Proceedings of the Conference on
Promoting Undergraduate Research in Mathematics

Undergraduate Research in Mathematics at the University of Akron

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1. The local setting

The University of Akron (UA) is a state-assisted institution. About 95% of our approximately 23,000 students come from Ohio, a state where relatively few adults have a college degree. The rest come from 41 other states and 68 foreign countries. Because UA has open enrollment, we will always fare poorly on many of the traditional measures that go into college rankings. Thus, to anyone who is too focused on bean-counting, we appear to be a far from ideal climate for conducting undergraduate research.

But despite what the averages say, we have many fine students. They come to us via a rapidly growing honors program, a large college of engineering, or other routes. For example, one of our top mathematics students of recent years came to Akron because of our dance program.

2. Early history

Intentionally or not, Judith Palagallo and Thomas Price founded our undergraduate research program. In each of 1998 and 1999, Palagallo invited one student to join her on a research project. The net results were an honors paper; presentations at the AMS/MAA Joint Meetings in 1999, the MAA MathFest in 2001, and at the Nebraska Conference for Undergraduate Women in Mathematics (2002); and two publications [4, 6].

The pace of activity accelerated in 2000, when Palagallo and Price conducted a summer undergraduate research program with local funding. This resulted in two MathFest presentations and one publication [9].

At the same time, we started encouraging some of our students to attend REU programs elsewhere. This was the start of a trend, and in recent years eight of our students (seven women, one man) have participated in eight different external REU programs, sometimes continuing their projects upon their return to Akron. So far, this effort has resulted in an article [7] and a technical report [2].

In the summer of 2002, Price established the Fibonacci Forum, which involved eight undergraduates over several years. One of the students was a McNair Scholar.

Received by the editor October 25, 2006.

When Palagallo and Price went on sabbatical, the Forum continued under the direction of one of the students. This activity led to several honors projects; three presentations at MAA meetings; and an article [8] that has been submitted for publication.

In the summer of 2003, our (still informal) program attracted a McNair Scholar from a nearby university. She worked with Palagallo, and presented her results at the 2003 MAA MathFest, winning a Best Presentation award.

Where are they now? Of the student participants in the activities described above, one has completed a Ph.D., and is now an assistant professor of mathematics. Another received an NSF Graduate Fellowship, and is expected to finish her Ph.D. in May. Our internal McNair Scholar is in graduate school in statistics. Five other students, including our external McNair Scholar, are presently in Ph.D. programs in mathematics or physics. (One of these was an invited graduate student participant at the 2006 Nebraska Conference for Undergraduate Women in Mathematics.) Two completed master's degrees in mathematics at UA, and now work as a mathematical modeler and a public school teacher. One spent a year working for the NSA, and recently completed a law degree. One is a college senior.

3. Formal REU program

Starting in the summer of 2005, we have operated an eight-week REU program with major support from the National Science Foundation, and additional support from our unit within UA, the Buchtel College of Arts and Sciences. Palagallo, Price, Jeffrey Riedl, and I run the show. In the summer of 2006, we used local funds to employ a graduate assistant. Not only could she help with clerical tasks, but her master's research was related to one of our summer research problems.

Each year, we recruit around eight students from around the country, including one local student. We received 131 applications the first year. The second year, partly in order to decrease this, we deliberately presented fewer sample projects on our web page. It worked: We received only about 100 applications.

Community. The students all live together in campus dorms, together with the students in UA's summer REU in polymer science. We organize a few social events and field trips, and the polymer science students join us for one or two of these. For example, during the first week, we attend a game of the Akron Aeros, our local AA baseball team. Later in the summer, we hear a concert at Blossom Music Center, the summer home of the Cleveland Orchestra. The students also tend to find plenty of social outlets on their own. It helps that one of the students is local.

We take a few field trips to local industrial concerns and research laboratories.

Since we want all of our students to understand each other's research problems, during the first week all attend all introductory lectures. Once a week, we require all students to give progress reports. This is followed by a communal lunch.

Visitors. We typically have four or five visiting faculty, who speak on a variety of topics in both pure and applied mathematics. (Examples: wavelets; coatings of nanofibers; Dirichlet's Theorem; working for the NSA; the Seven-Color Theorem; p-adic numbers; etc.) In addition to mathematical talks, we provide an introduction to LaTeX (sometimes in the form of Scientific Workplace), and discuss how to give a technical presentation.

Finances. In addition to free lodging, the students receive a stipend, food allowance, travel reimbursement, and a parking pass if necessary. When the travel budget allows (which so far it always has), we provide partial support to attend the Joint Meetings.

Problems.

- Gessel [5] proved the curious (to me) result that a natural number x is a Fibonacci number if and only if $x^2 \pm 4$ is a perfect square. More specifically, if F_n and L_n denote the nth Fibonacci and Lucas numbers, respectively, then the only integral solutions to
$$x^2 + (-1)^n 4 = y^2$$
are $(\pm F_n, \pm L_n)$. The first 2005 team proved an analogous theorem for generalized Fibonacci and Lucas numbers, as well as Fibonacci and Lucas polynomials.
- In order to prove their results, the first team required knowledge of the units in certain quadratic extensions of polynomial and other rings. Some of this was provided by the second team.
- A third 2005 team found the method used to encrypt several sixteenth-century letters to the king of Spain from some of his ambassadors in Italy. This result is of some historical interest, because our collection is only a small part of a larger one in the State Archive at Simancas, Spain. The task was complicated by the fact that, in the words of the students, "the documents are over four hundred years old, the ink has leaked through, and the handwriting finds its closest modern parallels on prescriptions."
- A 2006 team generalized a fundamental theorem of Bandt [3], concerning the construction self-similar periodic tilings, from Euclidean space to what according to Strichartz [10] should be its natural setting: nilpotent Lie groups having certain rationality properties. One reason for interest in such tilings is their connection with wavelets.
- Riedl has been able to boil down one of his own research problems, concerning the classification of monolithic subgroups of wreath product p-groups, into a combinatorial linear algebra problem. Seven students (spanning 2005 and 2006) collectively solved this problem. Since it's part of a much bigger problem, there is plenty left to do in 2007.

Rough assessment. All of the problems that we posed have been solved, sometimes in greater generality than we had expected. Three 2005 teams made well-received presentations at the 2006 AMS-MAA Joint Meetings, and I expect at least one 2006 team to present in January, 2007. One paper [1] on the 2005 results has been accepted pending revision, another is in preparation, and we expect to prepare a paper on the 2006 results in the spring.

Should all of these papers appear, this will make our program look good. However, there is always a potential conflict between the goals of producing the best research possible and producing the best educational experience possible. The latter is more important, but its degree of attainment is harder to measure.

Rigorous assessment. This must involve long-term tracking of our students. However, after only two summers, it is too early for that. So far, we must rely on anonymous student evaluations. But these have been positive, and have included

specific suggestions for adjustments. Many students reported a reinforced desire to attend graduate school in mathematics. None reported a decline in interest.

Two students reported having burned out on mathematics between their acceptance of our offer and their arrival in Akron. One is still in mathematics. In time, we will know the extent of our role in that.

Where are they now? The 2006 participates are back in school, of course, and the majority of them are applying to graduate school in mathematics. One intends to teach middle school. Of the 2005 participants who are still in college, one worked for the NSA last summer, and one attended another REU (and gave us lots of feedback!). Both intend to pursue graduate studies in mathematics. Of those who have graduated, two are in Ph.D. programs in mathematics, one works for the NSA, one is an actuary, one is applying to math graduate school after a year abroad, and one is preparing for law school.

4. Future

In the immediate future, our program will expand. First, several of our colleagues in applied math will get involved using their own grant money, thus increasing the number of both students and faculty involved. Second, we will bring in outside faculty mentors (one a professor at a nearby college, the other a former student of ours who will have just received her Ph.D.).

We will seek to work even more closely with UA's polymer science REU. For example, it makes sense for us to offer a joint GRE preparation workshop.

In the longer term, the nature of our program will depend on our ability to obtain continuing external financial support, and our ability to find replacements for faculty members who have retired recently. Among these are Palagallo and Price, although they will be involved for at least one more summer.

References

[1] J. Adler, R. Fuoss, M. Levin, and A. Youell, *Reading encrypted diplomatic correspondence: an undergraduate research project* (submitted).
[2] I. Averill and J. Gregoire, *Tilings of low-genus surfaces by quadrilaterals*, Technical Report 02-13, Rose-Hulman Mathematical Sciences Technical Report Series, 2002.
[3] C. Bandt, *Self-similar sets 5. Integer matrices and fractal tilings of* $\mathbb{R}^n$, Proc. Amer. Math. Soc. **112** (1991), 549–562.
[4] M. Breen and J. Palagallo, *Determining the area of fractal tilings*, Pi Mu Epsilon Journal (Spring, 2003).
[5] I. Gessel, *Problem H-187*, The Fibonacci Quarterly **10** (October 1972), 417–419.
[6] S. Hagey and J. Palagallo, *Complex bases and fractal tilings*, Math. Gazette **85** (2001), 194–201.
[7] B. Marko and J. Riedl, *Igusa local zeta function of the polynomial* $f(x) = x_1^m + \cdots + x_n^m$, Yokohama Math. J. **51** (2005), 117–133.
[8] L. McDonnell, B. Polovick, and T. Price, *Evaluation formulas for a family of integrals* (submitted).
[9] J. Palagallo and M. Palmer, *Analysis of an irregular Sierpinski triangle*, Fractals **8** (2004), 137–144.
[10] R. S. Strichartz, *Self-similarity on nilpotent Lie groups* (1992), 123–157.

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