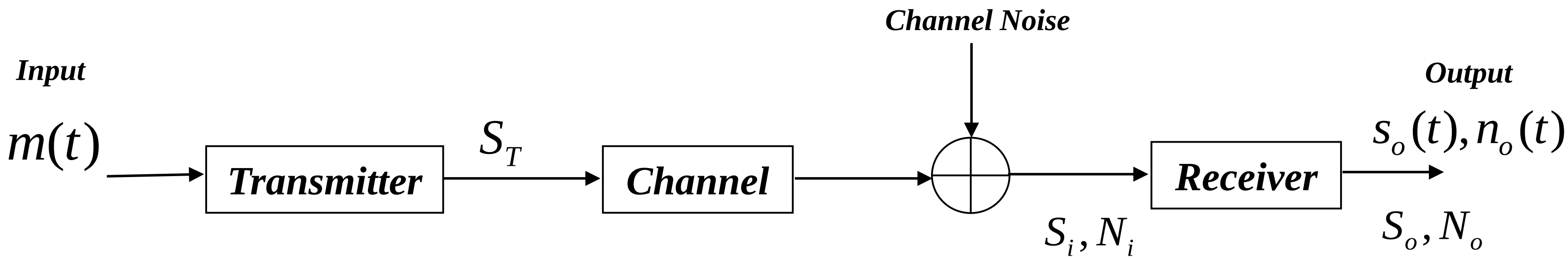
Impact of Noise on Communications Systems

Baseband System



$$S_o = S_i$$

$$N_o = 2\int_0^B S_n(f)\,df$$

$$= 2\int_0^B \frac{N}{2}\,df = NB$$

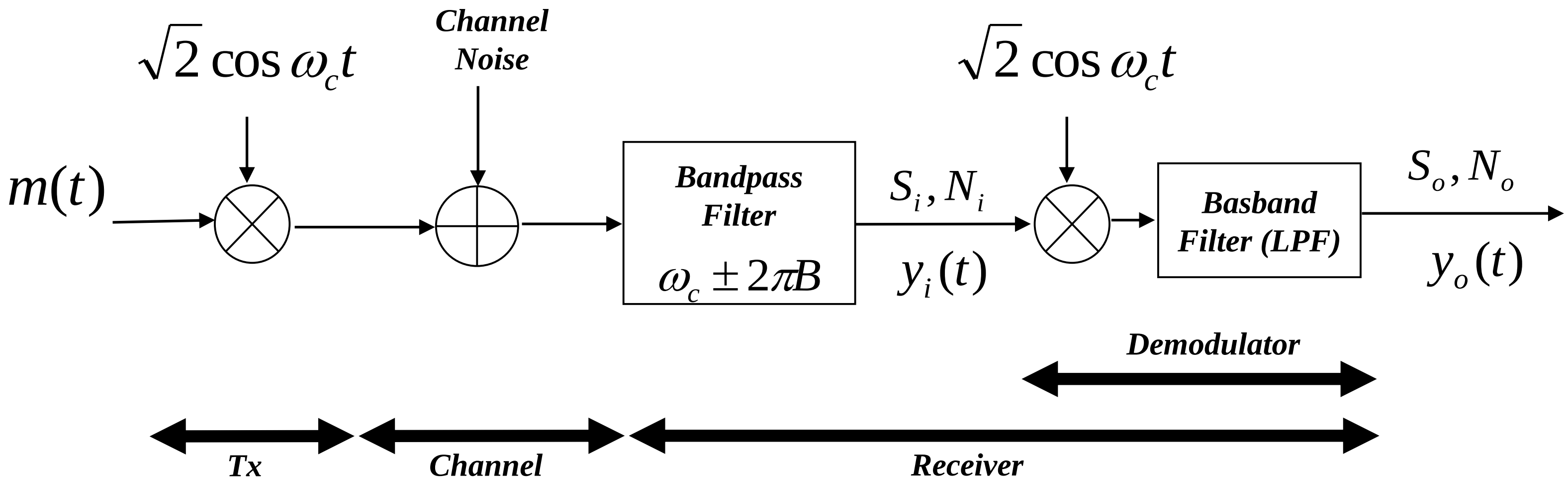
$$\frac{S_o}{N_o} = \frac{S_i}{NB}$$

Let $$\gamma = \frac{S_i}{NB}$$

Then $$\frac{S_o}{N_o} = \gamma$$

Note that $$S_o = \overline{m^2} = 2\int_0^B S_m(f)df$$

Impact of Noise on DSB-SC System



$$S_i = \overline{[\sqrt{2}m(t)\cos\omega_c t]^2}$$

$$= 2\overline{[m(t)\cos\omega_c t]^2}$$

$$= 2\frac{\overline{m(t)^2}}{2} = \overline{m(t)^2} = \overline{m^2}$$

To determine S_o, we start at demodulator input

$$y_i(t) = \sqrt{2}m(t)\cos\omega_c t + n(t)$$

We know that

$$n(t) = n_c(t)\cos\omega_c t + n_s(t)\sin\omega_c t$$

$$\Rightarrow y_i(t) = [\sqrt{2}m(t) + n_c(t)]\cos\omega_c t + n_s(t)\sin\omega_c t$$

Hence, at the output of demodulator

$$y_o(t) = m(t) + \frac{1}{\sqrt{2}}n_c(t)$$

$$y_o(t) = m(t) + \frac{1}{\sqrt{2}} n_c(t)$$

$$\Rightarrow S_o = \overline{m^2} = S_i$$

And

$$N_o = \frac{1}{2}\overline{n_c(t)^2}$$

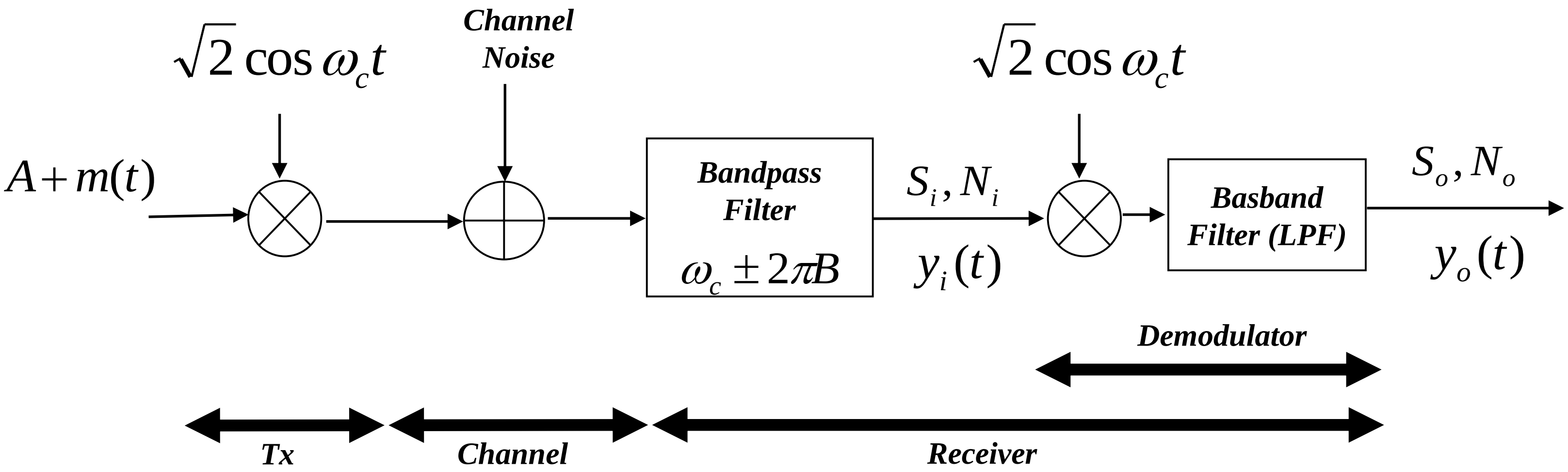
For white noise with PSD = N/2

$$\overline{n_c(t)^2} = \overline{n(t)^2} = 2NB$$

$$\Rightarrow N_o = \frac{1}{2} 2NB = NB$$

$$\Rightarrow \frac{S_o}{N_o} = \frac{S_i}{NB} = \gamma$$

Impact of Noise on AM System – Synchronous Demodulation



Please note that S_o and N_o will be same as in DSB-SC system

$$S_o = \overline{m^2}$$

$$N_o = NB$$

$$S_i = \overline{\left[\sqrt{2}[A+m(t)]\cos\omega_c t\right]^2}$$

$$= 2\overline{\left[[A+m(t)]\cos\omega_c t\right]^2}$$

$$= 2\frac{\overline{[A+m(t)]^2}}{2}$$

$$= \overline{[A+m(t)]^2}$$

$$= A^2 + \overline{m(t)^2} + 2A\overline{m(t)}$$

$$= A^2 + \overline{m(t)^2}$$

$$S_i = A^2 + \overline{m(t)^2}$$

$$S_o = \overline{m^2}$$

$$N_o = NB$$

$$\Rightarrow \frac{S_o}{N_o} = \frac{\overline{m^2}}{NB}$$

$$= \frac{\overline{m^2}}{A^2 + \overline{m^2}} \frac{S_i}{NB}$$

$$= \frac{\overline{m^2}}{A^2 + \overline{m^2}} \gamma$$

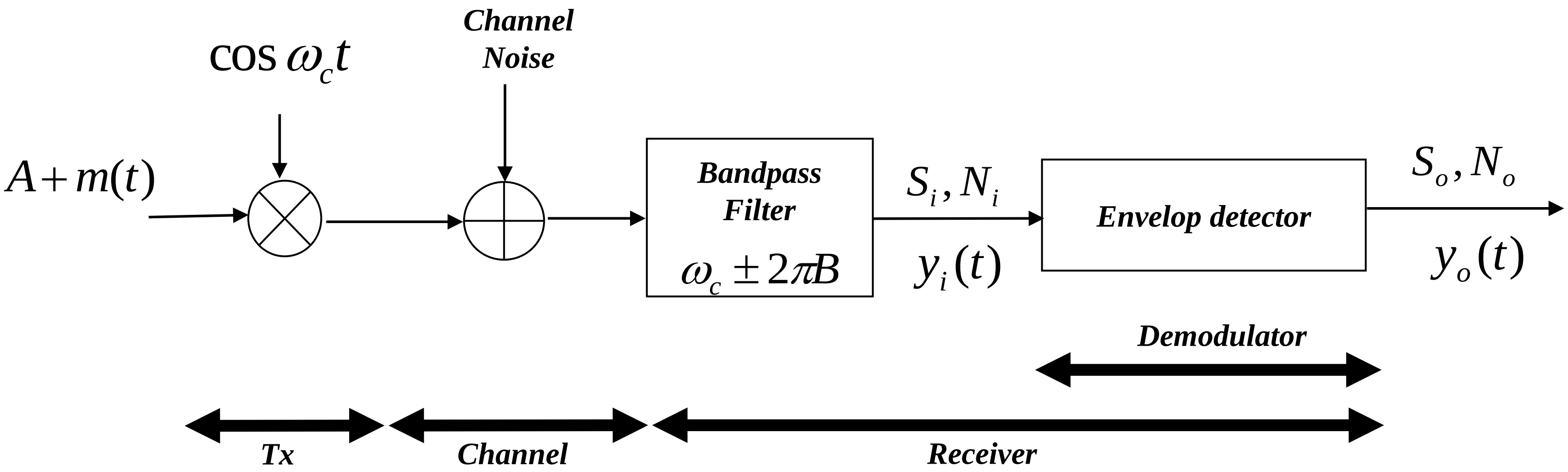
Remember $\boldsymbol{A \geq m_p}$

$$\Rightarrow \frac{S_o}{N_o} \leq \frac{\overline{m^2}}{m_p^2 + \overline{m^2}} \gamma$$

$$= \frac{1}{m_p^2 / \overline{m^2} + 1} \gamma$$

$$\Rightarrow \left(\frac{S_o}{N_o} \right)_{\max} = \frac{\gamma}{2}$$

Impact of Noise on AM System – Envelop Detection



$$s_i(t) = [A + m(t)]\cos\omega_c t$$

$$\Rightarrow S_i = \frac{\overline{[A+m(t)]^2}}{2} = \frac{A^2 + \overline{m^2}}{2}$$

$$y_i(t) = [A + m(t)]\cos\omega_c t + n(t)$$

$$= [A + m(t) + n_c(t)]\cos\omega_c t + n_s(t)\sin\omega_c t$$

Writing in envelop form

$$y_i(t) = E_i(t)\cos[\omega_c t + \Theta_i(t)]$$

Where

$$E_i(t) = \sqrt{[A + m(t) + n_c(t)]^2 + n_s(t)^2}$$

Small Noise Case:

if $[A+m(t)]>>n(t)$ ***for all t***

$$E_i(t)=\sqrt{[A+m(t)+n_c(t)]^2+n_s(t)^2}$$

$$\cong\sqrt{[A+m(t)+n_c(t)]^2}$$

$$\cong A+m(t)+n_c(t)$$

$$\Rightarrow S_o=\overline{m^2}$$

$$and \quad N_o=\overline{n_c(t)^2}=2NB$$

$$\Rightarrow \frac{S_o}{N_o}=\frac{\overline{m^2}}{2NB}=\frac{\overline{m^2}}{A^2+\overline{m^2}}\frac{S_i}{NB}=\frac{\overline{m^2}}{A^2+\overline{m^2}}\gamma$$

Large Noise Case:

if $[A+m(t)] << n(t)$ ***for all t***

$$E_i(t)=\sqrt{[A+m(t)+n_c(t)]^2+n_s(t)^2}$$

$$=\sqrt{[A+m(t)]^2+n_c(t)^2+2n_c(t)[A+m(t)]+n_s(t)^2}$$

$$\cong\sqrt{n_c(t)^2+2n_c(t)[A+m(t)]+n_s(t)^2}$$

$$=\sqrt{n_c(t)^2+n_s(t)^2+2n_c(t)[A+m(t)]}$$

$$E_i(t) = \sqrt{n_c(t)^2 + n_s(t)^2 + 2n_c(t)[A + m(t)]}$$

$$= E_n(t)\sqrt{1 + \frac{2[A + m(t)]}{E_n(t)}\cos\Theta_n(t)}$$

where

$$E_n(t) = \sqrt{n_c^2(t) + n_s^2(t)}$$

and

$$\Theta_n(t) = -\tan^{-1}\frac{n_s(t)}{n_c(t)}$$

$$\Rightarrow n_c(t) = E_n(t)\cos\Theta_n(t)$$

$$and \quad n_s(t) = E_n(t)\sin\Theta_n(t)$$

Further simplification

$$E_i(t) = E_n(t)\sqrt{1 + \frac{2[A+m(t)]}{E_n(t)}\cos\Theta_n(t)}$$

$$\cong E_n(t)\left[1 + \frac{[A+m(t)]}{E_n(t)}\cos\Theta_n(t)\right]$$

$$= E_n(t) + [A+m(t)]\cos\Theta_n(t)$$

For realistic case, SNR turns out to be

$$\frac{S_o}{N_o} = 0.916A^2\overline{m^2}\gamma^2$$

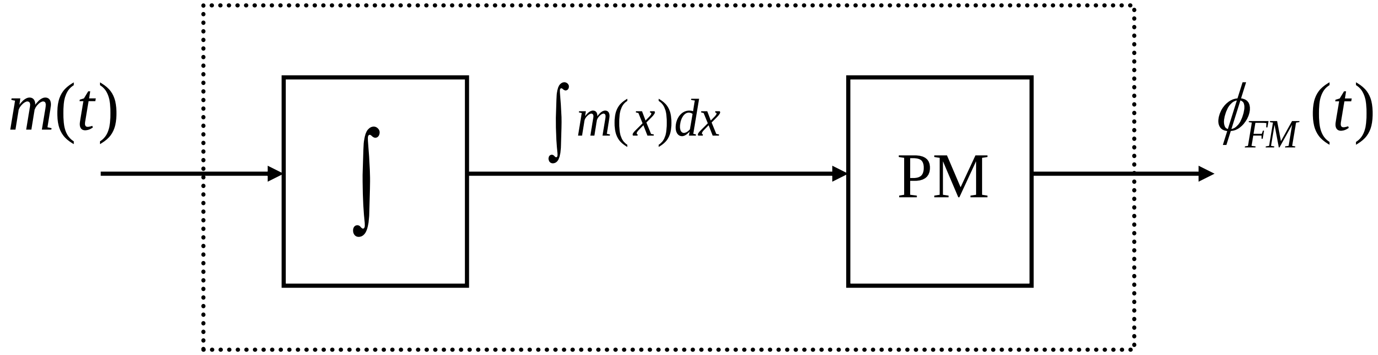
Impact of Noise on Angle Modulated Signals

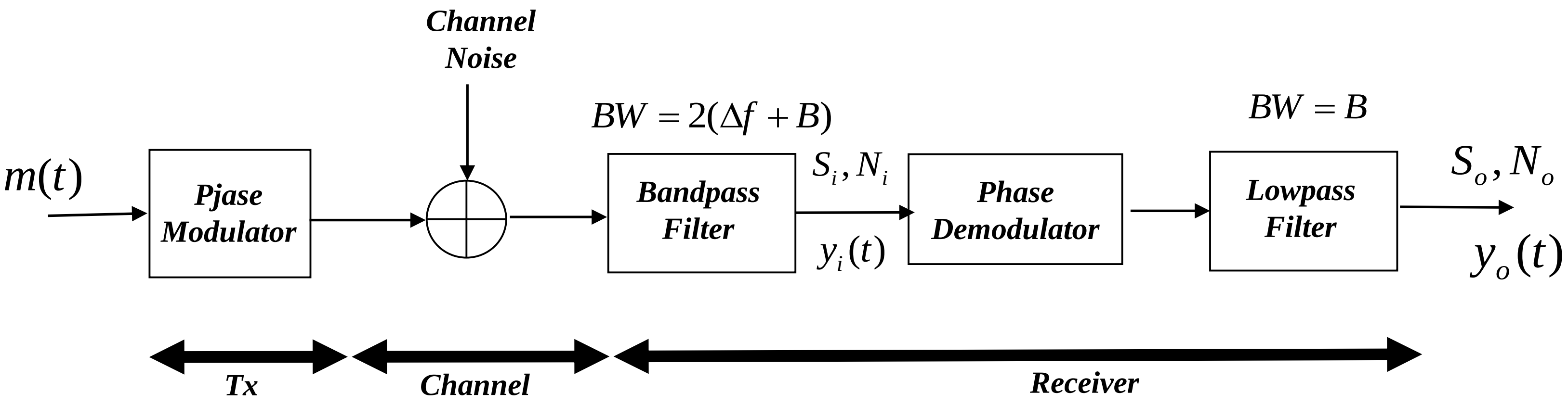
$$\phi_{EM}(t) = A\cos[\omega_c t + \psi(t)]$$

Where

$$\psi(t) = k_p m(t) \qquad \textbf{for PM}$$

$$\psi(t) = k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha \qquad \textbf{for FM}$$





At demodulator input:

$$s_i(t) = A\cos[\omega_c t + \psi(t)]$$

$$n(t) = n_c(t)\cos\omega_c t + n_s(t)\sin\omega_c t$$

$$= E_n(t)\cos[\omega_c t + \Theta_n(t)]$$

$$y_i(t) = s_i(t) + n(t)$$

PM – Wide Bandwidth Case

$$y_i(t) = A\cos[\omega_c t + \psi(t)] + n(t)$$

Where $\psi(t) = k_p m(t)$ ***for PM***

$$y_i(t) = A\cos[\omega_c t + \psi(t)] + E_n(t)\cos[\omega_c t + \Theta_n(t)]$$

$$= R(t)\cos[\omega_c t + \psi(t) + \Delta\psi(t)]$$

At demodulator output:

$$y_o(t) = \psi(t) + \Delta\psi(t)$$

$$= k_p m(t) + \Delta\psi(t)$$

By looking at the phasor diagram

$$\Rightarrow \sin[\Delta\psi(t)] = \frac{E_n(t)\sin[\Theta_n(t)-\psi(t)]}{R(t)}$$

For small noise case $\Delta\psi(t) << \pi/2$

$$\Delta\psi(t) \approx \frac{E_n(t)\sin[\Theta_n(t)-\psi(t)]}{R(t)}$$

$$\approx \frac{E_n(t)\sin[\Theta_n(t)-\psi(t)]}{A}$$

$$= \frac{E_n(t)}{A}\sin[\Theta_n(t)-\psi(t)]$$

At demodulator output:

$$y_o(t) = \psi(t) + \Delta\psi(t)$$

$$= k_p m(t) + \frac{E_n(t)}{A}\sin[\Theta_n(t) - \psi(t)]$$

BW of $\Theta_n(t)$ ***>> BW of*** $\psi(t)$

$$\Rightarrow \Delta\psi(t) \cong \frac{E_n(t)}{A}\sin[\Theta_n(t) - \psi]$$

$$= \frac{E_n(t)}{A}\sin\Theta_n(t)\cos\psi - \frac{E_n(t)}{A}\cos\Theta_n(t)\sin\psi$$

$$= \frac{n_s(t)}{A}\cos\psi - \frac{n_c(t)}{A}\sin\psi$$

$$\Delta\psi(t) = \frac{n_s(t)}{A}\cos\psi - \frac{n_c(t)}{A}\sin\psi$$

$$= \frac{\cos\psi}{A} n_s(t) - \frac{\sin\psi}{A} n_c(t)$$

$$\Rightarrow S_{\Delta\psi}(\omega) = \frac{\cos^2\psi}{A^2} S_{n_s}(\omega) + \frac{\sin^2\psi}{A^2} S_{n_c}(\omega)$$

$$= \frac{\cos^2\psi}{A^2} S_{n_s}(\omega) + \frac{\sin^2\psi}{A^2} S_{n_s}(\omega)$$

$$= \frac{S_{n_s}(\omega)}{A^2}[\cos^2\psi + \sin^2\psi] = \frac{S_{n_s}(\omega)}{A^2}$$

$$S_{\Delta\psi}(\omega) = \frac{S_{n_s}(\omega)}{A^2}$$

After the phase demodulator

$$\Rightarrow S_{\Delta\psi}(f) = \begin{cases} N/A^2 & |f| \le (\Delta f + B) \\ 0 & |f| > (\Delta f + B) \end{cases}$$

After the low pass filter

$$\Rightarrow S_{n_o}(f) = \begin{cases} N/A^2 & |f| \le B \\ 0 & |f| > B \end{cases}$$

$$\Rightarrow N_o = 2B\left(\frac{N}{A^2}\right) = \frac{2NB}{A^2}$$

$$N_o = \frac{2NB}{A^2}$$

Remember the starting point

$$y_o(t) = k_p m(t) + \frac{E_n(t)}{A}\sin[\Theta_n(t) - \psi(t)]$$

$$\Rightarrow S_o = k_p^2 \overline{m^2}$$

$$\Rightarrow \frac{S_o}{N_o} = \frac{k_p{}^2\overline{m^2}}{2NB/A^2} = (Ak_p)^2 \frac{\overline{m^2}}{2NB}$$

We know that

$$\gamma = \frac{S_i}{NB} = \frac{A^2/2}{NB} = \frac{A^2}{2NB}$$

$$\frac{S_o}{N_o} = (Ak_p)^2 \frac{\overline{m^2}}{2NB}$$

$$\gamma = \frac{A^2}{2NB}$$

$$\Rightarrow \frac{S_o}{N_o} = k_p^2 \overline{m^2} \gamma$$

Also for PM

$$\Delta\omega = k_p m'_p \Rightarrow \Delta\omega^2 = (k_p m'_p)^2$$ ***where*** $$m'_p = [\dot{m}(t)]_{\max}$$

$$\Rightarrow \frac{S_o}{N_o} = (\Delta\omega)^2 \left(\frac{\overline{m^2}}{m'^2_p} \right) \gamma$$

Example of Tone Modulation

$$m(t) = \alpha \cos \omega_m t$$

$$\overline{m^2} = \alpha^2 / 2$$

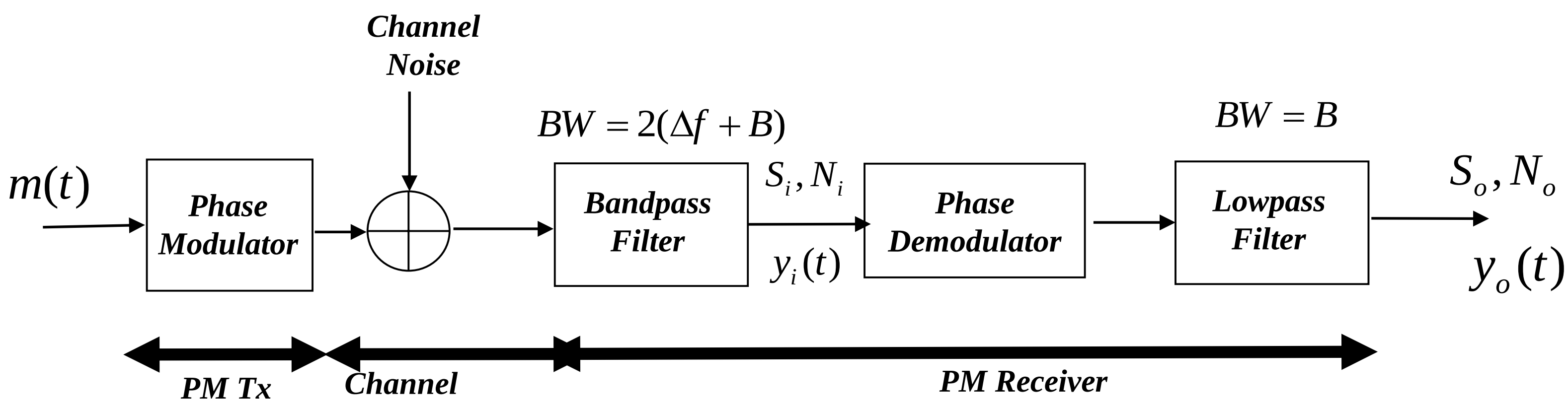
$$m'_p = \alpha \omega_m$$

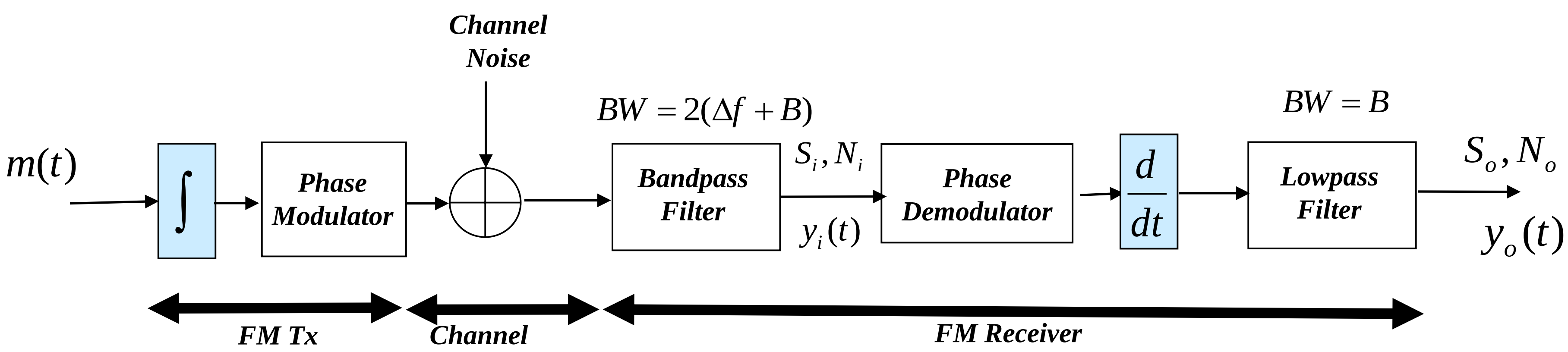
$$\frac{S_o}{N_o} = (\Delta\omega)^2 \left(\frac{\overline{m^2}}{m_p'^2} \right) \gamma = (\Delta\omega)^2 \frac{\alpha^2/2}{\alpha^2 \omega_m^2} \gamma$$

$$= \left(\frac{\Delta\omega}{\omega_m} \right)^2 \frac{\gamma}{2}$$

$$= \left(\frac{\Delta f}{f_m} \right)^2 \frac{\gamma}{2}$$

Impact of Noise on WBFM





$$\phi_{FM}(t) = A\cos[\omega_c t + \psi(t)]$$

Where

$$\psi(t) = k_f \int_{-\infty}^{t} m(\alpha)d\alpha$$

$$\Rightarrow S_o = k_f^2 \overline{m(t)^2} = k_f^2 \overline{m^2}$$

Noise Spectral density just before differentiator is

$$S_{\Delta\psi}(\omega) = \frac{S_{n_s}(\omega)}{A^2} = \frac{N}{A^2}$$

Noise Spectral density just before differentiator is

$$S_{\Delta\psi}(\omega) = \frac{S_{n_s}(\omega)}{A^2} = \frac{N}{A^2}$$

Noise Spectral density just after the differentiator is

$$S_{\Delta\psi}(\omega) = \frac{S_{n_s}(\omega)}{A^2} = \frac{N\omega^2}{A^2}$$

$$\Rightarrow N_o = 2\int_0^{2\pi B} \frac{N\omega^2}{A^2} d\omega$$

$$= 2\int_0^{B} \frac{N(2\pi f)^2}{A^2} df = \frac{8\pi^2 N B^3}{3A^2}$$

$$S_o = k_f^2 \overline{m^2}$$

$$N_o = \frac{8\pi^2 N B^3}{3A^2}$$

$$\frac{S_o}{N_o} = \frac{k_f^2 \overline{m^2}}{\frac{8\pi^2 N B^3}{3A^2}} = \frac{3A^2 k_f^2 \overline{m^2}}{8\pi^2 N B^3}$$

$$= 3\left(\frac{k_f^2 \overline{m^2}}{4\pi^2 B^2}\right)\left(\frac{A^2/2}{NB}\right)$$

$$= 3\left(\frac{k_f^2 \overline{m^2}}{(2\pi B)^2}\right)\gamma$$

$$\frac{S_o}{N_o} = 3\left(\frac{k_f^2 \overline{m^2}}{(2\pi B)^2}\right)\gamma$$

$$= 3\left(\frac{k_f^2 m_p^2 \overline{m^2}}{(2\pi B)^2 m_p^2}\right)\gamma$$

$$= 3\left(\frac{\Delta\omega^2}{(2\pi B)^2}\right)\left(\frac{\overline{m^2}}{m_p^2}\right)\gamma$$

$$= 3\left(\frac{\Delta f}{B}\right)^2\left(\frac{\overline{m^2}}{m_p^2}\right)\gamma = 3\beta^2\gamma\left(\frac{\overline{m^2}}{m_p^2}\right)$$

Example of Tone Modulation

$$m(t) = \alpha \cos \omega_m t$$

$$\overline{m^2} = \alpha^2 / 2$$

$$m_p = \alpha$$

$$\Rightarrow \frac{\overline{m^2}}{m_p^2} = 0.5$$

$$\Rightarrow \frac{S_o}{N_o} = 3\beta^2\gamma(0.5)$$

$$= \frac{3}{2}\beta^2\gamma$$

Comparison of SNR for WBPM and WBFM

$$\left(\frac{S_o}{N_o}\right)_{FM} = 3\left(\frac{\Delta f}{B}\right)^2\left(\frac{\overline{m^2}}{m_p^2}\right)\gamma$$

$$\left(\frac{S_o}{N_o}\right)_{PM} = (\Delta\omega)^2\left(\frac{\overline{m^2}}{m_p'^2}\right)\gamma = (2\pi\Delta f\,)^2\left(\frac{\overline{m^2}}{m_p'^2}\right)\gamma$$

$$\left(\frac{S_o}{N_o}\right)_{PM} \Big/ \left(\frac{S_o}{N_o}\right)_{FM} = (2\pi\Delta f\,)^2\left(\frac{\overline{m^2}}{m_p'^2}\right)\gamma \Big/ 3\left(\frac{\Delta f}{B}\right)^2\left(\frac{\overline{m^2}}{m_p^2}\right)\gamma$$

$$\frac{\left(\frac{S_o}{N_o}\right)_{PM}}{\left(\frac{S_o}{N_o}\right)_{FM}} = \frac{(2\pi\Delta f)^2\left(\frac{\overline{m^2}}{m_p'^2}\right)\gamma}{3\left(\frac{\Delta f}{B}\right)^2\left(\frac{\overline{m^2}}{m_p^2}\right)\gamma}$$

$$= \frac{4\pi^2 B^2 m_p^2}{3m_p'^2}$$