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Summer Mathematics Research Experience for Undergraduates (REU) at Brigham Young University

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1. Introduction

Brigham Young University (BYU) hosts an NSF-funded 8-week summer Research Experience for Undergraduates (REU) in geometric analysis and mathematical physics. The objectives of our program are: (1) to provide undergraduate students with the opportunity to experience doing mathematical research; (2) to encourage undergraduate students to attend graduate school in mathematics; and (3) to prepare participants to be successful in graduate school. We are specifically interested in recruiting female students and students who are from institutions without a graduate program in mathematics. Our experience with undergraduate research in mathematics has shown us that students are effective in learning about mathematics and in doing mathematical research if they are working on challenging problems in a supportive but structured environment. Hence, we offer many components in our program. To help students experience the process and excitement of doing research, students are first given a series of introductory lectures. Then they work individually and as a group on challenging research problems with faculty who have been successful in mentoring undergraduates. At the end of the REU, students present a research talk and write a final research report. Also, we want to provide the students with skills that will help them in their research and in graduate school. To help achieve this, we offer training sessions in computers, seminars on needed research skills, and discussions on graduate school. Further, we provide social and recreational activities that foster interaction and collaboration in a relaxing environment. All of this is done to ensure that the students have a meaningful research experience and develop the skills to help them be successful in graduate school.

2. The BYU summer REU

2.1. The research groups. During the BYU summer REU, there are two research groups each consisting of 4-5 undergraduate students, 1 faculty mentor, and 1 graduate student assistant. Each research group is further divided into two subgroups of 2-3 undergraduates; the subgroups work on a specific problem related

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to their main research area. There are two research topics selected from geometric optimization, minimal surfaces, and mathematical physics. The type of research problems the REU participants work on are graduate level mathematics.

In each research group, there will be a common overall problem or method. The problem will be divided into several components with each student working on a specific component. This setup has been effective in our undergraduate research groups – it fosters both collaboration and individual work. Because there is one common problem or method in each research group, students will naturally discuss their work with each other. However, having each student working on a specific component of the problem will provide each student with specific assignments to do.

2.2. Sample research problems.

Geometric Optimization: The basic problem in geometric optimization is to minimize length, area, or some other quantity, among curves or surfaces satisfying a given constraint. A well-known example is that a circle has least perimeter among curves enclosing a given area. Some problems that students have considered are: (1) the octahedron problem which asks what is the least area surface spanning the edges of the octahedron. The conjectured solution is a beautiful, piecewise-planar soap film consisting of twelve triangles and six kites; (2) large minimizing networks in the hyperbolic plane; that is finding the shortest path connecting a set of points in the hyperbolic plane; and (3) Melzak's conjecture which asks what polyhedron with unit volume in Euclidean 3-space has the shortest edgelength (for example, a cube with volume 1 has total edge length 12; there is a polyhedron that has a shorter edgelength–can you figure it out?).

Minimal Surfaces: At each point p on a regular surface $M \in \mathbb{R}^3$, we can compute a normal vector n. Any plane that contains n will intersect the surface in a curve c. For each curve c, we can compute its curvature. As we rotate the plane through the normal n, we will get a set of curves on the surface each of which has a value for its curvature. Let k_1 and k_2 be the maximum and minimum curvature values at p, respectively. The mean curvature of M at p is $H = (k_1 + k_2)/2$. Then M is a minimal surface if the mean curvature equals zero at every point. We can use ideas from complex analysis to investigate minimal surfaces. In particular: (1) we can use the Schwarz-Christoffel formula from complex analysis to derive analytic functions that map onto convex polygonal regions. We can then shear these functions and derive the corresponding minimal surfaces and see how they are related to Jenkins-Serrin minimal surfaces which project to convex polygons; (2) we can shear a specific elliptic integrals of the first kind to get a family of minimal surfaces that range from Scherk's doubly-periodic to the helicoid. What families of surfaces do we get, when we shear other elliptic integrals?; and (3) we can represent minimal surfaces with specific properties as solutions to a system of DE's. This system of DE's can be described as a manifold in $\mathbb{R}^n$. By computing its Lie symmetries with the help of Maple or Mathematica we get a continuous family of minimal surfaces that have the origianl specific property.

2.3. The schedule. The mathematical component of the REU can be somewhat divided into two parts. The first part occurs during the initial weeks and consists of activities to help the students be prepared to do research, such as introductory lectures on the research topic and problem sets related to the research lectures. Also, there are training sessions in computers (e.g., LaTeX, Maple, and Mathematica) and seminars on needed research skills (e.g., "How to read a math paper," "How to write a math paper," "How to give a math talk," "Available research tools on the web and in the library"). In addition, we start having discussions on graduate school (e.g., "Common misconceptions about graduate school in mathematics," "How to choose a graduate program," "What you should do now to prepare for graduate school.").

The second part begins after the students have learned enough to begin to do actual research (near the end of the second week). At this point, the formal introductory lectures transition into discussions of research problems and assignments of components that the students can work on. During this time, there are also afternoon colloquia on other areas of mathematics so to broaden the students' exposure to mathematics and presentations on graduate school. Some of these presentations are given by BYU mathematicians. Others are given by non-BYU female mathematicians whom we invite to the REU to not only give research presentations, but to also interact with the REU students and to answer their questions about graduate school. These colloquia have included such areas as geometric group theory, algebra, graph theory, mathematical biology, number theory, and algebraic geometry.

Also, we believe that social and outdoor activities are beneficial to the students participating in the REU. These activities are not only enjoyable, but also help the students to interact and promote collaboration. We arrange barbeques about every other week, several local hikes to waterfalls and springs in the nearby mountains, and one major weekend trip. In 2006, we organized an excursion to Arches National Park followed by a day-long white-water rafting trip.

2.4. Research presentation and research paper. Near the end of the REU, the students give a formal presentations about their research results. Giving research presentations is a skill that can be improved with practice. We give a list of positive aspects of their talks plus suggestions on how to improve specific aspects of it. Our experience is that this helps them to be better prepared to give a presentation at a regional or national conference, an activity that we feel is very beneficial for them as future mathematicians. We have found that undergraduates are excited about attending and participating in conferences. This is an important experience for them and we provide funds for the REU students to attend and present their research at such a conference. Also, we require the students to submit a final written report. Ideally, this will form the basis of a research publication for the students.

3. Recruitment and logistics

Students are recruited nationally through various ways. First, we mail a flyer announcing our program to mathematics departments at institutions in the local region. Also, we maintain a webpage (http://math.byu.edu/reu) that includes a flyer announcing the program, suggestions for background information about research topics, information about the previous programs, photos, and students' comments and information about applying for the program. Further, the proposed REU program is listed in the AMS and NSF listings of Summer REU programs. Recruitment has been successful. For the 2006 REU we received about 100 applications. Originally, NSF provided funds for 8 undergraduates, at least 6 of whom are non-BYU students, to participate in the BYU summer REU. Then in 2006 BYU agreed to separately fund a 9th student who is not from BYU.

Participants are given a stipend of $2750, up to $450 for travel to BYU, free housing in off-campus apartments during the 8-week program, and $400 travel allowance to attend a conference during the following academic year to present their research.

4. Project Evaluation and Reporting

Our evaluation plan has several components: (1) papers authored or co-authored by undergraduate participants; (2) presentations by participants at conferences; (3) percentage of participants who attend graduate school in mathematics; and (4) multi-stage surveys.

The multi-stage surveys are based upon the ideas of the statisticians Adhikari and Nolan[1] concerning the assessing of student learning in undergraduate research projects. The questionnaire contains both open-ended questions and questions that ask for a numerical rating. It is important to have a baseline survey that gives us an accurate reading of where the students are starting from and a way to measure long-term impact. Hence, we administer the questionnaire during the REU and one year later. Also, we send a brief survey to the faculty members who wrote letters of recommendation for the participants asking these faculty members to assess the effect the REU program had on the student's work in mathematics and their plans for attending graduate school.

In 2005, the first year of the BYU summer REU, there were 8 undergraduates (6 female) who participated. Before attending the REU, 36% of the participants were planning on attending graduate school in mathematics. After attending the 2005 REU, 100% of the participants had either applied to at least one graduate program in mathematics for next fall or are juniors who are planning on applying during their senior year. Also, 100% of the participants have presented their research at a mathematics conference. Last year, one group published their research result in the Illinois Journal of Math, in which mathematicians from research institutions publish.

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[1]A. Adhikari and D. Nolan, "But what good came of it at last?"–How to assess the value of undergraduate research, *Notices of the AMS*, **49** no. 10 (2002), 1252-1257.