^2}{2\sigma_n^2}} dn$$

$$= \frac{1}{\sqrt{2\pi}} \int_{A_p/\sigma_n}^{\infty} e^{\frac{-x^2}{2}} dx \qquad (x = \frac{n}{\sigma_n})$$

Let's define

$$Q(y) = \frac{1}{\sqrt{2\pi}} \int_{y}^{\infty} e^{\frac{-x^2}{2}} dx$$

$$\Rightarrow \Pr ob(E|0) = Q(\frac{A_p}{\sigma_n})$$

$$\Pr ob(E|1) = \int_{-\infty}^{A_p} p(n)dn = \int_{-\infty}^{A_p} \frac{1}{\sigma_n\sqrt{2\pi}} e^{\frac{-n^2}{2\sigma_n^2}} dn$$

$$= Q(\frac{A_p}{\sigma_n})$$

Now

$$\Pr ob(E) = \frac{1}{2}[\Pr ob(E|0) + \Pr ob(E|1)]$$

$$= \tfrac{1}{2}[Q(\frac{A_p}{\sigma_n}) + Q(\frac{A_p}{\sigma_n})]$$

$$= Q(\frac{A_p}{\sigma_n})$$

Error Probability for ON-Off Signaling

$$\Pr ob(E) = \frac{1}{2}[\Pr ob(E|0) + \Pr ob(E|1)]$$

$$\Pr ob(E|0) = probability \quad that \quad n > \frac{A_p}{2} = Q(\frac{A_p}{2\sigma_n})$$

$$\Pr ob(E|1) = probability \quad that \quad n < \frac{-A_p}{2} = Q(\frac{A_p}{2\sigma_n})$$

$$\Rightarrow \Pr ob(E) = \frac{1}{2}[Q(\frac{A_p}{2\sigma_n}) + Q(\frac{A_p}{2\sigma_n})]$$

$$= Q(\frac{A_p}{2\sigma_n})$$

Error Probability for Bi-Polar Signaling

$$\Pr ob(E) = \frac{1}{2}[\Pr ob(E|0) + \Pr ob(E|1)]$$

$$\begin{aligned}
\Pr ob(E|0) &= prob(|n| > \frac{A_p}{2}) \\
&= prob(n > \frac{A_p}{2}) + prob(n < -\frac{A_p}{2}) \\
&= 2[prob(n > \frac{A_p}{2})] \\
&= 2Q(\frac{A_p}{2\sigma_n})
\end{aligned}$$

$$\Pr ob(E|1) = prob(n < -\frac{A_p}{2}) \quad \textit{when positive pulse is used}$$

$$or \quad prob(n > \frac{A_p}{2}) \quad \textit{when negative pulse is used}$$

$$= Q(\frac{A_p}{2\sigma_n})$$

Now

$$\Pr ob(E) = \frac{1}{2}[\Pr ob(E|0) + \Pr ob(E|1)]$$

$$= \frac{1}{2}[2Q(\frac{A_p}{2\sigma_n}) + Q(\frac{A_p}{2\sigma_n})]$$

$$= \frac{3}{2}Q(\frac{A_p}{2\sigma_n})$$