# Mathematics Research by Undergraduates: Costs and Benefits to Faculty and the Institution

## MAA CUPM Subcommittee on Research by Undergraduates

Opportunities for undergraduates to do research in mathematics have increased dramatically over the past twenty years. The National Science Foundation sponsors programs such as Research Experiences for Undergraduates (REUs) to encourage involvement, and individual schools have supported local efforts. Administrators have seized on the idea of undergraduate research as a way to engage undergraduates in meaningful intellectual experiences, and departments are striving to increase the involvement of faculty and students in research activities. It is important to note that there are significant differences between undergraduate research in mathematics and undergraduate research in other disciplines.

This document seeks to address the following questions: What is mathematical research by undergraduates? What are the costs and what are the benefits? How is mathematical research by undergraduates different from research by undergraduates in other fields? Our goal is to provide a framework for students, faculty, and administrators to follow as they seek to participate in undergraduate research activities in mathematics.

### What is mathematical research by undergraduates?

For the purposes of this document, mathematical research by undergraduates is characterized by the following:

- The student is engaged in original work in pure or applied mathematics.
- The student understands and works on a problem of current research interest.
- The activity simulates publishable mathematical work even if the outcome is not publishable.
- The topic addressed is significantly beyond the standard undergraduate curriculum.

Undergraduate research in mathematics usually does not lead to publishable work in standard mathematical research journals. Publications are not a universal expectation for undergraduate research projects.

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Undergraduate research is an opportunity for students to explore open questions and to gain experience in making and testing conjectures or in building or testing models. It goes beyond learning what others have done.

Undergraduate research can be performed in pure mathematics, where the goal is to develop new mathematics and prove new theorems. It can also be essentially applied, where mathematics is used to solve problems of practical interest.

Excellent undergraduate research in mathematics draws the student close to the frontiers of mathematics, enabling the student to realize that mathematical research is a vibrant pursuit. This often includes reading recent journal or survey articles, doing computer searches for small examples, doing extensive background reading on the mathematics required to understand the problem, and writing the results in proper mathematical style. Undergraduates can and should present their work at regional or national mathematical meetings either in poster form or by giving a talk. There is a large Undergraduate Poster Session at the Joint Mathematics Meetings each year with many posters.

In summary, the activities by the undergraduates should simulate the activities of a mathematician, going through all the stages of a research project, with the expectations of the outcome generally more modest than for that of a professional mathematician. The stages include formulating and solving a problem, writing the paper, communicating results in a talk or poster, and possibly publishing a paper.

### How is mathematical research by undergraduates different from research by undergraduates in other fields?

Mathematical research by undergraduates should be distinguished from research by undergraduates in fields such as the natural sciences and the social sciences. Research in the natural and social sciences require many people to be involved, and undergraduates in those fields can participate in valuable ways toward the success of a project. Their names will rightly be included on publications based on their contribution.

Most areas of mathematics do not naturally fit into either of these models: we are not doing physical experiments, and we are not usually collecting data. Research in mathematics does involve much quiet thought, and communication so that ideas may be tested. Researchers in pure mathematics try to answer open-ended questions; these questions are in many ways similar to puzzles (like the Car Talk puzzler you might hear on the radio). But the "puzzles" mathematicians seek to solve, address deep questions about the structure of mathematical systems. Mathematics research is often misunderstood because of its introspective nature, and because some do not see any immediate application of the results. Mathematicians see their open questions, and their solutions to open questions, as beautiful and interesting in themselves, and occasionally applications do come about from their study.

In some applied areas of mathematics, mathematicians do construct models and run numerical simulations (frequently with real data.) In both pure and applied areas of mathematics, the work of undergraduate researchers may not contribute in significant ways to publishable work. But in some projects, publications do result with significant contributions from the undergraduates involved.

Furthermore, while undergraduate researchers in other disciplines often assist the project mentor in his/her own research projects, undergraduate research

projects in mathematics are often created specifically for undergraduates. Thus, time spent on leading an undergraduate research project is often time taken away from the mentor's own research endeavors. While undergraduate research is extremely beneficial to the student and to the department, it does not always advance the faculty member's own research program.

## What are the costs and what are the benefits?

The "costs" of undergraduate research in mathematics are primarily measured in time. Mathematics does not require expensive lab equipment, and most universities have access to adequate computing facilities and software. Time is a somewhat less tangible resource, but it cannot be underestimated as an obstacle to research activities. One typical format for undergraduate research is a year-long independent study in the senior year culminating in a thesis. Our estimates indicate that the cost in terms of faculty time averages about 3 hours per week for the year, including actual meeting time, time spent thinking about what direction to take the research, and time spent reading and correcting drafts of the thesis. It takes creative energy to formulate good projects for students. Another format for research is an 8 or 10 week summer program. In this case, the supervisor will spend between 5 and 15 hours per week per project. The number of hours can vary quite a bit depending on the backgrounds of the students and the difficulty of the project. Thus, the cost in terms of time to faculty is quite high.

Students receive tremendous benefit from this activity. Students get to be involved in a significant mathematics project under close supervision by a professor. They gain experience with independent learning, a skill that will prepare them for research in graduate school as well as prepare them to be productive members of a company. They get control over their education in ways that are impossible to duplicate in the classroom environment. Students come out of this experience significantly enriched in their understanding of modern mathematics. Presentation of the results in written and oral formats improves the communication skills of the student.

Professors, on the other hand, have less to point to as a benefit. Certainly there is personal satisfaction in working closely with students. Sometimes, these research experiences do not assist the professor with his/her own research agenda. Faculty who choose to participate may have less time to pursue research leading to publication. This can be overwhelming, particularly for young faculty who are in the early stages of establishing their research credentials. In fact, unless there is an institutional commitment to reward supervision of undergraduate research in mathematics, faculty will understandably be reluctant to agree to spend time working with undergraduates when they can see direct benefit from spending their time on other activities.

The mathematics departments of many colleges and universities provide research experiences for undergraduates. Some programs are during the summer and others occur during the semester. Some programs involve students from other universities and some involve only students at their own institutions. Some programs are supported by the National Science Foundation and the National Security Agency, while others receive support from their own administrations. [1, 2] See the Proceedings of the Conference on Summer Undergraduate Mathematics Research

Programs for a variety of types of programs. [3] See also the website of the Council for Undergraduate Research.[4]

We conclude that institutional support is a critical component for a sustainable undergraduate research program in mathematics.

Websites with further information.
1. `www.nsf.gov/crssprgm/reu/list_result.cfm?unitid=5044`
2. `www.nsa.gov`
3. `www.ams.org/employment/REUproceedings.html`
4. `www.cur.org/`

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