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Research Classes at Gettysburg College

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Introduction. I have been supervising undergraduate research projects in mathematics for over fifteen years. Three years ago I had the opportunity to formalize this (and give, as well as receive, course credit) by introducing three levels of research courses at Gettysburg:

Math 201: *Introduction to Research in Mathematics*,

Math 301: *Intermediate Research in Mathematics*, and

Math 401: *Advanced Research in Mathematics*.

Introduction to Research in Mathematics has no course prerequisites, and is usually taken by students whose career plans do not involve mathematics, who but like the subject and have a desire to take the course as an elective. *Intermediate Research in Mathematics* has Math 215 (*Abstract Mathematics I*) as a prerequisite; it is taken mostly by students majoring or minoring in mathematics. Finally, *Advanced Research in Mathematics* is taken by math majors who already took Math 301 and intend to base their capstone experience in mathematics (a requirement for all majors) on their research project. Roughly speaking, Math 201 expects students to learn new material and make discoveries; Math 301, in addition, asks them to provide proofs for their conjectures; and Math 401 focuses on writing a professional paper as well.

First we offered these courses once a year; last year, we began to offer each course every semester. However, due to some obvious reasons (e.g. not having sufficient faculty resources), we offer the courses in "one-room-schoolhouse"-style: students are together in the same class. The combined enrollment in the three classes usually ranges from fifteen to twenty.

I have been largely satisfied with these courses and, more importantly, the students seem to be as well. Below I attempt to summarize my experiences and contrast our program with more traditional undergraduate mathematics research programs.

The sums. Each research class is really the sum of three classes, and each class is, in essence, the sum of several independent research courses. I truly believe that what we have is greater than the sum of the parts. However, special care needs to be taken when coming up with research projects.

An ideal research project needs to be

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- based on substantial topics – students should be engaged in the study of non-trivial and not-too-esoteric mathematics;
- challenging at a variety of different levels – students with different backgrounds and interests will be able to engage in each project;
- approachable with a variety of different methods – students interested in theoretical, computational, abstract, or concrete work will be able to choose their own approaches;
- incrementally attainable, where at least partial results are within reach yet complete solutions are not easy – no student would want to spend long hours of hard work and not feel productive; and
- new and unsolved – the results attained by the students would have to be, at least in theory, publishable.

During the past thirteen years, I have had an opportunity to work with a very large number of research students (perhaps as many as one hundred students). Coming up with so many research projects is not easy. The topics of the research projects have come mainly from my own research interests in algebraic and geometric combinatorics, finite set theory, and additive and combinatorial number theory – but they also included questions from probability, statistics, computer networks, game theory, and quite a few other topics. I believe that my carefully selected projects yielded meaningful and enjoyable research opportunities for many students.

The differences. Our program structure is rather unusual and thus requires unusual approaches. Some of the main differences are the following.

- Our courses run parallel to other classes that students take. This gives them a somewhat longer time frame (fourteen weeks), but prevents them from truly immersing themselves in their research. (Nevertheless, students report spending a very large number of hours with the class – occasionally in excess of twenty hours a week.)
- The courses are taught together, and therefore the students in the same room have a very diverse set of objectives, interests, backgrounds, and talents. This gives special challenges for the instructor.
- Since the research classes yield course credit in mathematics, an important feature needs to be that students learn a substantial amount of new mathematics. Therefore, especially during the first month or so during the semester, I lecture on topics which are typically beyond regular courses.
- Students receive a grade in the course. Since research results can be unpredictable, most components of the grade need to come from effort, proficiency in learning of new material, frequency and level of class presentations, and willingness and ability to participate in the projects of other students.

I believe that these differences, while necessitating certain limitations on the research, provide unique opportunities for a diverse group of students.

The products. The primary products of the courses are the research results achieved by the students. These results must be communicated both verbally and in writing. Therefore, I ask each student to

- give weekly presentations on his or her progress;
- give a final formal presentation on the research – this presentation is open to the public;

- write a formal research paper – these papers appear in the proceedings of the course entitled *Research Projects in Mathematics*, published locally at the end of the course.

In addition, each student is encouraged to

- give an off-campus presentation on his or her research;
- submit (an improved version) of his or her paper to a refereed journal.

We have had numerous students give presentations at conferences, including the national meetings of the American Mathematical Society, sectional meetings of the Mathematical Association of America, the Nebraska Conference for Undergraduate Women in Mathematics, the Rose-Hulman Undergraduate Mathematics Conference, and the Moravian College Mathematics Conference. Two students also gave talks at a research conference where they were the only undergraduates present. Additionally, two students gave a poster presentation at the annual meeting of the American Mathematical Society; their work won Honorable Mention.

Although I strongly encourage students to revise their paper so that it can be submitted to a refereed journal, no student has been able to do this so far. Even if they don't win awards or get applauded after giving a successful presentation, it is very important to give recognition to every student who performed quality research.

The quotients. I have not carried out a systematic assessment of these classes; to be honest, I am not entirely sure how to perform such an assessment. For now, here are some statistics on the students so far.

- about 1 out of 25 students at Gettysburg takes a research course in mathematics, including about a third of those minoring and a half of those majoring in mathematics;
- about 4 out of 5 students generate new research results that can be considered non-trivial (although, in all honesty, most results can be expected from a talented and motivated student);
- about 1 out of 3 students receives an A in the course, and about 3 out of 4 students receive a B or higher;
- about 3 out of 4 students rate the course as "excellent"; all have rated the course as "very good" or "excellent";
- about 2 out of 3 students who did not originally intend to major or minor in math decide to do so after having taken the course.

I intend to develop appropriate assessment tools and evaluation criteria for these courses.

The powers. Our corridor in front of the Department has a huge poster with a fist and the sign "Math is power", and I believe that few courses empower students as much as a research course. In addition to learning about exciting, current, and lively topics, students have the opportunity to perfect a wide variety of skills from dealing with setbacks to presenting their accomplishments.

Ways in which research activities benefit students with a serious interest in mathematics – especially those intending to go to graduate school – have been described by others. But I feel strongly that every student should engage in an undergraduate research experience. Whether they will go on to graduate school, enroll in professional studies, or take jobs in education, government, or industry,

students will benefit from the opportunities for perfecting a variety of skills that a research experience provides.

An invitation. I am looking forward to hearing anyone's comments on this article whether before, during, or after the conference. It would be particularly interesting to hear from colleagues who are engaged in similar mathematical research experiences at other institutions.

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