Proceedings of the Conference on
Promoting Undergraduate Research in Mathematics

The Rose-Hulman REU in Mathematics

Kurt M. Bryan

1. Overview

In the past 18 years 126 undergraduates have participated in Rose-Hulman's NSF-REU program in mathematics. The program began in 1989 under the direction of Gary Sherman, who mentored six students in computational group theory each summer. In 1996 Allen Broughton began co-directing students, and subsequent renewals from 1997 to 2003 under Dr. Broughton expanded the program to eight students per year, working in teams of four with two faculty, including John Rickert and Kurt Bryan on a rotating basis. Kurt Bryan took over as director in 2004, and faculty members Tom Langley and David Finn began mentoring students. The list of research topics was also enlarged based on the new faculty research areas, to include hyperbolic geometry, number theory, inverse problems, and geometric analysis. The program has evolved from a single mentor to one in which faculty rotate in and out from summer to summer, offering a wider range of problems for undergraduates to explore.

Although the areas of research we offer are quite diverse, we do have a common approach: the significant use of computational tools like Magma, Matlab, or Maple to help students understand the problems at hand, begin making conjectures or designing algorithms, and then follow up with rigorous analysis. The problems are carefully designed so that students can start work immediately, without extensive background beyond standard undergraduate course work in the area of interest. However, we do not give them "canned" or "toy" problems, and students always find that they need to master new mathematical tools as the summer progresses.

We emphasize collaboration and communication throughout the summer. Students typically work in pairs, and the four students in each research group share a common work area. The students make regular presentations of their progress to the other REU participants and faculty, and research results are also published in the Rose-Hulman Mathematics Technical Report series. Many papers from our REU have appeared in professional journals.

Students are selected based on having adequate preparation to succeed in the program, and we particularly seek students who express interest in a career involving research, but who have not had the opportunity to engage in research at their home institution.

Received by the editor October 20, 2006.

2. Goals, Philosophy and Problem Selection

2.1. Goals. The goal of our program is simple: to give undergraduates the chance to play the game of mathematics at the professional level. Specifically, students

- Tackle research problems which start out a bit nebulous, and for which no "answer key" exists. Emphasis is placed on clarifying and simplifying the difficult initial problem as a means of gaining understanding, analyzing specific examples, making and proving conjectures, and adding additional complication once simpler versions are thoroughly understood. Computational tools are frequently useful here.
- Improve their collaborative skills by working intensively with other students and professional mathematicians.
- Develop their speaking skills by giving frequent formal presentations of their results during the program, as well as at conferences after the program ends.
- Improve their written communication skills, by preparing a report in the Rose-Hulman Mathematics Technical Report series, and when appropriate, refining and submitting the work to a professional journal.
- Develop their ability to be independent learners, by mastering relevant material in professional texts or journals (including those written by previous REU participants).
- Develop a sense of identity as mathematicians, as being capable of original insight and contribution to mathematics.
- Develop a sense of belonging in the mathematics community, by forming relationships with professional mathematicians, in and out of our REU program.
- Deepen and broaden their knowledge of one or more areas of mathematics.

2.2. Student Recruitment. Each year we mail notices of the program to several hundred schools nationwide, and in particular to schools in which students may lack access to the kind of research program we can offer, perhaps because of a lack of sufficient faculty involved in research. The mailing list is updated regularly to reflect any good leads we get on appropriate students, and of course our large web of prior year REU participants and their institutions can be a good source of applicants. Faculty also use professional contacts at other institutions in order to identify appropriate applicants.

The students we seek will generally have completed their junior year, though well-qualified sophomores or even freshmen may be accepted. The desirable student will have completed a year of differential equations (ordinary and/or partial), or a year of abstract algebra, depending on the research area of interest to the student (which they specify on the REU application form). They should also have experience in some computer programming language, though not necessarily any of those we use, an ability to work with others, and of course a drive to achieve in mathematics. Of particular interest to us are students who have not had a significant research experience, but wish one, as a basis for making a career decision. To demonstrate these qualities students must submit a letter of interest, academic resume, transcripts, and two letters of recommendation.

2.3. Program Approach and Philosophy. Students are selected for the program by mid-March and arrive in early June. In that interval we make available to them, via email or the web, a small amount of preliminary material to look over, to familiarize themselves with the problems and past results, though we don't impose any great workload, since most students are still in class.

The problems we give the students are typically an integral part of the faculty mentor's research program. The problems are thus "real" research questions, of significant interest to mathematicians, but which can be initially simplified if necessary to gain understanding and begin making progress. Some involve extending techniques which worked in one setting to new situations. In all cases we emphasize experimentation with software to allow students to build insight and begin making conjectures or trying algorithms.

We put a strong emphasis on getting students to work on the problems very quickly, preferably within the first week. The first few days of the program are split between lectures from the faculty mentor followed by students working together on simple versions of the problems (workable in an afternoon or less), as a means of building the students' understanding of what the research question is, if not an approach to the solution. The students are expected to have settled on a specific problem early in the second week. At this point the students have generally decided with whom they would like to work, and are strongly encouraged to team up with at least one other person.

Each group of four students has exclusive use of one classroom, outfitted with tables, chairs, whiteboards, and computers, at least one workstation per student. All computers run up-to-date versions of TeX/LaTeX, Magma, Maple, Matlab, and Femlab (a powerful 3D finite element PDE solver). No fixed daily schedule is imposed, but the students have 24 hour access to the room. The faculty mentor has an office next to the classroom and wanders in and out during the day, talking with students, suggesting things to try, giving lectures when the students need to know something. Lunch is taken as a group almost every day. The close physical proximity of all eight students is a powerful factor in building collaboration and preventing anyone from feeling isolated.

Presentations are required from each problem team, about every two weeks starting in the third or fourth week. These presentations are made to the entire REU group of eight students and faculty, and are frequently attended by other Rose-Hulman faculty as well. In addition, during the final week of our program in the summers of 2002-2006, the faculty at nearby Indiana University staged a conference for all mathematics REU participants in Indiana (30 to 50 students, 8 to 12 faculty), in which all of our students presented their work, a tradition we intend to continue. These regular presentations and final presentation to an outside group encourages students to keep plugging away at the problems. Each student must also hand in a draft of a technical report by the final day of program. The faculty mentor reviews the report and students make revisions after the program ends (these are usually fairly minor).

The relationship we strive for with the students is one of junior and senior colleagues, working as a team to understand a difficult problem. Although we choose problems on which we can be reasonably certain students will make progress, we do not steal the problem from the students by telling them how to solve it. This approach, along with the polished technical report and presentations, gives the students a sense of pride and ownership of the results.

3. Project Evaluation and Reporting

Our central goals are to improve the participants' communication skills, both oral and written, to develop their collaborative skills, to help them develop intellectual independence, and get them excited about a career involving mathematical research.

The most immediate assessment of our success occurs by the end of the program itself: We expect each student, collaborating with others, to contribute to a first draft of a technical report which advances mathematical thought, however modestly. We expect every student to give a minimum of three formal presentations during the course of the program, incorporating feedback to iteratively improve each succeeding presentation. We also conduct exit interviews and have students fill out a questionnaire to obtain immediate feedback on the quality of that summer's program. The students have been overwhelmingly positive about their experience in our program, and most feel it helped them make a more informed career decision.

In the longer term, a primary measure of our success is that every participant complete a polished technical report and give at least one presentation at his or her home institution or at a conference, concerning the results obtained during the REU. Indeed, a significant portion of our budget is devoted to supporting students who travel to conferences after the program has ended; in recent years many students have presented the REU work at the national AMS/MAA joint meetings. The REU mentors also usually attend these meetings and hence have a chance to follow-up with students on research and career plans. We also gauge success in improving student mathematical and writing skills by the number of reports which prove suitable for publication in professional refereed journals.

After the program ends we maintain close contact via email with almost all of the students (for at least a couple years, since we serve as important letter writers and advisors for graduate schools). Because of this continued contact we have built up and maintain a database of student career choices and/or graduate schools, as evidence for the impact of our program in producing independent mathematical scholars. Of those 82 participants from 1995-2005, 64 went to graduate school, 7 went into industry, 4 are still undergraduates, and there are 7 on which we have no data.

Department of Mathematics, Rose-Hulman Institute of Technology, Terre Haute, IN 47803

E-mail address: `Kurt.Bryan@rose-hulman.edu`