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Research With Students from Underrepresented Groups

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Introduction

In 2003 the *MAA National Research Experience Program for Undergraduates* was created for students from underrepresented groups. The program provides grants to institutions that support four or five students and a project mentor. To insure that minorities are chosen, prospective students must be named in each incoming proposal. In 2005 Darren Narayan was awarded one of twelve grants to run this program at the Rochester Institute of Technology (RIT). Dr. Narayan conducted his project for six weeks during the summer with two African-American and two Hispanic student researchers.

Although RIT has a diverse population of students, it was difficult to find four mathematics majors from underrepresented groups. Two outstanding mathematics majors were selected from Dr. Narayan's discrete mathematics and graph theory courses: Rachell Ashley, a 4th year Applied Mathematics Major from Ft. Worth TX and Carol Callesano, a 2nd year Applied Mathematics major from Ganesvoort, NY. Two more were selected from his calculus classes: Aisosa Ayela-Uwangue, a 2nd year Electrical Engineering major from Benin, Nigeria and Frances Cabrera, a 1st year Environmental Management major from Virginia Beach, VA

The four students bonded quickly despite different backgrounds and being in different stages of their academic careers. Each student brought their unique talents to the table and a strong research group developed. Not only did the group nearly solve an open problem in combinatorics, but their work culminated in an award-winning poster session at the National Meetings in San Antonio in 2006. Publication of their work in refereed research journal is also in the works.

Selecting a Research Problem

Since two of the students were not mathematics majors, Dr. Narayan searched for a project that was easy to explain to non-mathematicians. On the first day he told them, "I want you to have a research problem that you can explain to your mother, father, sister, brother, aunt, and uncle. While I am not all that interested in whether your family members understand, I do care if one of your potential employers does. This research experience will be a prominent item on your resume

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that could come up during an interview. You might have a 30-second window to explain your research. You have to make it count."

Our project in combinatorics is a rectangular tiling problem using tiles with dimensions 4×6 and 5×7. For a proper tiling the entire rectangle must be covered so that no tile extends beyond the rectangle and no pair of tiles overlap.

We considered the following problem: Determine all natural numbers m and n such that an $m \times n$ rectangle can be tiled with 4×6 and 5×7 rectangles.

It was important that the students become interested and excited about solving this problem. Six weeks is a short period of time for research. Potential applications gave this problem extra appeal. Many companies use two-dimensional load software to obtain optimal packing configurations for shipping. Another application unique to the dimensions 4×6 and 5×7 involves optimal cutting of photo paper. For example it was shown in [**2**] that a 34×33 rectangle can be tiled and hence a 34×33 sheet of photo paper can be partitioned into popular sizes 4×6 and 5×7 with no waste. Since the methods of solving the problem could be grasped quickly, the students could produce results quickly which greatly added to their enthusiasm. One student, Frances Cabrera, commented "I never imagined I'd get so excited, even passionate, over a seemingly obscure theoretical combinatorics problem, but I did. This made the entire experience much more enriching. It made me realize how much I enjoy mathematics and decide not to let math leave me."

The student's excitement was not confined to the group. They wanted to share their experiences with others. In two cases friends of the student researchers mentioned that they were intrigued by the problem. One non-NREUP student commented. "My friend was excited about the research problem she was working on. She explained it to me and I got excited."

The Research

Tiling problems of this specific nature were introduced on the 1991 William Lowell Putnam Examination. Problem B-3 asked "Does there exist a real number L such that if m and n are integers greater than L, then an $m \times n$ rectangle may be expressed as a union of 4×6 and 5×7 rectangles, any two of which intersect at most along their boundaries?" Clearly some rectangles, such as the 7×9 and 8×10 rectangles, can not be tiled. Allen Schwenk and Darren Narayan showed that any rectangle with both dimensions at least 34 can be tiled [**2**]. However this does not cover cases involving rectangles, such as 23×91, 17×200, etc. where one dimension is less than 34. For any fixed width, the length can be arbitrarily large so the group had to consider infinitely many rectangles. This was in no doubt an ambitious project.

Allen Schwenk mentioned that he had considered working on this particular problem years ago, but was hesitant with the large number of cases involved. However the students were enthusiastic about tackling a challenging problem.

After four weeks, we were able reduce the number of rectangles under consideration from an infinite number to a mere 54. This was a tremendous accomplishment. A computer search could now be brought in to finish the work. However, we wanted to obtain a solution that was independent of a computer. However many of the cases proved to be quite difficult. We contacted Allen Schwenk, notified him of our progress, and offered to include him in the project. He was delighted and provided tips on further reducing our list of 54 cases.

During the final two weeks we focused our attention on these cases. Our conjecture is that many of these rectangles can not be tiled, but this is difficult to prove. Using various combinatorial arguments, we made steady progress and showed one by one that many cases could not be tiled. This process gave the program an incredibly nice feature. We would write the number of remaining cases up on the board, and watch as the number steadily decreased. We would often be asking each other, "What number are we down to?" Not only did this add excitement, but it also gave us an additional measure of our progress.

Program Activities

In addition to the program's research component, we spent a considerable amount of time talking about graduate school. We discussed various programs, as well as graduate studies in general. Midway through the program we attended a master student's defense. As a result the student researchers were not as intimidated by the thought of producing and defending a thesis. They commented afterwards that a defense should not be scary. By that time the student knows their work better than anyone.

We also benefited from interactions with more than 100 other undergraduates and mentors working on research projects in the College of Science at RIT. Activities included a seminar series featuring student presenters. The NREUP students enjoyed presentations from other scientific fields, and gave their own dynamic presentation during the program's final week to an audience of nearly 100 students and faculty.

The last day of the program was spent on a field trip to Letchworth State Park. This provided an informal setting to talk about graduate school, interviewing for permanent jobs, and future career paths. It also gave us an opportunity to reflect on everything we had accomplished.

Follow Up Activities

The program officially ended in the summer, but we continued working on the problem. There are now only 18 cases to resolve. The four students presented their work at the 19th Midwest Conference on Combinatorics, Cryptography, and Computing, held October 7-9, 2005 at RIT. They presented their work again in the 2006 AMS-MAA-SIAM Special Session on Research in Mathematics by Undergraduates in San Antonio, TX. The group also presented in the Undergraduate Student Poster Session at the same meeting. To cover the students' travel expenses Dr. Narayan secured funds from RIT alumni, the RIT Honors Program, and the students' home departments. In addition, the students received support from the *MAA Diversity Initiative*, a program designed to increase the number of women and minority students attending the Joint Mathematics Meetings.

The national meetings were a tremendous success. Both their oral presentation and poster presentation were well received. The students were named as one of the winners in the Undergraduate Poster Session and received a $100 cash prize. A picture of the group celebrating their win appeared on the cover of March Issue of FOCUS, the newsletter of the MAA. We are currently preparing their results for publication in a refereed journal [**1**].

References

[1] R. Ashley, A. Ayela-Uwangue, F. Cabrera, C. Callesano, D. A. Narayan, and A. J. Schwenk, Necessary and Sufficient Conditions for Tiling with 4×6 and 5×7 Rectangles, to be submitted.
[2] D. A. Narayan and A. J. Schwenk, Tiling Large Rectangles, Mathematics Magazine, 75, No. 5. (2002), 372-380.

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