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Introducing Undergraduates from Underrepresented Minorities to Mathematical Research: the CSU Channel Islands/California Lutheran University REU, 2004 - 2006

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History and Funding

In 2003 the Mathematical Association of America (MAA), through its Strengthening Underrepresented Minority Mathematics Achievement (SUMMA) program, obtained funding from the NSF and the NSA to initiate summer REUs for mathematics majors from underrepresented minorities. Three sites were funded in 2003; in 2004 California Lutheran University (CLU) joined the original sites and two others in hosting an REU. CLU again hosted an REU in 2005 (and the Moody's Foundation also began providing funding), and in 2006 the REU migrated to CSU Channel Islands. In 2004 and 2005 four undergraduates worked on individual (but related) research projects under the direction of a faculty member, assisted by a volunteer graduate student. Six students participated in 2006: they worked in pairs on research projects, receiving guidance from two faculty members. This report will focus on the 2006 REU, as lessons learned during 2004 and 2005 were implemented in 2006, making 2006, in the opinion of the faculty involved, the most successful REU (in terms of student learning).

Program Goals

The following program goals were developed by the faculty and shared with the students on the first day of the REU. After group discussion and modification, the goals were posted prominently and referred to periodically throughout the REU.

(1) Raise the mathematical maturity level of the program's participants.
(2) Get participants excited about doing mathematical research.
(3) Create a learning community.
(4) Help participants develop the confidence to succeed in ongoing mathematical studies.
(5) Increase participants' skills in communicating mathematics.
(6) Extend the participants' abilities to read, understand, construct, and write proofs.

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(7) Acquaint participants with the culture and activities of research mathematics.
(8) Develop participants' skills in reading professional-level mathematics.
(9) Give the participants technical tools for future mathematical learning and research (e.g. MathSciNet, LaTeX, presentation software).

Goals #1 - 7 and #9 were met substantively; some progress was made towards #8[1].

Participant Recruitment and Selection

The MAA (SUMMA) REUs follow an atypical procedure for proposal submission. A call for proposals goes out in the fall; proposals are due in January or February. At the time the proposal is submitted, the proposer must have identified the students who will participate. Qualified students[2] are sought through e-mails to faculty and undergraduate advisors at colleges and universities in the region, and through announcements via the web and at sectional MAA meetings. Funding for travel is not included in the MAA (SUMMA) grants, so only regional students are recruited. Given that the MAA (SUMMA) wishes to reach minority students at a critical point in their career path - midway through their undergraduate programs - the ideal student is a sophomore who is at a transition point between lower and upper division mathematics.

Students apply by providing basic information about their mathematical background and motivation. Once the proposal has been approved, "finalists" are asked to provide letters of recommendation from faculty. For various reasons, not least of which is the scarcity of students from underrepresented minorities who have both sufficient mathematical background to undertake intensive research as well as the desire to do so, recruiting a minimum of four qualified students annually has been very challenging. Each year, at least one student named at the time of proposal submission has dropped out, and replacements were found only through strenuous efforts. Selectivity has thus been low: each year every student who actually completed the application process was accepted to the program - in some cases, despite faculty reservations about the student's readiness. In 2006 the most advanced courses taken by participants were Calculus II (one student), one proof-based course (two students), and two or three proof-based courses (three students).

Description of Research Area

Students addressed open questions in graph pebbling during the 2004 and 2005 REUs; in 2006 the questions were in the area of graph labeling, specifically radio labeling. Radio labeling is motivated by the FM channel assignment problem, which - in greatly simplified form - states that two channels that are geographically close must receive channel assignments with a large frequency difference. We model this situation with (simple) graphs: a radio labeling of a graph G is an assignment c of positive integers ("labels") to the vertices of the graph satisfying the condition $d(u,v) + |c(u) - c(v)| \geq \operatorname{diam}(G) + 1$ for every pair of vertices u, v of G. The

[1]Overall, participants' skills in reading professional-level mathematics was only somewhat enhanced: students struggled with journal articles early in the REU, and for most this aspect was put aside as they began tackling their own research problems.

[2]The call for proposals specifies that students must be members of the following minority groups: African Americans, Latino Americans, American Indians, and Native Pacific Islanders.

radio number of a graph is then the smallest integer used as a label, taken over all radio labelings of the graph. Graph labeling provides an excellent source of research questions for beginning researchers: students can almost immediately begin creating examples, seeking patterns, and making and testing conjectures. After determining the radio number of some "easy" families of graphs, and exploring what it took to prove these numbers were correct, the group as a whole brainstormed over a dozen research questions. Students then formed pairs to work on three different research questions; after exploring a few, each group decided to work on determining the radio number of one or more families of graphs. Two pairs of students completely resolved their problems; the third pair had resolved all but a few small cases by the end of the REU. Some approaches and proof techniques developed by one pair of students or by the faculty mentors were found to have applicability in the work of other pairs. It should be noted that between start-up time (about one week of reading literature, getting a feel for the research area, brainstorming and exploring potential research questions) and wind-down time (about a week of writing up results and polishing presentations), students actually had only a little over four weeks to concentrate on their problems.

Nature of Activities

An observer arriving on a typical day in the middle of the program would see students working on their research questions, whether by generating examples on the boards or on paper, discussing ideas with their partners, constructing or revising proofs, or reading literature looking for ideas they could adapt to their own problems. The observer would see the two faculty strategically spending time with each pair of students, providing feedback, guidance, and encouragement[3]. The observer might notice that one student seemed to be in charge, reminding the other students to take breaks, or to get back to work as necessary. In fact, the students' time was highly structured: the faculty recognized that it's unrealistic to ask anyone to concentrate deeply for eight hours daily, and so created daily and weekly schedules featuring blocks during which students were to have different "focus" levels[4]. A different student served as team captain each week, responsible for some logistics and also for keeping the entire REU team on task.

On days 13, 19, and 24 (of 29), faculty from other institutions visited the REU. These faculty gave informal presentations of their own research, so that the REU students gained knowledge of other active areas of research. The students then made formal presentations of their work-in-progress to the faculty visitors. Following the presentations and some discussion, the entire group went to lunch together. The faculty visitor aspect provided students practice presenting formally as well as talking informally and answering questions about their research. Visitors gave feedback on presentations, suggested ideas for next steps, and shared their own research topics. Over lunch and informally the visitors also provided different

[3]The two faculty typically spent 10 hours (total) daily working directly with the students, and other hours reading students' work and working privately on ideas that would help the students make progress on their research questions.

[4]From the document **Expectations of Students**, discussed and posted: "Focus Level 1: maximum intensity concentration and work. No social conversations, cell phones calls, e-mail, etc. Focus Level 2: OK to do work that requires less intensity here (e.g. solicit teammates' opinions, literature search, write up results, etc.)."

perspectives on becoming and working as mathematicians and gave advice on applying to and choosing graduate schools, doing mathematical research, succeeding in upper division courses, etc.

The REU schedule also included breaks in the form of social events and workshops on various topics. July 4 occurred during the middle of the REU: mathematics was put aside as most of the participants, the faculty, and associated friends enjoyed a barbeque and relaxation time at a nearby beach. Students frequently ate lunch together and some planned after-hours get-togethers. The faculty hosted workshops on several topics, e.g. applying different proof techniques to some well-known results in graph theory (three workshops), why we bother with proofs, deadlines and application procedures for graduate study, learning LaTeX, using PowerPoint effectively, searching with MathSciNet, creating posters, preparing and delivering talks, the nature of the mathematical community, and a viewing of *N is a Number*.

Evaluation

Evaluation of all the MAA (SUMMA)-sponsored REUs is being conducted at a program-wide level. An independent researcher at the University of Oregon has developed and implemented an evaluation plan consisting of student pre-program and post-program surveys, project manager surveys, e-mail contact with students, interviews with students, and selected site visits. Preliminary results suggest that the MAA (SUMMA)-sponsored REUs are having a result in terms of increasing the likelihood that participants pursue graduate study in the mathematical sciences. Personal interviews and tracking of the participants in the CLU/ CSUCI Mathematics REU correlate with these preliminary results and are summarized below.

2004 Participants

- Post-bachelor's plans upon applying to the REU: unclear (3), grad school (1)
- Post-bachelor's plan upon finishing the REU: grad school (3), work with possible grad school later (1)
- Current status: in grad school (3), working (1)

2005 Participants

- Post-bachelor's plans upon applying to the REU: unclear (1), grad school (2), teaching credential (1)
- Post-bachelor's plan upon finishing the REU: unclear (1), grad school (2), teaching credential (1) (Interestingly, two students interchanged "unclear" and "grad school" plans.)
- Current status: in grad school (1), working on teaching credential (1), finishing bachelors (2)

2006 Participants

- Post-bachelor's plans upon applying to the REU: unclear (3), grad school (1), teaching credential (2)
- Post-bachelor's plan upon finishing the REU: grad school (5), credential and possible grad school later (1)
- Current status: finishing bachelors (6)

A second measure of the effectiveness of an REU is the participants' progress on their research questions. Each year students made sufficient progress to present

their results at sectional MAA meetings[5] and at the Southern California Conference on Undergraduate Research. In 2006, all three groups, working jointly with the faculty mentors, solved problems significant enough to submit for publication. Specifically, one team resolved the radio number of gear graphs, another determined the radio number of three types of generalized prism graphs, and the last team found the radio number of cross products of cycles. Additionally, the techniques developed in this work show promise in terms of adaptability for finding radio numbers of other families of graphs.

Lessons Learned

The faculty director's practices have evolved over the three years of this REU program, guided by reflection on what worked and what didn't, by conversations with REU participants of this and other programs, and by advice - through consultations, articles, and conference presentations - of other REU directors. The following lessons learned apply specifically to REUs whose student participants are midway through their undergraduate mathematical studies, and whose faculty recommenders rate them in the top half, but not the top quarter, of their peer groups. Some of the lessons apply as well to students from underrepresented minorities, particularly those who may be the first in their family and/or neighborhood to attend college.

- Be explicit about both goals and expectations. Discuss each with students and modify when appropriate. Return to goals and expectations midway through the REU and discuss progress.
- Structure the students' working time carefully. Build in intense working time, more moderate working time, interaction, and breaks, and be clear about what activities are encouraged during each.
- Spend time getting to know the participants as people.
- Expect a lot of the participants, and encourage them to set high goals for themselves.
- Take advantage of friendly faculty at the host institution and at nearby institutions: ask them to listen to talk with students about their mathematics, to ask students questions about their projects, and to provide the students advice about everything from upcoming courses to graduate study to careers especially about developing the habits of mind of a mathematician.
- Have students work in teams of 2 - 3 per project. Students seem to work more consistently and enthusiastically when responsible to a teammate for carrying out their share of a project. This also allows faculty to rotate between fewer projects, with the benefit of being able to spend more time with each team of students than would be possible to spend with individuals.
- Provide the students a working space of their own. Ideally it would have computers, bountiful white boards and tables, and a couch or two.
- If size of the REU permits, have multiple faculty serve as co-directors. Directors can then share the load, cover for each other should one be called

[5]A 2004 participant won the top prize for his presentation.

away, bounce ideas off each other, and more easily maintain enthusiasm for the work required to keep the students making progress on their projects.
- Should the students' results be of sufficiently quality to submit for publication, recognize that helping the students write to the level necessary will take countless hours, even after the REU ends.

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