Proceedings of the Conference on
Promoting Undergraduate Research in Mathematics

The NSF REU at Central Michigan University

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1. Introduction

The summer NSF REU at Central Michigan University is directed by Sivaram Narayan and Ken Smith, focusing on research topics in matrix analysis, graph theory and algebraic combinatorics.

Narayan and Smith have been invloved in undergraduate research in mathematics since the early 1990s, originally working with strong honors students from Central Michigan University (CMU). During the 1990s, Narayan worked individually with students in an independent study format and began seeking internal and external funding for his students. Initially Narayan received support from the Dean of the College of Science and Technology at CMU to support the students during summer projects. Beginning in 1998, some of his students received funding through the CMU Summer Research Scholarship program. Meanwhile, in the summer of 1992, Smith worked with undergraduate students at the Director's Summer Program at the National Security Agency in Maryland. This was an exciting experience for him and he returned to Central Michigan eager to work with undergraduate math majors in research projects in algebra and combinatorics. He also guided honors projects and undergraduate independent studies.

In 2000, Narayan wrote a grant application to the NSF for an REU site and received funding for the summer of 2002. During that eight week summer program, Narayan, Smith and a colleague, Yury Ionin, worked with six students supported by the NSF and five students supported by Central Michigan – three faculty working on five projects with 11 students!

Since 2002, CMU has been funded each summer to run an NSF-REU site. (Our current grant runs through 2008.) Each summer, eight students are selected to participate in the program with three faculty mentors. The NSF-supported students are joined by two to four students who are supported by CMU. The students work on research problems in teams of two or three. Each faculty mentor supervises one or two research teams. The program begins on the first or second Monday in June and runs for eight weeks.

Recently CMU has partnered with Coppin State University, the University of Richmond and Olin College to create the LURE (Longterm Undergraduate Research Experience) program, funded by the NSF. In that program, each institution

Received by the editor November 28, 2006.

will work with their own students in two-year projects. (The LURE program is described more fully in a separate article.)

2. NSF-REU site program

The goal of the CMU REU program is to stimulate talented undergraduate students to pursue graduate work in science and mathematics by providing accessible, challenging, and unsolved research problems in mathematics. Emphasis is placed on learning new mathematics, the excitement of discovery, and interactions with other students and faculty mentors. Communication of ideas and results are emphasized through oral presentations and written reports.

In summer 2002 Narayan and Smith and Ionin worked with eleven students on five projects. Lisa DeMeyer, Narayan and Smith mentored thirteen students on five research projects in summer 2003. Ionin, Narayan, and Smith mentored twelve students on four research projects during summer 2004. During the summer of 2005, fifteen students were mentored by DeMeyer, Narayan and Smith on six research projects. This past summer, Narayan, Smith and Boris Bekker mentored ten students on five projects. Over five summers, our program has worked with 61 students (28 men, 33 women). Thirty-eight students (18 men, 20 women) have been supported by the NSF, 21 have been supported by CMU and two have been supported by another institution. The NSF students come from a variety of institutions, some large (Berkeley, Rutgers, Yale, Princeton, Michigan, Arizona State, Kansas, Georgia, Nebraska) and others small and private (Susquehanna University, Ashland University, Bates College, Grinnell College, Providence College, Oberlin College.) We have deliberately mixed students from small colleges with those from larger schools. We have also worked to take relatively new and inexperienced students (freshmen, sophomores) and mix them with one or two students who have previously been to an REU.

Central Michigan University's REU program has been very successful. The following is a quote from the external evaluation report of Professor Carl Cowen (IUPUI). He states, *"Overall, the students and the mentors seem happy with the program. Moreover, the results in terms of progress on the project problems, increasing the interest of the students in research mathematics, and providing support for further development of the students as future scientists all seem to be meeting the project goals. I think the Department of Mathematics, Central Michigan University, and the National Science Foundation should all be pleased with this project and interested in continuing to support it in the future."*

3. Mathematical activities of the NSF-REU site

The REU problems involve basic mathematical research in the areas of algebra, combinatorics, graph theory, matrix theory, and number theory. The students work on unsolved problems that focus on improving our understanding of the very structure of mathematics, problems whose solutions will give insight into the nature of commutative and noncommutative mathematics, the structure of graphs, properties of numbers, and the theory of linear algebra.

Every effort is made to involve students with problems that are considered significant by both the students and the faculty. Problems are contemporary, important, and usually of interest to a wide mathematical audience. Only a modest mathematical background at the sophomore level is necessary to understand most

of the open questions that are presented to the participants. Some problems require a knowledge of abstract algebra. This allows the participants to be at different stages of undergraduate mathematical preparation. All the problems require that the students rapidly learn a fair amount of mathematics.

During the past five summers, students have found the minimum semi-definite rank of a variety of graphs (under the supervision of Narayan) used group representations and algebraic number theory to construct difference sets (Smith), solved conjectures in graph pebbling (Narayan) and computed the "summability number" of families of graphs (Narayan). Other teams investigated zero divisor graphs of semigroups (DeMeyer), tight subdesigns of symmetric designs (Ionin), addition graphs and nonabelian Cayley graphs (Ionin), matrix stability (Narayan), distance regular graphs (Smith), cellular automata (Smith), and Polya's fenced garden problem (Bekker).

On the first two days of the REU program, there are introductory lectures by faculty mentors followed by descriptions of four or five targeted problems. On the third day there are usually one or two lectures to provide needed background material followed by discussions about how to conduct mathematical research. The students are encouraged to ask questions and participate in these discussions. At the end of the third day, the students are asked to describe their two favorite problems and the reasons for their choices. Based on this information, the students are divided into teams. A typical team consists of one faculty mentor and two students. Students then meet with their mentor for morning lectures which examine topics in matrix analyis, group representation theory, finite geometries, and graph theory.

Typically, the students work on problems from 9:00 a.m. to noon and 1:30 to 4:30 p.m. on weekdays. Faculty mentors are actively involved with the students during these working hours. The day typically begins and ends with the team reviewing the students' progress.

We make every effort to involve students in publishing their research. Although there is no guarantee that any particular research leads to publication, students are required to write their results and submit a final report at the end of the program. The written report will use some version of TEX and follow the article format of the most appropriate journal. Moreover, students are asked to present their work as talks and/or posters at the Mathematical Association of America meeting (Math Fest) in August or at the American Mathematical Society-Mathematical Association of America joint meetings in January or at the annual undergraduate mathematics research conferences held at different locations.

4. Additional student activities at the NSF-REU site

Participants live on campus in university apartments. These apartments are across the street from Pearce Hall, which houses the Mathematics Department and the computer labs. The Park Library and the Bovee University Center are a five-minute walk from Pearce Hall. The Multicultural Center located in the University Center sponsors various activities throughout the year. The apartments are also within walking distance of the Student Activity Center, tennis and basketball courts, and downtown Mount Pleasant.

Participants live in two-bedroom apartments with a shared living room, kitchen, and bathroom. Two male or two female students share an apartment. Each apartment is furnished with a bed, mattress, dining table, chairs, dresser, cable TV access, and telephone. Each student is provided with a notebook computer during the eight weeks of the program. They have access to the Internet and are therefore able to conduct library searches from their apartments.

Apart from working on their research projects, the students participate in seminars given by faculty mentors, seminars by guest speakers on topics of interest (such as "millennium problems" or applications of math in biology) and seminars given by other students. REU students participate in a one-day mid-summer conference with undergraduates similarly engaged in mathematics research at other Michigan institutions (Grand Valley State University and Hope College). This conference takes place at a location convenient for all schools. This event provides a good opportunity for the REU students to interact with students doing research in other areas. The Michigan REU conference was hosted by CMU in 2003 and 2006, by Hope College in 2004, and by Grand Valley State University in 2002 and 2005.

REU students also participated in many social activities including a day trip to the Mackinac Island. Anant Godbole (East Tennessee State), Carl Cowen (IUPUI) and Harriet Polletsek (Mt. Holyoke College) were external evaluators of the CMU REU site in 2002, 2003 and 2004 respectively. Charles Johnson (College of William and Mary) and Hugh Montgomery (Michigan) were the keynote speakers at the 2003 and 2006 Michigan REU conferences held at CMU.

Every participant in our REU program has given an oral or poster presentation at one or more regional meetings (such as the Michigan Undergraduate Mathematics Conference, Young Mathematicians Conference at Ohio State, Hudson River Valley Math Conference or the Nebraska Conference for Women in Mathematics) or at national meetings (such as the joint meetings of the American Mathematical Society and the Mathematical Association of America, the MAA Summer Math Fest or the annual meeting of the American Association for the Advancement of Science). Published or submitted journal articles based on the REU projects from 2002-05 are listed below.

5. Publications and Awards

Awards

As of fall 2006, six articles from our program have been accepted for publication.

- J. Muntz, S. K. Narayan, N. Streib, K. VanOchten, *Optimal Pebbling of Graphs*, to appear in Discrete Mathematics.
- Y. Jiang, L. Mitchell, S.K. Narayan, *Unitary Matrix Digraphs and Minimum Semidefinite Rank*, to appear in Linear Algebra and its Applications.
- P. Hackney, B. Harris, C. R. Johnson, M. Lay, L. Mitchell, S. K. Narayan, A. Pascoe, K. Steinmetz, B. Sutton, W. Wang, *On the minimum rank among positive semidefinite matrices with a given graph*, to appear in SIAM J. of Matrix Analysis and its Applications.
- S. Klee, L. Yates, *Tight subdesigns of the Higman-Sims design*, RHIT Undergraduate Math Journal, Volume 5(2), Fall 2004.
- C. Berkesch, J. Ginn, E. Haller, E. Militzer, *A Survey of Relative Difference Sets*, RHIT Undergraduate Math Journal, Volume 4(2), Fall 2003.

- B. Cheyne, V. Gupta, C. Wheeler, *Hamiltonian Cycles in Addition Graphs*, RHIT Undergraduate Math Journal, Volume 4(1), Spring 2003.

At least six manuscripts based on the results of REU projects are in preparation for submission in the academic year 2006-07. A number of students have won awards for their poster presentation.

- Christine Berkesch, *Relative Difference Sets*, MAA Undergraduate Poster Session, Baltimore, MD, January 2003.
- Josh Whitney, *Subgraph Summability Number of Graphs*, MAA Undergraduate Poster Session, Phoenix, AZ, January 2004.
- Margaret Lay, Amanda Pascoe, *The Minimum Positive Semi-definite Rank of a Graph*, MAA Undergraduate Poster Session, Atlanta, GA, January 2005.
- Steve Klee and Leah Yates, *Tight Subdesigns of the Higman-Sims Design*, MAA Undergraduate Poster Session, Atlanta, GA, January 2005.
- Jessica Muntz and Kelly VanOchten, *The Optimal Pebbling Number of a Graph*, Student Poster Session, American Association for Advancement of Science, Washington D.C., February, 2005.

6. Summary

Five summers of energetic math research with eager undergraduates have been exciting. It is an art to find good undergraduate research problems! The problems need to be genuine, arising out of one's own explorations, and they need to be pitched at the right level. The students need to be given the tools: math concepts, broad mathematical principles and a survey of recent results, yet this must be provided rapidly in a format which does not overwhelm the students. It is important that faculty monitor student progress and provide the appropriate amount of support (in individual conversations or in group seminars). At times, this daily mentoring can be exhausting and time-comsuming.

There is a strong social component to undergraduate research. Bright eager students may need to be directed into productive areas of study; timid insecure students need encouragement and instruction. At times, the faculty member may need to work a little ahead of the students, scouting the research terrain, while still leaving the problem unsolved. Occasionally the faculty mentor must admit that the research team is at a dead end, and must suggest alternative problems – while encouraging the team members that "dead ends like this are typical in research!"

An effective undergraduate research program requires faculty mentors who love mathematics and enjoy working with undergraduates. It also requires significant institutional support. We are grateful to the support of both the NSF and CMU for our program.

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