**Proceedings of the Conference on
Promoting Undergraduate Research in Mathematics**

The University of Chicago's VIGRE REU and DRP

J.P. May

The University of Chicago has been called "the place where fun comes to die," but it is more accurately called a place where students come to learn. This article will describe programs that are intended to entice students into wanting to teach and to do mathematics. We like them to create mathematics now, but we are far more interested in their wanting to do mathematics always. Our programs are leading significant numbers of students to make such lifelong commitments.

For background, Chicago is now entering the seventh year of an NSF VIGRE Program. (VIGRE stands for Vertical InteGration of Research and Education). This has large graduate student and postdoctoral components, but the most significant effects have been on the undergraduate level. We have run an eight week summer VIGRE REU for the past seven years. It is restricted to University of Chicago undergraduates who have just completed their first, second, or third year of undergraduate studies. We originally budgeted for 18 participants. In the seven years, we have had, in order, 22, 36, 45, 42, 59, 65, and 70 such participants. The 70 this year were chosen from an applicant pool of over 90, and some of the 70 participated without funding since we could not afford to support that many students. In addition, for the past four years our graduate students have run an academic year Directed Reading Program (DRP) for undergraduates, and this has had around 15 undergraduate participants each quarter. In combination, these programs offer year round mathematical enrichment.

This large number of participants comes from a small undergraduate college, now approaching 5000 students after a period of expansion, ensconced within a major graduate university. The number and percentage of graduating mathematics majors has increased substantially and for the past five years has numbered between 75 and 81 BA's, accounting for over 7% of the total graduating class. That puts Mathematics in a statistical dead heat with English as the fourth most popular major at Chicago, bested only by Economics, Biological Sciences, and Political Science. Moreover, an average of 12 students per year have gone on to graduate study in mathematics over the past five years, with more going into other mathematical sciences, especially computer science and physics. These current mathematics graduate students (or in a few cases new PhD's) include 4 at Harvard, 7 at MIT, 7 at Berkeley, 5 at Stanford, 4 at Michigan, 4 at UCLA, 3 at Rutgers, 3 at Texas, and 1 or 2 at each of Chicago, Yale, Columbia, CalTech, Wisconsin, Cornell, Brown,

Received by the editor October 8, 2006.

Indiana, Northwestern, Duke, Rice, Virginia, Ohio State, Maryland, San Diego, Florida, Missouri, Tufts, and a few schools in other countries. A quarter of these mathematics graduate students from Chicago are women.

I wish that other schools had similar records. As director of graduate admissions, I have to report a static or somewhat diminishing pool of very highly qualified applicants over these same years. A striking feature of our first year graduate classes is that the few University of Chicago undergraduates permitted to take them are among the very strongest and best prepared of all the students.

What does this have to do with undergraduate research? The truth, perhaps, is not very much. The NSF has been persuaded to allow the "R" in "REU" to be interpreted broadly. Chicago is a center of advanced mathematical research. Most of our faculty, including many but not all who teach in our REU, have little idea of how to give genuine research problems to undergraduates. For example, I teach in the REU, and nothing related to my own research could possibly be farmed out to undergraduates: far too much background is required. As a teacher of graduate students (40 have obtained PhD's under my supervision), I like students to find their way to their own problems. As a teacher of undergraduates, at least in the REU, I like students to find things to learn on their own, with the cooperation and help of a graduate student or faculty mentor. It does not much matter to me whether or not they do anything genuinely new, as long as it is new to them. Fortunately, others who teach in our REU know better how to give research problems. Lazlo Babai, in particular, has written joint papers with several participants and has even featured some joint work on "the Abelian sandpile model" in a Distinguished Lecture Series at Emory University. While we can point to other original research, that is not the central focus of our program.

Our REU is deeply intertwined with our outreach teaching programs, which are organized and run by Paul Sally, Diane Herrmann, and John Boller. The regular REU participants serve as counsellors for part of the time that they are in the REU, working with either our YSP program for middle school and high school students or our SESAME program for grade school teachers. It came as a welcome surprise to us how much the participants enjoy the dual experience of learning and teaching that our program provides. In the first few years of the REU, our evaluation forms had leading questions aimed at soliciting complaints of too much teaching. There were none. In our dual program, students develop a simultaneous commitment to teaching and research. This is crucially important to the effectiveness of our program in attracting people into graduate mathematics as a career.

Our REU also includes an apprentice program tailored for all but the most advanced of those participants who have completed only one year of college. This is officially a four week program, but some of its participants spend the entire eight weeks. The apprentice program has only been in operation for three summers. It was created in large part to meet the growing popularity of the REU. There is a limit to how many participants can productively be used to teach in the outreach programs, since a ratio of fewer than five to one of outreach participants to student counsellors seems to become counterproductive. This past summer there were 48 participants in the regular REU and 22 apprentices. Interestingly, applicants were asked to select which program they preferred, and most of those who attended as apprentices had expressed a preference to participate in that program. There were also 3 exceptional participants, all women, who were not part of the prescribed pool

(a high school student, a student who had just graduated, and a graduate student in another field), making a total of 73 active participants.

The reader interested in a detailed description of the various components of our VIGRE program and of the REU and DRP can find them on our web site

http://www.math.uchicago.edu/ may/VIGRE/index.html.

Links to the successive yearly operation of the REU can be found there. The direct link to the full description of the 2006 REU is

http://www.math.uchicago.edu/ may/VIGRE/VIGREREU2006.html.

Detailed course descriptions, hour by hour schedules, and the organization of the mentorship pairings can be found there. These give an idea of how intensive this eight week program is. Nobody, or at least nobody in his right mind, would start out with the idea of organizing and running a program with over 70 participants, mentored by 28 graduate students, and taught by 11 faculty members. The program has evolved over time, and the best piece of advice to anyone starting out as an organizer of any such program is to listen to the participants at all levels, and learn from them. Evaluations and requests for suggestions have been made every year, and the program has been changing in response to suggestions. This year, for the first time, there was a requirement of either an end of program presentation or a written report for all regular participants. Next year, a written report will be required of everyone. The presentation must of course be optional, since there simply isn't time to listen to 70 presentations.

Most of the 2006 undergraduate papers are also posted on the web site. They range over a wide spectrum of mathematics at varied levels. Some of them are seriously sophisticated. Others are playful. Some are elementary learning experiences. The students are urged to work at their own level of comfort, always mentored, but free to explore any direction. Many of the papers are tied to one or another of the course offerings, others are not. I urge the reader to read a few of these papers, and at least to look at their titles. They run the gamut, including serious algebra, geometry, topology, analysis, probability theory, and logic. Several of them contain interesting original research; many of them include interesting surveys and perspectives on fields. The amount of mathematics the students have absorbed is prodigious. The reader is also urged to look at the course listings. The faculty teaching them include Dickson instructors, assistant professors, and tenured faculty. The subjects include topics in discrete mathematics, algebraic geometry, algebraic topology, geometry, analysis, and even a segment on mathematics and music.

This year's evaluations asked even more explicitly than usual for suggestions for future improvement. The answers seem to indicate that we are getting it right. A few of the responses nicely sum up the concensus and make clear that the respondents know that their answers are taken seriously.

"Overall I thought the program was better this year for two reasons: the mentoring program was much more coordinated, and the requirement of a paper/presentation was enjoyable and worthwhile. These two additions made the program more organized and should definitely be retained for the future."

"Having done the program for three years, I feel that I am in a good position to comment on how effective the current structure is. I felt that in this year, more than in any other, everyone participated in the class/research aspect of the REU. I feel that the paper was largely responsible for this. All in all I really would not change anything about the REU."

"I have done the REU three years and enjoyed it greatly each time, but this year even more so. This year I felt that much more of the material presented was accessible, a lot of which I think has to do with the two additional years of math I've had since my first year in the program. In that vein, I really like how there are classes offered specially for apprentices and more advanced ones as well.

I also liked the addition of a final project requirement. Not only did it force me to learn TeX (a definite plus), but it gave me a goal to work for, which helped give my work more direction. Also, it's nice to have something tangible to present or have at the end of the REU that is representative of the work I put in over the summer.

Another aspect I like is the teaching aspect (I did SESAME this year), as I really enjoy that and feel it's an important part of mathematics.

Those are the things that stood out this year, though of course other aspects are excellent and I think should stay the way they are: the mentorship program, a wide variety of classes on many subjects not usually covered in "regular" math classes, and the positive environment and freedom to learn whatever type and as much mathematics as you like.

My grad student mentor was [name], and I thought he did an excellent job. He was enthusiastic and helpful whenever I had questions.

Overall my experience in the REU has been very positive and has solidified and confirmed my desire to further pursue mathematics (and teaching it) as a career. Thanks for the opportunity and for providing such an excellent program."

Our REU works, but on this scale it is very expensive. I would be delighted to see it emulated at other major research universities. On a smaller scale, aspects of it, at least, could be emulated in many other places. The DRP, in contrast, is very inexpensive, and all it requires in the way of organization is willing and able graduate students. It can easily be emulated elsewhere. In fact, one past participant has organized a replica of the program at Rutgers. The program at Chicago is entirely organized and run by a self-perpetuating committee of graduate students, under very minimal supervision. A detailed description may be found at

http://www.math.uchicago.edu/drp.

The student and mentor agree on a topic of mutual interest and the student learns and discusses all that he or she can. Undergraduates apply to and are screened by the committee. Participants are required to meet with their mentor once a week during the quarter and to put in at least four hours of work on their topic per week. They are required to give a presentation at the end of the quarter. Projects must be approved by the committee. The program description advertises the following benefits of the program. "Participating undergraduates will learn to work independently through studying a topic of their choice, well-suited to their interests. They will develop relationships with graduate student mentors and receive a good deal of personal attention focused on their mathematical studies. Finally, they will gain valuable experience in mathematical communication by giving a presentation on their work to an audience of their peers."

It is to be emphasized that undergraduates who participate obtain no course credit and are not paid, and they do the work on top of their usually heavy course loads. Many of them are also working to help defray their college expenses. The fact that more and more are participating makes a powerful argument that the advertised benefits are being delivered. The graduate student mentors are also

volunteering their time, and they receive just token payment (currently $300 per quarter, plus $100 for books for the participants, with a little more for the organizing committee). This is a true labor of love on both sides.

The REU and the DRP both depend on close mentoring, and often joint learning, between undergraduates and graduate students. Students learn an enormous amount of genuinely deep and interesting mathematics that way. The students may or may not do research, but they get a taste of it, and they get a serious feeling for the world of graduate mathematics and beyond, serious enough to make many of them aspire to that as the way to spend their entire lives.

In both the REU and DRP, rigorous mathematics and a sense of community are combined. The presentations are coupled with dinners and are social occasions. Students bring their friends. Mathematics is many Chicago students' view of fun. Once critical mass has been achieved, it is amazing to see just how contagious a love of mathematics can be. There is one prerequisite. The mathematics must be real. It must not be dumbed down. Rigor is essential. Our students, at all levels, learn what a proof is and how to appreciate a beautiful argument or an interesting conclusion. Here are some quotes from this year's applications to the REU, in answers to the request to "Explain briefly why you want this position". Incidentally, the answers show why we no longer have to make any real effort to recruit applicants. Word of mouth is enough.

"I am in love with and absolutely addicted to mathematics and plan to continue its study my entire life. This program is an excellent opportunity both to further my knowledge and to begin exploring this new* area — research. I want to play, I want to learn, and I want to do this with others who are just as excited and passionate about mathematics as I am.
*new to me, that is."

"... the YSP counselor work I've done this year has been some of the most fun I've had. Taking such young, malleable, minds that are unaware of what they "cannot" do, setting them in front of esoteric mathematics, and watching them go forth has been an utter joy this year so far; doing this for an extended period of time over the summer would, I believe, be amazing."

"I have heard from friends that it is an excellent program. I have heard glowing reviews of the participating faculty. In short, I believe that this is as close to an ideal math-learning situation as I can ever hope to encounter."

"I have heard wonderful things about the REU at Chicago ... and have wanted to do it since my first year. Finally, I have obtained my parents' permission to not come home this summer (for the first time), and so I am jumping on a chance to take an intensive math program."

"I love mathematics." [That is a leit motif running through the applications.]

"I could say 'I love mathematics,' but that is cliché. Math is for me not a simple 'love' but more of an all-consuming fascination, something that I just need to do."

"I really truly enjoy mathematics. It is what I plan on doing the rest of my life."

"Well, *briefly*, because I had a blast in the REU last year."

"Because it's a summer full of math."

"Because it means spending my summer studying, researching, and teaching math, which sounds like an amazing way to spend the summer."

"This program brings together two of my favorite things: teaching mathematics and learning mathematics."

"Of all of the classes that I have taken in my life, Honors calculus has probably been the most stimulating. This course made math not only interesting, but also made it challenging for me for the first time in my life. Seeing the proofs that π and and the square root of two are irrational was one of, I have to say, the most thrilling moments of my life. I think that this program offers me a unique opportunity to pursue this recently discovered passion."

Department of Mathematics, The University of Chicago, Chicago, IL, 60637
E-mail address: may@math.uchicago.edu