**Proceedings of the Conference on
Promoting Undergraduate Research in Mathematics**

Undergraduate Research during the Academic Year

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I am an unabashed proponent of undergraduate research in mathematics. I want to share with my undergraduate students what I find so joyful about mathematics. The NSF-sponsored Research Experiences for Undergraduates are a wonderful way of providing a focused immersion experience in undergraduate research. The summer REUs, however, serve a very small number of those mathematics majors who wish to explore mathematical questions and produce original results in mathematics. In addition to the national REUs, I advocate undergraduate research done at our own institutions with our own students during the academic year.

1. Goals of undergraduate research

1.1. Graduate studies. There is substantial agreement that one of the goals of undergraduate research in mathematics is to encourage students to pursue graduate studies. The University of Dayton has long been one of the higher-ranked schools for mathematics in its category in the Franklin & Marshall Baccalaureate Origins of Doctoral Recipients. Yet, the University of Dayton was an REU Site for mathematics for only a handful of years in the early 1990s. In fact, very few of the top-ranking 4-year colleges and Master's-granting institutions mentioned in this report have been NSF-REU Sites. Although it is possible that the students from these schools who go on to get Ph. D. degrees in mathematics all went to summer REUs, I believe that the more likely explanation is that most of these schools have means in place to support students who wish to undertake serious investigations in mathematics beyond the setting of a course.

There are many formats for undergraduate research established at various schools. Some are formal, such as University-wide Honors or Scholars programs, or a senior thesis or project requirement. Then there is the far less formal one-on-one interaction between a student and a faculty member that leads to exploring some mathematical question. Since the summer research programs available to mathematics undergraduates are few, they are highly competitive, and the students who come to them have likely already considered advanced study in mathematics. In contrast, one of the advantages of undergraduate research experiences in one's own institution is that it can reach students who would not have considered their own

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This article is based on the opening address I gave at this conference. I had the honor of sharing the opening address with Joe Gallian.

talent and ability sufficient for applying to mathematics graduate school. Such students build up confidence in their mathematics abilities through their academic-year research experiences and often end up applying to mathematics graduate schools.

1.2. Introduction to our profession. It is generally accepted that goals for doing research with undergraduates include introducing undergraduates to our profession, and advancing knowledge in a particular field of mathematics. These goals are often met by undergraduate research efforts in our own institutions. Such research definitely introduces students to our profession, but it may or may not lead to new and publishable results. Whether or not a publication results from the experience, I believe strongly that the experience is valuable for the student and for the mathematics community.

The value to the student's mathematical and intellectual growth is undisputed. The value of undergraduate research to the mathematics community lies in a new member of our community seeking advanced preparation in mathematics, or in gaining an addition to the workforce who is knowledgeable about the workings of mathematicians and about the potential of mathematics.

2. The Time Factor

At the University of Dayton, I have had the experience of conducting year-long academic research in addition to having co-directed an NSF-sponsored REU. I think that the luxury of time is a major benefit of working with students on research during the academic year. The pace is far more leisurely than in an intensive summer program, and hence the student and I can spend some more time on various important aspects of undergraduate research. Typically, I advise students after they have heard from me about specific open questions, and after they have expressed interest in at least one of those. We spend some time getting to know the background of the problem, and this allows us to adjust the question so that this particular student can make some progress on it. I suggest readings to the student that provide more background in the subject as a whole (not just on the particular problem). In this manner, the student is always learning, even during the times when the problem doesn't seem to be getting solved. Converging to a problem that is interesting to the student, and that provides the student with a challenge while still not immobilizing her/his creativity, is facilitated by the year-long setting. All the background learning and the fine-tuning of the problem in question usually results in us being able to carve out a niche in mathematics in which the student is an expert.

Academic year research has a feature that is both an advantage and a challenge for the student. Students have to learn to manage their time so that they make sustained progress on the research question while taking a full load of courses. This is a more natural state of affairs than that of devoting an entire summer exclusively to mathematics research.

3. Formats of academic year undergraduate research

From an institutional point of view, a selling point of year-long academic research is that no (or minimal) funding is required, and hence no (or minimal) funders' expectations need to be met. Let me hasten to add that I am in favor of finding funding so that the student can engage in research to earn money to defray

expenses. Funding for faculty release time, or as added compensation to attend a meeting, perhaps, is also desirable and welcome. My point here is to emphasize that funding is not a necessary condition for academic-year undergraduate research.

Academic year undergraduate research in mathematics takes place in many institutions. The Fall 2000 CBMS Survey (Table Ar. 12, Statistical Abstract of Undergraduate Programs in the Mathematical Sciences in the United States, by David J. Lutzer, James W. Maxwell, Stephen B. Rodi, AMS website) provides the percentage of mathematics programs that report having undergraduate research opportunities: 84% of Ph. D. granting institutions, 67% of Master's degree granting institutions and 52% of B.A. granting institutions.

At this conference, many formats and rationales of in-house undergraduate research experiences are represented. I provide examples of some of those whose work I know about, and urge you to contact these faculty if you are interested in knowing more about their versions of undergraduate research.

3.1. Communicating mathematics in paper and poster sessions. Some faculty help students with their research and encourage them to present their work at regional or national conferences. These faculty are involved in providing much-needed guidance to inexperienced students in communicating mathematics effectively. Most students benefit from such attention and their presentations are often noticeably more interesting than those who did not receive faculty help. Some of the faculty who are known to work with students on presentations and who are in attendance at this conference are Ermelinda Delavina of University of Houston-Downtown, Doug Faires of Youngstown State University, Lew Ludwig of Denison University, James Sellers of Penn State University, Dan Schaal of South Dakota State University and me.

3.2. Directing undergraduate research after participating in it. Some of the more recent Ph. D.s in our profession are now directing undergraduate research because they felt it was an interesting and valuable part of their own undergraduate experience. At this conference, some such faculty members are Joel Foisy of State University of New York Potsdam, Stephen Hartke of the University of Illinois Urbana-Champaign, Dan Isaksen of Wayne State University and Darren Narayan of Rochester Institute of Technology.

3.3. Starting locally and building to a funded program. Some faculty or institutions built up a record of working with undergraduates and have now received grant monies to continue on a larger scale, either nationally or with their own students. At this conference, some of those institutions and their representatives are Brigham Young University (Michael Dorff), California State University Channel Islands (Cindy Wyels), Rochester Institute of Technology (Darren Narayan) and Valparaiso University (Zsuzsanna Szaniszlo).

4. Undergraduate research is "hot"

There is a trend in undergraduate mathematics departments to do, or consider doing, undergraduate research. You can find a historical perspective in "Research by Undergraduates is Hot!" by Gallian and Higgins, MAA FOCUS 2002, March Vol 22, #3, 16-17. Evidence of this trend abounds.

- For the past decade or so, Joe Gallian and I have presented an MAA minicourse at the January Joint Mathematics Meetings on getting your students involved in undergraduate research. Each year, we have about forty participants.
- For the last twelve summers, I have presented a course at the Project NExT workshop. (Project NExT - New Experiences in Teaching - is a professional development program of the MAA for new and recent Ph. D.s in the mathematical sciences who are in the first two years of a full-time, post-Ph. D. college or university position.) The four-hour course is entitled "Undergraduate Research: How to make it work." It is capped at twenty-five participants in order to allow for interaction, but that is inadequate to allow everyone who wants it to take it. The seventy or so incoming Project NExT Fellows are asked to rank order six Project NExT courses by interest. About 35% designate the undergraduate research course their first choice, and another 35% request it as their second choice.
- Both Joe Gallian and I are frequently invited by colleges or universities and MAA Sections to provide a workshop on getting started in undergraduate research.

5. Undergraduate research as a pedagogical enterprise

Why is undergraduate research so popular these days? In my opinion, a large part of the phenomenon is due to the broadening of the understanding of the term "undergraduate research." The term no longer refers exclusively to work done by undergraduates that increases the current state of mathematical knowledge in a way that the research community might consider significant. In fact, undergraduate research is now valued as a pedagogical enterprise, with all the ensuing benefits and challenges.

5.1. Institutional point of view. From an institutional point of view, undergraduate research often serves as a recruiting and retaining tool. Several schools have institutional initiatives where first-year students join faculty in research. In mathematics, I am aware of such initiatives at The University of Louisiana at Monroe and at Valparaiso University. Pairing up students with individual faculty mentors in research is often used in conjunction with other retention efforts for minority students.

5.2. Departmental point of view. From a departmental point of view, undergraduate research allows faculty members to stay professionally active at schools with large teaching loads, helping to accomplish the increasing demand of "scholarly activity" at these schools. Curricularly, some interesting capstone experiences can be created for students based on undergraduate research. The department faculty may also be able to bring in small grants for their work with students.

5.3. Showcasing results. In addition to traditional mainstream research outlets, there are many more venues for undergraduates to present the results of their research. Some of these are listed below.

- **Mathfest** (the summer meeting of the MAA) At the Knoxville Mathfest 2006, there were forty-one papers presented in the Pi Mu Epsilon student paper sessions and fifty-one papers in the MAA Student Paper Sessions.

For many years, student paper presenters have accounted for about ten percent of total number of attendees at Mathfest.

- **Joint Mathematics Meetings** (in the winter) There is a poster session on research by undergraduates at the winter meetings. There were over one hundred fifty posters last year, and the number of posters is limited only by the size of the room. More information on the poster session can be found in articles by Mario Martelli on the MAA website. Also, there are paper sessions dedicated to research by undergraduates.
- **MAA Regional Undergraduate Mathematics Conferences** The MAA received a grant from the NSF to partially fund regional undergraduate mathematics conferences. The MAA website has more information on these, including links to the funded conferences.
- **Other Undergraduate Conferences** Many colleges and universities host annual undergraduate conferences in mathematics. Several universities with NSF-VIGRE grants also host such conferences.
- **Conferences for Women** Conferences that are targeted to supporting women in undergraduate mathematics are funded by the MAA Tensor Grants.

5.4. Discovery Learning. A case can be made that some current trends in undergraduate mathematics education favor the implementation of undergraduate research. One such trend is that of discovery learning. This form of learning is championed in K-16 mathematics education. Many undergraduate students are being taught in ways different from lectures. They are engaging in active learning, and some are being taught using modified Moore methods. This could mean that our current students are more attuned to independent learning and discovery than those of past years.

Many departments have instituted an undergraduate research or senior project requirement. When undergraduate research is part of the required curriculum, the faculty at these schools are advocating that every mathematics major is capable of undertaking some measure of research. Consider that most undergraduate mathematics have one of three career plans: being a K-12 teacher, entering the workforce, or going to graduate school in a mathematical science. For pre-service teachers, going through guided discovery is very useful so that they can use that experience to guide their own students through discovery. Learning what mathematics research is also helps them answer questions regarding what mathematicians do. For those majors joining the workforce after graduation, engaging in mathematics research provides them with a heightened ability as problem-solvers. The benefits of doing undergraduate research are very obvious for those majors who intend to go to graduate school.

6. Promoting Undergraduate Research

As we open this conference, we try to state its purpose. Although it is clear that all attendees of this conference are engaging students in undergraduate research, the organizing committee wanted to bring all of us together so we could become aware of the diversity of ways and formats in which undergraduate research in mathematics is conducted. So, the hope of the organizing committee, the hosts and the funders is that we learn from each other about different ways in which we can engage our students in undergraduate research.

7. Thanks

Thanks go to the National Security Agency, and to Barbara Deuink in particular, for funding PURM in both 1999 and 2006, to the AMS (John Ewing, Ellen Maycock and all the staff) for making all the abstract arrangements concrete, and to Joe Gallian for agreeing to edit the Proceedings. I feel immense gratitude to each of you for being willing to share your successes (and not-quite-successes), and for coming to this conference to increase your awareness of the diversity of mathematics undergraduate research.

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