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Promoting Undergraduate Research in Mathematics**

The Hope College REU Program

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Overview

Undergraduate student research has long been the hallmark of the Hope College Science Division with over 150 students involved in research each summer. This research is supported by a number of grants including a divisional HHMI grant and NSF-REU awards in Biology, Geology, Chemistry, Physics, Mathematics, and Computer Science. Excepting one year, the Department of Mathematics has been supported by NSF-REU funds each year since 1991 resulting in twenty papers published or submitted.

The objective of our program is to give students the opportunity to "get their mathematical wings." That is, over the course of the summer we aim to guide them through the entire research process - selection of a problem, literature research, reading and understanding mathematical articles and texts, grappling with and solving a problem, effective oral explanation of mathematical ideas, the writing of a mathematical paper suitable for publication, and presentation of the paper. That is much to do in eight weeks. It may or may not result in a published paper, but from our experience it nearly always results in students who have a new understanding and appreciation of – and confidence in their ability to do – mathematical research. This is the first step towards a successful career as a scientist/mathematician.

Significantly, such an experience is not typically a part of the undergraduate education of most mathematics students. Our objective is to give this opportunity to students who 1) have the potential to benefit from it (which means that they have requisite ability, are self motivated, and can work well with others), and 2) do not otherwise have the opportunity for such an experience. In particular, we look for students who have not yet participated in an REU and who are from schools where research experiences are not available.

Along with advanced undergraduates who have chosen to major in mathematics, we hope to attract and encourage some of the students who have finished just their first year of higher education and have not yet committed themselves to a particular discipline. We have found from past experience that some of the brightest and most talented students spend time exploring various interests until they are generally obliged to choose a major in their sophomore year. Providing them with a positive research experience at the appropriate level can serve to attract some of

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these students by showing them an appealing side of mathematics they would not otherwise see. Thus some of our projects will be tailored for students who have not yet taken advanced undergraduate courses.

Summer research projects supported by the NSF and Hope College are offered to 6-10 students in probability (involving pebbling and geometry), analysis (involving dynamical systems and computational mathematics), and algebra. Students work in pairs with faculty mentors for eight weeks beginning the first Monday of June. They receive a stipend of $3200 and are provided with free housing and transportation.

Two intentional trends occur throughout the summer. The first is a trend from dependence to independence in reading literature and solving problems. The second is a trend from giving informal presentations to the writing and formal presentation of a paper suitable for submission for publication. As such, the student has experienced a microcosm of graduate research. The ability and confidence so gained is transformational as students feel the flush of accomplishment at successfully investigating, solving, and presenting their research problems.

Nature of Student Activities

The goal of the summer research program at Hope College is to help talented and motivated students develop ability and confidence as mathematical researchers by achieving significant mathematical results in partnership with faculty mentors. In particular, students will 1) learn new mathematics, 2) gain mathematical independence by learning to read math texts and journals, 3) learn how to do library/electronic research, 4) solve an original mathematical problem, 5) learn to communicate mathematics verbally through informal and formal oral presentations, 6) write a mathematical paper suitable for submission to a journal and present it at a conference, and 7) learn the use of LaTeX, Maple, PowerPoint, and MathSciNet.

Students will initially learn about their own projects and those of the other students through introductory lectures given by the faculty researchers and/or by reading requisite background material provided by their mentors. Then, for the bulk of the summer, the students will be given their own project or research problem to solve - typically working in pairs. (We find from experience that pairs offer the best dynamic. Working with another student is generally more fun and enhances the communication skills. Groups larger than two allow for one to be left behind.) During this time students work on their problems with regular visits and help from their mentors as needed. Since students live and work together, they share ideas and strategies throughout the summer continually on an informal basis as well.

Our particular REU model has students working on projects in distinct areas of mathematics. This seems to work well; students do their own research in one topic and are exposed to a couple other areas via a weekly research seminar in which student researchers present their ongoing work. These presentations, which begin as informal progress reports and evolve over the course of the summer into more polished and professional presentations, hone the oral communication skills of presenters and critical listening skills of students in the audience. **Learning to effectively communicate mathematical ideas both formally and informally is a primary goal of the research experience.** In particular, students should learn how to be critical and creative in constructing and presenting a well-honed formal presentation. But they must also learn how to give informal, impromptu

expositions. In both cases, the key is not to give a slick, flawless delivery, but to effectively communicate mathematical ideas. This involves focusing on the listener rather than on the material/presentation. It involves asking the question, "Did they understand?" rather than "Did I make any mistakes?" The difference is subtle but important, and we stress the distinction. Through formal and informal critique, presenters are evaluated on the clarity of their exposition and their ability to engage the attention of the audience. Students assess their own growth in effectively communicating mathematical ideas in an end-of-the-summer questionnaire.

Finally, for the last 1-2 weeks of the project students concentrate on writing their results in a paper suitable for publication. **Developing skills in writing mathematics is another primary goal of the research program** at Hope for several reasons. First, writing contributes to clear, precise thinking. Second, the ability to communicate well is an essential component of higher education. Third, since many students have no opportunity to write an extended technical paper as part of their undergraduate experience, it is one of the best preparations we can provide them for graduate school and/or a professional career.

Generally speaking, research projects have been inspired by problems in the faculty members' own research programs and will ideally generate original results. Past experience has indicated that it is best to find projects with a range of partial outcomes, and the projects have been chosen with this in mind.

The student workday is from 9 a.m.– 4:30 p.m. We encourage them to spend their evenings and weekends recreating. This helps avoid two possible pitfalls. First, sometimes even disciplined, motivated students who are working independently for the first time on a long-term problem can fall into a relaxed summer slump. Setting regularly scheduled work hours helps avoid that and allows students to work together most effectively. On the other extreme, conscientious students sometimes need to be encouraged to balance their research with leisure activities. Students should leave the program fully satisfied and proud with what they have accomplished, but also refreshed and energized for a new school year.

During the final week of the project, students give formal presentations of their research at a Michigan Mathematics REU Conference which we put on jointly with two other math REU in the west Michigan area (GVSU and CMU). Each summer ends with a Michigan Mathematics REU Mini-Conference (rotating between the campuses) which provides opportunity (and audience) for all of the REU students to present their research results. In addition to the student presentations, the conference includes a guest speaker or a mathematics game competition.

We also encourage our participants to present their results at regional or national mathematics meetings in order to gain a sense of the wider mathematical community, and our budget includes funds to support such travel. Over the years, many of our REU students have presented papers and posters at the Joint AMS/MAA Meetings, MathFest, various regional conferences, and at their home institutions.

Since an overarching goal is to promote mathematics research as a career, at some point in the project we often involve an outside research mathematician to give talks on his/her own research in particular and the nature of graduate mathematical research in general. Alternatively, we sometimes take the students to Chicago to visit the departments at the U of Chicago and Northwestern University.

The departments with REUs also take turns hosting seminars to which all research students are invited. Recent examples include: "Preparing for the GRE", "The ethics of scientific research", "Do dogs know calculus? - an example of the scientific method", and "Research similarities and differences across the disciplines."

Although our goals are centered around research, the success of the research experience depends heavily on the social aspects of the summer. First of all, such activities help foster good student relationships and friendships. Since students spend considerable time each day studying together, the increased intensity and enjoyment that comes from working with friends is an essential component to a productive summer of research. Secondly, providing the students with a wide variety and constant stream of activities for their free time helps ensure an enjoyable and memorable summer.

The first week is especially important since it sets the tone for the summer. Also, since students are coming to a new and unfamiliar dwelling for a relatively brief time, we want them quickly to get to know each other and the area's recreational opportunities (and to have little opportunity to feel lonely or homesick). So on the Sunday evening before research begins, we meet for pizza and a trip to the beach and sand dunes of Lake Michigan. We have a grill-out on the first Monday, and spend the first Saturday afternoon at another beach and visiting area attractions.

We find that if we keep them fully engaged the first week, by then they are comfortable and excited to go off on their own adventures. Our present group of students just returned from a weekend together in Chicago and they have already organized several bonfires at the Lake Michigan beaches and several bowling and movie nights. We plan events as well for the July 4th holiday weekend, some faculty mentors host board game nights, and the PI, being a classic movie buff, invites the students to his home for *Casablanca*, *The Sting* and other great movies.

Departments take turns hosting Thursday afternoon ice cream socials and Tuesday night beach volleyball picnics. Ultimate frisbee is available on Wednesdays and Sundays. On Wednesdays the math REU has lunch together. Students organize scavenger hunts and late night capture the flag. Between planned and spontaneous events, almost every day offers the opportunity for a social/recreational activity.

Student Recruitment and Selection

We aim to recruit and select students who i) have the requisite talent and interest, ii) are self motivated and disciplined, and iii) can work well with others. We identify such students through application materials which includes their college grade transcript, two letters of recommendation from professors, and their own statement of interest in mathematical research. We also note whether they have taken full advantage of extra-curricular mathematical opportunities available to them - such as competitions and participation in departmental responsibilities such as tutoring.

We target students who have not yet participated in an REU and who are from schools where research experiences are not available. We realize that some students enjoy and benefit from multiple REU experiences, but with the demand greater than the number of available positions, our site operates under the philosophy that the NSF-REU program should give as many students as possible an experience

in mathematical research. For those wanting more, graduate school awaits. We also give lower priority to very good candidates who have already had substantial research experiences from their home institution. Some submit published papers with their applications. These students are to be congratulated; they are very deserving, but we give others higher priority. The ideal student is the one (often from a small college or regional university) who is described as being the best student in many years, who has taken advantage of everything the department has to offer, and who is still hungry for more.

Increasingly, students learn about our program through the NSF and MAA web pages. However, we still target certain schools, sending them a full information packet describing the Hope Undergraduate Research Program, with program posters, application forms, and project descriptions. These include regional and peer institutions as well as a smattering of colleges and universities throughout the United States in order to attract students from various institutions and different parts of the country.

Also included in the target group are a number of women's colleges and institutions with significant minority enrollment (such as Morehouse College and Spelman College in Atlanta) which have strong mathematics programs and/or an interest in undergraduate research.

Prospective participants can apply via an online application form. Deadline for submission is February 28, and we typically begin making offers within the second week of March.

Project Evaluation and Reporting

At the end of the project, students are given an evaluation form which asks the following questions:

(1) **Local Arrangements**
- Were housing accommodations satisfactory?
- Were office, computing, and library facilities sufficient?
- Were the number and nature of organized social and recreational activities satisfactory?

(2) **Individual Projects**
- Was the topic at a level appropriate for your background?
- Was the amount of guidance you received from your advisor too little, too much, or about right?
- Are you satisfied with what you learned both about mathematics and the nature of mathematical research?
- Did you learn to effectively communicate mathematical ideas - both orally and in written form?
- Were the opportunities to learn about the work of others adequate and helpful?
- Are you more/less likely to pursue a career involving mathematics research after this research experience?
- If you were directing the research/program, what would you do differently?
- What advice would you give to a friend coming next year?

(3) **Any other comments?**

These evaluations have provided valuable feedback over the years, and we have implemented many of their ideas and suggestions. Long term evaluation occurs informally through continued contact with students as we revise papers for publication and prepare talks for major meetings. Students also write to ask for letters

of recommendation, and still others stay in close contact through the friendships which have developed.

Feedback from student participants has been very positive. Many respondents indicated that they were more likely to pursue graduate study in mathematics because of the program. Some examples:

- "The REU was a good experience for me. I had already considered graduate school in mathematics, but the program gave me more confidence and direction with that decision. I learned skills which helped me greatly during my first year of studies here at the University of Texas at Austin. I also made friends whom I still keep up with to this day."
- "The REU program was the one experience that solidified my goals to pursue a higher degree. This was my first exposure to academic research. The experience was so enjoyable and challenging that I knew I wanted to seek a position that would keep me close to the study of mathematics."
- "It was one of the most interesting and most fun summers I have ever had . . . This REU was conducive to thought, creativity and play - which is an excellent combination." (Ph.D. from U of Michigan, post doc at U of Oklahoma.)
- "This experience convinced me that I will definitely follow a career in research mathematics. This was a tremendously great experience. I cannot imagine a better way for a mathematics undergraduate to spend a summer. The program was excellently run and the advisors did a great job of advising us."
- "The Hope College REU was my first crack at mathematics research, and got me started in the right direction. If I hadn't gone to it, I believe I would not be where I am today. It was truly helpful to get an idea of how math research goes at such an early stage in my career. (Ph.D. from Stanford, post doc at U of Arizona.)

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