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Promoting Undergraduate Research in Mathematics at the University of Nebraska – Lincoln

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The Department of Mathematics at the University of Nebraska – Lincoln (UNL) has several programs which promote undergraduate research in a variety of ways. Two of these are summer programs which draw from a national applicant pool: The Nebraska REU in Applied Mathematics (Section 1) is a traditional NSF-funded REU site, and Nebraska IMMERSE (Section 2) offers a summer "bridge" program (with a research bent) for students about to start graduate school in mathematics. IMMERSE is a relatively new program, started in 2004 as part of the department's Mentoring through Critical Transition Points (MCTP) grant from NSF. The MCTP grant also is now the primary source of funding for two conferences involving undergraduate research which the department launched in 1999: The Nebraska Conference for Undergraduate Women in Mathematics (NCUWM) (Section 3) and the Regional Workshop in the Mathematical Sciences (Section 4). The bulk of the program at NCUWM consists of talks by undergraduates on their own research, and, while the original goal of the Regional Workshop was to forge and maintain ties between faculty at smaller college and universities, it has recently been expanded to provide a forum for undergraduates to present their research. Finally, we offer several opportunities for our own undergraduates to do research: the MCTP Undergraduate Scholars program (Section 5), the Research for Undergraduates in Theoretical Ecology (RUTE) program (Section 6), the Undergraduate Creative Activities and Research Experiences (UCARE) program (Section 7), and two upper-level undergraduate courses which aim to give students a taste of mathematics research (Section 8). This article provides a brief overview of each of these programs; more details can be found online at `http://www.math.unl.edu`.

1. Nebraska REU in Applied Mathematics

The department has offered the Nebraska Research Experience for Undergraduates in Applied Mathematics as an eight-week NSF-funded REU Site each summer since 2002. We have offered an average of three projects each summer. A typical project group includes three or four undergraduates, one or more UNL faculty members, and a UNL graduate student. Our goal is to give the students as full a research experience as possible, including how to define a good problem, how to investigate

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the problem, how to come up with solutions and/or models for the problem, how to write mathematics, and how to give a talk. All of the projects are in applied mathematics, most have an interdisciplinary component (with an emphasis on biology), and many emphasize the role of computer exploration in gaining insight into mathematical problems. A few past project titles have been: "The Spread of Information and Social Interactions," "Exponential Stability of Dynamic Equations on Time Scales," "Chaos Theory in Food Chains," "Dynamics of Fish Populations Dependent on Cannibalism," "Game Theory and Population Dynamics," "Dynamics of Tumor Growth," and "Control Theory Techniques Applied to Population Problems."

Detailed project descriptions, including prerequisites, are posted on the web by the end of the preceding December. The students choose those projects they wish to apply for and rank them according to preference in their applications. Students are selected for individual projects before the Site starts, allowing for preliminary communication between the mentors and the students. When choosing students, we put a premium on how much value our Site can add to their education, rather than making admission decisions based solely on the quality of their previous work. We have had an approximately even gender balance among the student participants. A bit more than half have been from non-Ph.D. granting institutions, almost half have been from the midwest, and about 7% (fewer than one each year) have been from UNL.

There are certain features common to all of the projects. We want the students to have an experience which is very different from taking a course. The students are expected to work together closely. We want them to dive in head-first, learning material as needed. For the first few weeks, the students meet with their faculty and graduate student mentors daily for at least an hour, with the mentors giving informal lectures, exercises, and possibly reading assignments. We do not assume much expertise (if any) in the research topic, and so the material given in these first few weeks provides the students with the background necessary to do their research. The projects are quite demanding, often involving advanced material, and we've learned that most mentors find it desirable to continue to meet most weekdays throughout the eight weeks, initially to help the students stay on track and ask the right questions, and then to guide them in preparing their reports and presentations. During the last week of the Site, we hold a mini-conference, where the students give an oral presentation of their work. In most cases a paper is written, with work being finished after the Site is officially over. If the paper is submitted to an undergraduate journal, most of the writing is done by the students. If the paper is submitted to a professional journal, the faculty mentor takes a larger role in the finished product. Most of the students present their results at conferences during the year after the Site, with their travel being funded by the department.

2. Nebraska IMMERSE

As part of the department's MCTP grant, we run a summer program called Nebraska Intensive Mathematics: a Mentoring, Education and Research Summer Experience (IMMERSE). In a sense, Nebraska IMMERSE is really two programs: one that develops the teaching, research, and mentoring skills of graduate students and early-career faculty, and one that strengthens the preparation of students as they begin the transition from being undergraduate students to being graduate students.

We focus here on the "pre-grad" program; for information on the "early-career faculty" portion of IMMERSE, see the departmental website.

Sixteen "pre-grads" — students who will be starting graduate school, either at UNL or elsewhere, in the fall — comprise the participant list for IMMERSE. The pre-grads emerge from this program with a strong foundation on which their first-year graduate courses can build. They gain experience in working in groups on problems. They develop a solid understanding, with examples in hand, of several of the topics they will encounter in their graduate courses. They gain exposure to mathematical research and start to develop the skill of learning through mathematical research literature. They are also exposed to some of the issues which they will face as teachers, and they develop a network of peers and mentors on whom they can continue to rely as they begin graduate school.

The program lasts six weeks, and the main component consists of two intensive courses: one in algebra and one in analysis. The courses are at the advanced undergraduate/beginning graduate level, with, for example, Herstein's "Topics in Algebra" and Rudin's "Principles of Mathematical Analysis" as resources. However, rather than working through a textbook, the courses are structured around the reading of research papers. The instructors (pre-tenure faculty at four-year colleges who are participating in the early-career faculty portion of IMMERSE) select papers that use as tools some of the topics that typically appear in first-year graduate algebra and analysis courses, and then structure the IMMERSE courses around the material of the papers. The papers are typically relatively recent and reasonably short, and have a single easily-understood result.

For example, the algebra course from IMMERSE 2006 was structured around the 1995 *Houston Journal of Mathematics* paper "Parametric decomposition of monomial ideals (I)" by Heinzer, Ratliff, and Shah [**1**]. This paper is concerned with existence and uniqueness of decompositions of monomial ideals, that is, ideals generated by monomials in a fixed regular sequence $x_1, \ldots, x_n$ in a commutative ring R with identity. The decompositions under investigation are finite intersections of parameter ideals, which are particularly simple monomial ideals — those of the form $(x_1^{a_1}, \ldots, x_n^{a_n})R$. It is straightforward to show that in order for a monomial ideal J to admit such a decomposition, it is necessary that J contain a power of each element x_i. The authors show that this condition is also sufficient and prove that irredundant parametric decompositions are unique. (Think of unique factorization of integers or polynomials in one variable.) One feature of the article is a geometric interpretation of the parameter ideals occurring in each such decomposition, based on the "staircase diagram" associated to the ideal J. After a bit of time spent on a review of undergraduate-level material, there was an introductory lecture which introduced commutative algebra and explained why decomposing monomial ideals is interesting. The students then read through the paper, probably not understanding much of it beyond the statement of the main result. The intensive algebra course then began, covering the basics of commutative rings, ideals (including the operations of intersection, sum, product, radical, and colon), and the Noetherian property (including the Hilbert Basis Theorem). Notice that while these topics were inspired by the paper chosen by the instructors, most of them are still standard material in a first-year graduate course in algebra. Every so often, the students re-read the paper, understanding more and more each time. By the end of the

course, the students had learned quite a bit of algebra, had some good examples in mind related to the topics covered, and completely understood a research paper.

Each course is team-taught by a pair of early-career faculty participants, with a mixed group of first-year and advanced graduate students serving as teaching assistants. Each course has a daily one-hour lecture plus an afternoon problem session run by the graduate student assistants. Students are encouraged to work together on problems and take turns presenting solutions on the board to each other. They also using LaTeX to formally write up their solutions to additional problems. There are several special presentations, colloquia and workshops throughout IMMERSE, each of which provides an additional forum for exploring issues the pre-grads are likely to face as they begin graduate school.

We solicit applications nationwide for thirteen of the sixteen pre-grad positions; the remaining three slots are reserved for incoming UNL graduate students who will be supported the following academic year as First-Year MCTP Graduate Trainees. Selection is based on transcripts, letters of recommendation, and a personal essay on what the student hopes to gain from the program. All acceptances are provisional, with the understanding that the student must actually be accepted to, and commit to attending, a graduate program in mathematics starting that fall.

3. NCUWM

The annual Nebraska Conference for Undergraduate Women in Mathematics (NCUWM) has been held in early February each year, starting in 1999. The conference brings together women undergraduate math majors from all over the United States. The main part of the program is a series of talks by the undergraduate women about their own research. Two prominent women mathematicians give plenary research talks and there are panel discussions relating to graduate school and mathematics careers. When possible, we also have NSF and NSA representatives give presentations on opportunities available through their agencies for undergraduates and graduate students.

Close to 200 students from across the country participated in the most recent (2006) conference. Roughly 50 of the students presented their research, either via talks or posters. Plenary speakers for the first eight conferences have included four members of the National Academy of Sciences, at least three former Vice Presidents of the American Mathematical Society, at least five former Presidents of the Association for Women in Mathematics, at least one former President of the Society for Industrial and Applied Mathematics, and one winner of an individual Presidential Award for Science, Mathematics and Engineering Mentoring.

The conference is now funded by the NSF (via our MCTP grant) and the NSA. Previous conferences have been funded by a mixture of conference grants from both the NSF and the NSA, internal UNL funds, and an NSF grant associated with the 1998 Presidential Award for Excellence in Science, Mathematics, and Engineering Mentoring. In fact, the conference was started in celebration of this award, which the department received in recognition of its success with female graduate students.

4. Regional Workshop in the Mathematical Sciences

UNL hosted its first Regional Workshop in the Mathematical Sciences in the spring of 1999 and we have continued to host a workshop in each fall semester since 1999. The primary goals of the conference are two-fold: to foster research contacts

among faculty and students at colleges and universities in the region, and to expose undergraduate students to research-level mathematics across the discipline, while providing them with information about graduate education in the mathematical sciences. The workshop features six 45-minute plenary talks on Friday afternoon on a broad range of mathematical topics, followed by a banquet, a panel discussion on a topic related to graduate education, a social gathering on Friday evening, and a series of parallel sessions of talks on Saturday. Beginning with the 2004 conference, at least one of these sessions has been filled with talks by undergraduates on their own research. All talks at the workshop are aimed at a level appropriate for advanced undergraduates. Undergraduates at the Workshop attend talks by faculty, graduate students, and other undergraduates, and leave with valuable information about how to choose a graduate program.

5. Undergraduate MCTP Scholars

The department's MCTP grant supports five UNL undergraduate students each year as Undergraduate MCTP Scholars. Selection is based on students' interest in attending graduate school in mathematics as well as on their transcripts and letters of recommendation. During the academic year, these students have a research experience (similar to an REU funded as a supplement to an individual investigator NSF grant) under the direction of a UNL faculty member, and they are specifically encouraged to apply to REUs or other programs around the country for the summer. Undergraduate MCTP Scholars are paid up to $2400 for each academic year of participation (typically we expect a two-year commitment) and funds are also available to allow each Undergraduate MCTP Scholar to attend at least one conference to present the results of his or her research.

Undergraduate MCTP Scholars who are seniors also participate in a Graduate School Application Workshop, the Mathematical Landscapes Seminar (required of first-year graduate students), and the Introduction to Teaching Seminar (required of graduate students teaching a non-recitation for the first time).

6. RUTE

The Research for Undergraduates in Theoretical Ecology (RUTE) program is funded by an NSF grant to the Mathematics and Biological Sciences programs at UNL. The purpose is to significantly increase the number of students pursuing advanced degrees in areas that combine mathematics and biology. The primary component of the RUTE program is a structured research experience undertaken by a team consisting of two undergraduates in each of mathematics and biology, at least one faculty member in each of mathematics and biology, and a graduate student in biology. The students begin their project in the spring semester of their sophomore or junior year with a reading course designed by their mentors to familiarize them with the ideas and methods of the project research area. The students do intensive field or laboratory work during the summer under the direct supervision of the graduate student mentor and oversight of their biology faculty mentor. The Cedar Point Biological Station in western Nebraska, managed by the School of Biological Sciences, is an ideal location for such research, but projects can also be conducted elsewhere. The students earn academic credit and a $3500 stipend for their summer work. In the following academic year, the students do mathematical and statistical analyses of their biological data and also work on mathematics problems motivated by their

biological work. Some students may be able to obtain academic-year support from the MCTP (see Section 5) or UCARE (see Section 7) programs for their RUTE work. Each project is expected to result in at least one professional paper, along with presentations at conferences. The RUTE program also includes a transitional component designed to introduce students to the possibilities of interdisciplinary research in math and biology while they are still in the early phase of their undergraduate experience. This component takes the form of a 5-week, 3-credit course called "Research Skills in Theoretical Ecology" that is taught in the summer to approximately 12 students who are new high school graduates or have just finished their freshman year of college. We pay for the students' tuition and living expenses, and we also pay a $1500 stipend. The course is team-taught by a biologist and a mathematician. The students conduct a variety of laboratory experiments in population dynamics and learn the mathematical and statistical techniques necessary to develop a mathematical model, fit it to their data, and use it to make testable predictions. To complete the research experience, the students prepare a paper or poster to present their results and write an abstract for a research proposal on a topic of their choice inspired by their summer project. These proposals could lead to an MCTP or UCARE project (see Sections 5 and 7); alternatively, some of these students will participate in the main RUTE program later in their undergraduate career.

7. UCARE

UCARE (Undergraduate Creative Activities and Research Experiences) is a UNL-funded effort to support creative activities and research efforts across all disciplines. This program provides up to $2000 per year for up to two years for juniors and seniors to experience the creative activities of their chosen discipline. Students apply by submitting a proposal approved by a faculty mentor. Students are encouraged to present the results of their work at the annual UNL Research and Creative Activities conference, either orally or as a poster. At least two mathematics students, and sometimes as many as five, participate in UCARE each year.

8. Other local efforts

The department has many students who write honors theses each year; most of these contain some measure of undergraduate research and not all are supported through one of the programs described above. Moreover, we have recently introduced two upper-level undergraduate courses which give students a taste of mathematics research. On the applied side, "Math in the City" gives students the opportunity to develop relevant mathematical models in cooperation with local area businesses. Our "Introduction to Mathematics Research" course focuses on more traditional mathematics research, and aims to help students develop the ability to think creatively and independently on open-ended problems.

References

[1] W. Heinzer, L.J. Ratliff, Jr., and K. Shah. Parametric decomposition of monomial ideals. I. *Houston J. Math.* 21:29–52, 1995.

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