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Research Experience for Undergraduates in Numerical Analysis and Scientific Computing: An International Program

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1. Introduction

In the summer of 2006, five male and five female undergraduates from nine universities across the United States participated in an eight week Research Experiences for Undergraduates (REU) program in Hong Kong that was directed by the Department of Mathematical and Computer Sciences (MCS) at the Colorado School of Mines (CSM). This program was supported by the National Science Foundation (NSF) (DMS-0453600) and is the first of three such opportunities supported through the project, "United States–Hong Kong REU in Numerical Analysis and Scientific Computing"[1]. The participating Hong Kong universities were Hong Kong Baptist University (HKBU), The Chinese University of Hong Kong, and City University of Hong Kong. This REU program is one of forty-seven currently funded NSF mathematics REU site grants nationwide[2]. Only one other site has an international component, Rutgers University, which is collaborating with Charles University in Prague, Czechoslovakia[3]. The mathematical focus of the Rutgers REU is discrete mathematics, computer science, statistics, biomathematics, and biomedical applications. The current paper describes an REU site in Hong Kong, the mathematical focus of which is numerical analysis, scientific computing, and applications in applied science and engineering

The purpose of establishing an REU site in Hong Kong is to provide undergraduate mathematics students with the opportunity to contribute to the exciting research that is being conducted in numerical analysis and scientific computing at an international level. Many students incorrectly assume that researchers in mathematics work in relative isolation. This international REU program is designed to challenge this misconception which may discourage talented students who desire a highly interactive and culturally rich experience from pursuing degrees in mathematics. A growing national concern is that there are few students pursuing graduate studies in science and mathematics [**1**]. In today's world, mathematics students as

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[1] `http://www.mines.edu/reu-mcs`

[2] `http://www.nsf.gov/funding/pgm_summ.jsp?pims_id=5517&from=fund`

[3] `http://www.nsf.gov/awardsearch/showAward.do?AwardNumber=0138973`

well as students in other fields must be able to assimilate into different cultures and learn how to function in a global environment [**1, 2**]. The Hong Kong REU program is designed to encourage more students from diverse backgrounds to pursue higher degrees in the mathematical sciences with an international perspective.

MCS has sponsored prior REU sites on its own campus. The impetus for expanding overseas came from support from NSF's Division of International Programs in the form of a supplement to a previous REU site grant, to investigate the establishment of an REU site in East Asia, an area of strategic importance to the United States.

The purpose of this paper is to describe the first implementation of this international REU program, the specific goals of which are:

(1) to increase undergraduate students' interests in pursuing advanced degrees in mathematical sciences;
(2) to provide participating students with a high quality international research experience in the mathematical sciences.

For a student perspective of the program, see [**3**].

2. Student Participants

Ten U.S. undergraduates were selected from thirty-three applicants through a competitive application and review process. All participants had completed their third year of college and had a declared major in mathematics. Participants were also required to provide evidence of a strong grounding in ordinary differential equations, linear algebra, and numerical methods, together with knowledge of a programming language such as C, Fortran or Matlab. The students selected were from the following institutions: Colorado School of Mines, Colorado State University at Pueblo, Davidson College, Illinois Institute of Technology, Loyola College in Maryland, New College of Florida, Regis University, Taylor University, and the University of California at Berkeley.

3. Program Design

3.1. Local Logistics. Before departing for Hong Kong, the students were assigned to research teams comprising a faculty member from a participating Hong Kong university and two or three U.S. undergraduates. These individuals began to interact electronically before the trip. In preparation, students were provided background material regarding their project and addressing practical issues, such as cross-cultural understanding, travel arrangements and insurance matters, accommodation and food, and safety and health concerns.

The program was centered at HKBU, where the students stayed at the Ng Tor Tai (NTT) International House, which is conveniently located for shopping, dining and transportation. This on-campus housing provided a safe, pleasant and convenient environment for the REU participants and was within 15 minutes of the business center of Hong Kong via public transportation. A wide variety of food was available, including Western food, at cafeterias on the participating campuses. Since English is an official language in Hong Kong, language barriers were minimal.

The grant covered the students' travel and living expenses, and provided each with a stipend of $2,500. An MCS faculty member traveled to Hong Kong with the students as their U.S. faculty adviser. The host institutions provided each student team with office and computing facilities.

The program ran for eight weeks, beginning in the last week of May 2006. After weeks 3 and 6, the teams met as a group and each team presented an oral progress report, with the U.S. adviser moderating. These reports included a discussion of accomplishments, problems encountered and their solutions, and the remaining goals of the project. Each team was also required to prepare a final written report which was submitted at the end of the program. At that time, each team gave a 30 minute presentation on the completed project to the group and their research mentors as well as other faculty and students from HKBU.

3.2. Research Projects. The four research projects on which the students worked were: 1) the development of wavelet algorithms for high-resolution image reconstruction, 2) subspace clustering methods for high-dimensional categorical data, 3) spectral analysis of differentiation matrices and applications, and 4) numerical challenges for resolving spike dynamics for reaction–diffusion systems. Each project is described on the program's webpage: www.mines.edu/reu-mcs

3.3. Additional Experiences. The students had the opportunity to participate in additional academic activities, such as attending the 2nd International Workshop on Structured Matrices at HKBU. This experience provided them with exposure to international mathematical research that was being conducted beyond their team projects, and a chance to interact with internationally recognized researchers. They also attended seminars presented by visiting speakers and a Ph.D. thesis defense, and had tea with renowned numerical analyst, Dr. Gene Golub of Stanford University.

4. Assessment and Evaluation

Before the program began, the students submitted an essay describing what they hoped would be their future contributions to mathematics and the mathematics community. On their return to the U.S., they submitted an essay on how they would revise their original essay based on their Hong Kong experience. At the end of each of the first seven weeks in Hong Kong, students submitted written responses to the following: "Reflect on the past week's experiences. Explain what you learned both mathematically and culturally." As mentioned earlier, each team was required to submit a final written report and give a presentation which were evaluated by their Hong Kong mentor and the U.S. adviser. This section describes the results of the assessment and evaluation effort. Since the number of participating students was small (ten), only descriptive analyses were completed on the data.

4.1. Pre and Post Program Essay. In their original essay, all students indicated their intention to attend graduate school in a mathematical field (i.e., mathematics, computer science or physics), and half of the students expressed interest in acquiring a Ph.D. in one of these fields. The majority of students also indicated that they had an interest in seeking a career that included college level instruction (seven students) and/or mathematical research (six students).

The majority of the post project essays indicated that the experience had not changed their future aspirations, but rather reinforced them. Six of the eight students felt that the REU experience confirmed that graduate school and a mathematical sciences career was appropriate to them. Two students questioned their mathematical ability as a result of the experience. Neither of these students indicated a change in their desire to pursue a mathematical sciences career, but rather

that they needed to acquire a better understanding of the field before making a final decision.

Of the five women who participated in this project, three returned from the REU experience expressing a greater understanding of the impact that they could have on the mathematics community as females. One woman explained, "Their [sic] was only one woman participant in the conference on Structured Matrices, and some of the Chinese professors said that they did not take female Ph.D. Students. The latter fact concerns me the most; how can women feel accepted in the field if they have difficulties finding a professor to take them on as a student based on their gender?"

4.2. Weekly Reflections. At the end of the first week, most students primarily reported cultural observations. For example, "The people here are very nice and friendly", and "People here don't hesitate to cram into the train or a crowded elevator". Very few students addressed what they learned mathematically and those who did, reported that they were still in the start-up process, e.g., "We spent the past week understanding our research project and learning the background material."

In the second week, the students attended the 2nd International Workshop on Structured Matrices at HKBU. The majority of the cultural and mathematical reflections were based on this experience. Students reported that they were surprised that English was the language of the conference and that they felt lucky and somewhat embarrassed to be native speakers. They also reported that interacting with practicing mathematicians allowed them to see the "human" side of the profession.

By the end of the third week, the students' reflections began to focus more on mathematics and on problems they were experiencing with their research, e.g., "We keep trying new things, and sometimes we get a little bit better results, sometimes we get worse results, but never the results we really want." With respect to culture, students began to document differences between the Hong Kong culture and their own, "Asian women wore much more conservative swimwear and Asian men actually wore much less fabric than other men".

In weeks four and five, students began to express satisfaction with the progress of their research, "Our project seems to be moving forward which makes me feel good because there was a while I didn't think it was going anywhere". Another interesting outcome reported in the fourth week was that the students were beginning to judge the culture in which they were participating. Several students expressed concern about the recruitment of Filipino women to Hong Kong to act as poorly paid night club workers or maids.

During the sixth week, a major concern of the students was group work. Several students reported that they were used to being group leaders, and rarely depended on the efforts of others. This was not the case in their research. Instead, they found their own efforts delayed, while waiting for results from other team members.

By the seventh week, the students began expressing satisfaction with the outcomes of their research, e.g., "This past week has really been about tying up loose ends" and "Work-wise, I wish we had another two weeks. I think that we are finally getting some results that make me wish that there were more time...". A number of students were also excited about their meeting with Dr. Golub. Students further expressed excitement and regret at the prospect of returning home.

Excellent	Above Average	Average	Below Average	Unacceptable
This international REU program was well organized as a whole.				
7	3	0	0	0
The international component(s) of the program added value to the underlying scientific experience.				
8	1	1	0	0
This program helped me to improve my research skills.				
8	2	0	0	0
This program helped to increase my general knowledge of mathematics.				
5	5	0	0	0
I will continue my pursuit of a career in mathematics as a result of this international REU experience.				
4	3	3	0	0

TABLE 1. Summary of student responses to a subset of questions from the end of program survey

4.3. Hong Kong Faculty Evaluations. As part of the evaluation process, the research mentors in Hong Kong were asked to rate each of their students with respect to the following: contributions to the research team, final submitted research project, and future research potential. Only two of the mentors completed the feedback form, accounting for six of the ten students. All of these students were evaluated as "Average" or above with respect to each question, the majority being "Above Average. In response to the question concerning research contributions and future research potential, the majority of students received a rating of "Excellent."

5. End of Program Survey

At the conclusion of the REU program, the participating students completed an end of program survey. This survey was based on [**4**] and contained a total of 52 questions. Given the length of this survey, it cannot be summarized here in full. Table 1 provides the results of five key questions that addressed the overall effectiveness of the REU experience. As this table suggests, the majority of students either agreed or strongly agreed with each statement.

6. Discussion

As a result of implementing and evaluating this REU, we are obliged to reconsider goal (1), stated in section 1. The participating students did not uniformly display an increase in their interest in pursuing advanced degrees in the mathematical sciences. In fact, most of the students maintained their original intentions, i.e., to pursue an advanced degree and a career in the mathematical sciences. Students entered the program because they already had interests in this area. Participation in this REU experience did help the students to clarify their career objectives. At the conclusion of the REU, two of the participants questioned their mathematical ability and their future intentions of pursuing a mathematical sciences career. Both of these students indicated that they would seek additional experience and information with respect to mathematics before making their final career decision.

As educators, we must interpret these results as a success. After all, we do not wish to encourage students to enter the mathematical sciences for the simple purpose of increasing the pool. Our goal is, more appropriately, to guide students into a satisfying career that is consistent with their personal goals. With this in mind, we reconsider the first goal of this project and refine it as follows, "To provide

students who are interested in pursuing an advanced degree in the mathematical sciences with a better understanding of the nature of mathematical research and mathematical careers". The data presented in this paper supports the assertion that the revised version of the first goal was attained.

The second goal was to provide the participating students with a high quality international research experience in the mathematical sciences. Based on the student feedback as well as the evaluations of the research mentors, this goal appears to have been met. Furthermore, based on the work of [**2**], the student weekly reflections suggest that the participating students were progressing toward global competency. Downey et al. [**2**] proposed the following criteria for the evaluation of minimal global competency in engineering students.

- Students will demonstrate substantial knowledge of the similarities and differences among engineers and non-engineers from different countries.
- Students will demonstrate an ability to analyze how people's lives and experiences in other countries may shape or affect what they consider to be at stake in engineering work.
- Students will display a predisposition to treat co-workers from other countries as people who have both knowledge and value, may be likely to hold different perspectives than they do, and may be likely to bring these different perspectives to bear in processes of problem definition and problem solution.

In order to apply these criteria here, we must replace "engineers" with "mathematicians" and "engineering" with "mathematics". The analysis of the weekly reflections suggests that the students in this REU program progressed through the above criteria in almost a linear manner. They began by analyzing similarities and differences, than began to examine the life experiences within the given country and, by the conclusion of the REU program, began to recognize the valuable contributions of all involved in the project.

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