**Proceedings of the Conference on
Promoting Undergraduate Research in Mathematics**

Undergraduate Research in Mathematics at Grand Valley State University

Steven Schlicker

Introduction

The Department of Mathematics at Grand Valley State University (GVSU) has a long and successful history of research with undergraduate students. Since 1994, faculty in the department of mathematics have mentored 65 students in funded research projects in mathematics. These projects have resulted in many publications in refereed journals and award winning presentations at the national level. In this article, we describe the undergraduate research programs at Grand Valley and why we believe these programs have been so successful.

Grand Valley State University

Grand Valley State University is a mid-sized, comprehensive regional university with an emphasis on providing the feel of a small college. GVSU enrolled 22,565 students in the 2005-06 academic year. The Department of Mathematics at GVSU is home to 34 tenure-line faculty. Our emphasis is on undergraduate education. Within that context, our faculty is active in research, with faculty members regularly publishing books and papers. This department- and university-wide focus on undergraduate education combined with active research and scholarship provides a unique environment for undergraduate intellectual growth and achievement. We see undergraduates as active participants in the scholarly life of our department and university.

Undergraduate Research at GVSU

GVSU has a history of successful undergraduate research. Formal research programs with undergraduate students in mathematics began at Grand Valley State University in 1994 when the Division of Science and Mathematics established the Summer Undergraduate Research Program (SURP). This program was created to foster and support collaborative faculty-student research projects in science and mathematics. The SURP program started out small, funding 5 proposals (1 in mathematics) in its first year. In 2003, the SURP was expanded to the Summer

Received by the editor January 20, 2007.

Student Scholars (S^3) program, a campus-wide program supporting scholarly activities in all disciplines. Between 1994 and 2006 the SURP and S^3 programs funded 17 student-faculty collaborative research experiences in mathematics. Examples of past SURP and S^3 mathematics research topics include Sierpinski polyhedra, the differentiability of certain area functions, differential games, polynomial root-dragging, investigation of middle school students' understanding of step and linear functions, spherical geometry, separable voting preferences, bioinformatics algorithm development, dynamics of the dual billiard map in the hyperbolic plane, hypergeometric summation and the Wilf-Zeilberger (WZ) algorithm, and circle packings.

The Grand Valley State University REU Program

The SURP program was intended to serve as "seed" money for the development of undergraduate research programs. With that in mind, we submitted a Research Experiences for Undergraduates (REU) proposal to the National Science Foundation (NSF) to support our research activities with students. Our first REU grant allowed us to run an REU program in the summer of 2000. Since then we have offered REU programs in every summer except 2001. We just completed (in 2006) the second summer of a five-year REU grant.

Overview: The goal of our REU program is to provide a complete research experience for student participants, including literature searches, background reading, focused work on one or more research problems, and both oral and written communication of results. Developing and honing students' communication is a critical component of our REU. We provide a broad range of research choices in various areas of pure and applied mathematics that reflect the diverse interests of the many department faculty who serve as mentors.

Research Groups: The GVSU REU program is an eight-week program that provides support for eight students. We recruit nationally and give priority in our recruiting to students for whom this REU will be their first research experience, and we actively recruit students from groups that are traditionally underrepresented in the mathematical sciences. The eight students work with four faculty mentors and form research teams of two students and one faculty member. Examples of research projects in past REUs include polynomial root-dragging and the location of critical numbers of polynomials, Lanczos' derivatives, wavelets, spans of derivatives of polynomials, Hausdorff metric geometry, bifurcations in dynamical systems, San Gaku problems in spherical and hyperbolic geometry, Lie theory, and Mandlebrot sets for ternary number systems.

Format of the Program: We begin the summer with an opening picnic on a Sunday afternoon. This provides an opportunity for both the students and faculty to get acquainted. After a group breakfast (supplied by the faculty) the following Monday morning, each faculty member gives a presentation on his/her research area. That same evening, each student sends his/her preference rankings of the four research areas to the faculty who use the rankings to determine the research groups. The research groups are announced on Tuesday morning, and the students immediately get to work on background reading, literature searching, "warm-up

problems", and further preparation for research. All students are actively pursuing research problems in the first week of the program. During the summer, each research group meets regularly (almost every week day) to discuss progress on the research, brainstorm, and plan for the future. In addition to their own research, through a weekly program of outside speakers our REU students are exposed to a variety of mathematical topics and learn a bit about what life is like as an academic mathematician. We also engage our students in many events in addition to their research (e.g. workshops, student talks, group breakfasts, group social activities) to create a student-faculty team environment.

Communication is an important component of our program. During the first week the faculty mentors provide a workshop for the students on using LaTeX. Throughout the program, students are required to write and submit weekly reports on their research progress in LaTeX. As these weekly reports are updated, they provide a solid foundation for the final technical report each group will produce. When appropriate, these technical reports are used by the research teams to prepare manuscripts to submit for publication. In addition, students are required to give short bi-weekly presentations to the entire group. Not only does the frequent writing and speaking hone their communication skills, it also helps clarify and solidify in students' minds the work they have been doing. In addition, the presentations help our students prepare presentations for two conferences: the Michigan REU conference and MathFest.

We initiated the first annual Michigan REU conference in the summer of 2000 to expose our REU students to the research being conducted at the nearby Hope College REU site. We expanded the conference in 2002 and served as hosts to the Hope College REU and the new REU program at Central Michigan University (CMU). During this day-long event, all of the REU students (as well other students engaged in mathematical research) give presentations of their work in parallel sessions. The Michigan REU Conference now rotates among the three institutions (CMU hosted the last conference in 2006). As a culminating experience for our REU group, we plan our program so that it ends at MathFest, the annual summer meeting of the Mathematical Association of America (MAA). We take all of our REU students to this conference at which they give presentations in the MAA or Pi Mu Epsilon student sessions.

Philosophy: Our philosophy of research with undergraduate students is founded on four basic principles. The first is Intellectual Independence. We understand that research is much different than a typical classroom experience. Our program is designed to help students make the transition from the more structured environment provided by classroom instruction to the open, independent work required in research. In the beginning of our program, the faculty mentor takes the lead and provides a framework for the research experience. As the summer progresses, the student researchers are gradually led to more independence and the students and faculty work as research collaborators and colleagues. The second principle is Student-Student Communication. An important component of the research experience is communicating one's results. This is a new experience for most students, so we create a non-threatening environment in which students regularly prepare and practice presenting their work to their peers. Through the presentations, students receive feedback on their work and their communication skills from both students

and faculty. These presentations also expose our students to the work of their colleagues and areas of mathematics other than their own research area. One advantage to the regular student presentations is that the presentations increase in formality as the summer progresses, and by the end of the summer our students are well-prepared to give presentations at the Michigan REU conference and MathFest. The third principle on which our program is based is Community. To build community, and to foster informal communication among the students, we house our REU group together in 4 townhouses on campus (2 students per townhouse). Living together encourages cooperation among the students and helps them form close bonds. We also foster community through regularly scheduled social events, seminars by outside speakers, group breakfasts (provided by the faculty each Monday), and other informal activities. We feel that providing community is important for students for many reasons. In particular, most of our REU students are new to the area and it is vitally important to have support when students are struggling to learn new mathematics. Our fourth principle is Student-Faculty Interaction. Our faculty mentors are co-participants in all of the REU activities. We format our program to foster independence in our students and strive to create an environment in which students ultimately see themselves as colleagues and collaborators with the faculty.

Research Projects: Selecting appropriate research projects for undergraduate students is a challenge. Many faculty mentors in our REU have used this challenge as an opportunity to become students themselves, reading books and journal articles to learn new mathematics outside their own areas of expertise and find problems that are mathematically interesting and substantial but still accessible to talented undergraduates. Since the faculty are not experts in these new research areas, they are able to convince their students that the research the students and faculty do in the program is truly collaborative. We search for problems that are significant enough mathematically so that a full solution will be publishable in a refereed journal, but at the same time are accessible enough to allow for substantial progress to be made during the summer. Ideally, a good research problem will come in layers - for example, early progress can be made on specific cases while the problem proves more challenging as the students progress in their mathematical sophistication. We want our students to be able to take gradual ownership of their research and be confident at the end of the program that their contributions have been substantial and real.

Post-Program: We expect our REU students to continue their work after the program has formally ended. Faculty work with their students to prepare manuscripts for submission to journals, and encourage students to give presentations on their work at their home departments and at local, regional, and national conferences.

Evidence of Research Success

Students and faculty in the Grand Valley State University REU, SUPR, and S^3 programs have produced publications in a variety of journals. A list of publications includes:

- "Sierpinski Polyhedra," *Pi Mu Epsilon Journal*

- "Integral Functions Whose Right Derivatives Are Average Values of Periodic Functions," *Pi Mu Epsilon Journal*
- "On the Differentiability of Certain Integral Functions," *Mathematics Magazine*
- "Period-3 Orbits via Sylvester's Theorem, and Resultants," *Mathematics Magazine*
- "When Lines Go Bad in Hyperspace," *Demonstratio Mathematica*
- "Breaking the Holiday Inn Priority Club CAPTCHA," *College Mathematics Journal*
- "Symmetry in Bifurcation Diagrams," *Pi Mu Epsilon Journal*
- "Quadratic Dynamics in Matrix Rings: Tales of Ternary Number Systems," *Fractals*
- "A Singular Introduction to the Hausdorff Metric Geometry," *Pi Mu Epsilon Journal*
- "Wavelet-Based Steganography," *Cryptologia*
- "Lisa's Lemonade Stand: Exploring Algebraic Ideas," *Mathematics Teaching in the Middle School*
- "A Note on PIPCIRs," *Pi Mu Epsilon Journal*
- "On the Ratio Vectors of Chebyshev and Equispaced Polynomials", *Missouri Journal of Mathematical Sciences*

Our students have also been recognized for their presentations:

- Christy Heideger and Amanda Taylor (REU 2006) - best student talk award in an MAA session at MathFest 2006.
- Samuel Kolins (REU 2005) - best student talk award in an MAA session at MathFest 2005.
- Chantel Blackburn (REU 2005) - Council on Undergraduate Research (CUR) award for best student research talk at the Pi Mu Epsilon sessions at MathFest 2005.
- Alex Zupan (REU 2005) - CUR award for best student research talk at the MAA sessions at MathFest 05.
- Matthew Katschke (REU 2004) - best student award in an MAA session at MathFest 04.
- Christopher Bay (REU 2003)
  - Best student talk award in an MAA session at MathFest 03.
  - Best poster award in the Undergraduate Poster Session at the 2004 Joint Mathematics Meetings.
- Kris Lund (S^3 2003) - best student talk award in an MAA session at MathFest 03.
- Micah TerHaar (S^3 2003) - presented a poster at the prestigious Undergraduate Research Posters on the Hill session sponsored by CUR in Washington D.C.
- Lisa Driskell (REU 2000) - won the Annual Greg Mellen Memorial Cryptology Scholarship Prize from Cryptologia for her article "Wavelet-Based Steganography".

Reasons for Success. There are many factors that we feel have made our undergraduate research programs so successful. One is institutional support. From the original Summer Undergraduate Research Program to the current Student Summer Scholars program, Grand Valley for years has encouraged faculty and undergraduate students to be collaboratively involved in research. Students are expected to communicate their work to others during Student Scholarship Day (SSD), a day-long campus wide event at which students present results of their scholarly activities to a university-wide audience. GVSU also provides financial support for students

to give presentations at conferences. All of this support creates a climate on campus that acknowledges this type of scholarship as important for both faculty and students and builds an expectation that many students will be involved in research or scholarly activities beyond the classroom.

Another factor that contributes to the success of our undergraduate research programs is the emphasis the mathematics department places on student-faculty interaction. Almost half (15 of 34) of the regular faculty in our department have participated in senior theses or research projects with our students. We encourage faculty who have not had research experiences with undergraduates to participate in the S^3 program and, at the same time, to engage in activities with the REU program and to consult with the REU faculty. In this way the department mentors new faculty in this area. We recognize this type of scholarship as vital to the department and university mission and encourage faculty and students to be involved. We expect students to share the results with their peers and others at our department seminar, at SSD, and at outside conferences. We also hang student posters through our building to publicize their work.

Other than our senior thesis course, the mathematics department doesn't have any specific courses directed toward undergraduate research. Rather, in all of our courses we emphasize communication skills, both verbal and written. Our courses include writing assessments, from short essay questions to extensive projects, and student presentations, even as simple as having students present solutions to homework problems. As a result, GVSU students who participate in research programs have already developed the skills to allow them to effectively communicate the mathematics they learn. In addition, most mathematics faculty at Grand Valley expect students to work cooperative in small groups on class projects and activities. In this way our students come to understand how to work effectively in groups and can then be productively contributors to any research group. In a similar vein, we work in depth with our REU students who are not from Grand Valley to develop these communication skills during our REU program. REU students write weekly reports on their progress, give bi-weekly presentations, and speak at two major conferences. Throughout the program, the faculty work with the students, coaching them on their presentation skills and providing specific, directed feedback and advice on their writing. As the results indicate, our students give well-received talks at major conferences and their final reports at the end of the program often become published papers.

One other factor that we believe contributes to the success of our research with undergraduates is the nature of the research problems themselves. Our research problems are specifically chosen to be substantial and accessible to undergraduates. This has required most of our faculty to learn about new fields of mathematics, different from their thesis areas. An interesting consequence of this is that the research projects we conduct with our students are truly collaborative in nature. The projects are just as much research for the faculty as they are for the students. While the faculty of course have broader and deeper mathematics backgrounds than the students, the fact that we are often new learners in the research areas makes it possible, as often happens, for our students to truly discover new mathematics for themselves instead of just being led to the mathematics.

Department of Mathematics, Grand Valley State University, Allendale, MI 49401
E-mail address: schlicks@gvsu.edu