Industrial Mathematics and Statistics Research for Undergraduates at WPI

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For the past nine years, the National Science Foundation has funded the REU program in Industrial Mathematics and Statistics at Worcester Polytechnic Institute (WPI) in Worcester, Massachusetts. During the summers of 1998–2006, the program has hosted 96 students from 28 states and Puerto Rico who have worked on 27 different industrial projects sponsored by 13 different companies. Half of the 96 students were women and almost half of the students came from schools without a PhD program in mathematics.

The goal of the WPI REU program is to provide a unique educational experience by introducing students to the ways that advanced mathematics and statistics are used in the *real world* to analyze and solve complex problems. The students work in teams on problems provided by local business and industry. They work with a company representative to define the problem and to develop solutions of immediate importance to the company; they work closely with a faculty advisor to maintain a clear focus on the mathematics and statistics at the core of the project. When students work with a company on an industrial problem, the problem is real and the company needs a solution. This is usually the first time that students are placed in a situation where someone is going to make a decision, perhaps an expensive decision, based on their mathematical work.

The WPI REU program provides a glimpse of the many career possibilities which are open to students with a strong mathematical background. The hope is that by the end of the summer, the students will have better answers to the questions: (i) *What is the role of a mathematician in business and industry?* (ii) *What is it like to work with technical experts on a problem that requires significant mathematics but also must satisfy real-world constraints?* (iii) *What kind of mathematical and statistical tools are used to solve problems in business and industry?*

The program provides challenges not faced in standard undergraduate programs and strengthens skills not always developed in traditional educational programs. The *SIAM Report on Mathematics in Industry* [1] provides a comprehensive study of these special skills, and they include: (a) *communication at several levels, including reading, writing, speaking, and listening;* (b) *problem formulation as an interactive, evolutionary process;* (c) *the ability to work with a diverse team.*

The REU program at WPI provides an excellent experience for advanced undergraduate students going on to graduate school, whether they choose to specialize in applied mathematics or not. The experience is certainly valuable for students interested in following nonacademic career paths, but it is just as valuable for students who enter "traditional" graduate programs and go into academic careers.

WPI Project-Based Undergraduate Program WPI has a special infrastructure and a long history that supports intense project activities like those in REU's. Project-based learning has been central to the WPI educational program for more than 30 years. In 1971, the **WPI Plan** marked a departure from the conventional approaches to undergraduate education. It introduced as degree requirements three types of projects: the Humanities Sufficiency, the Interactive Qualifying Project (IQP) and the Major Qualifying Project (MQP). The last is a senior-year project completed in the major field of study. It is often the work of a team and spans over $3/4 - 4/4$ of the academic year. The purpose of the MQP is to provide a capstone experience in the student's chosen major that will develop creativity, instill self-confidence and enhance the student's ability to communicate ideas and synthesize fundamental concepts. In completing the MQP, students are expected to:

- formulate a problem, develop a solution and implement it competently and professionally,
- interact with the outside world before starting their careers,
- work in teams and communicate well both orally and in writing.

This project activity has been highly successful at involving WPI undergraduates in significant research with faculty. In 1987, NSF began funding Mathematics REU programs. In 1988, the WPI Mathematical Sciences department hosted its first REU program and was then able to involve non-WPI students in undergraduate research as well.

In order to enhance the industrial project experience for our students and to help make new contacts with business and industry, the Center for Industrial Mathematics and Statistics (CIMS) was established at WPI in 1997. Members of the Center work to establish contacts with industry, businesses and government labs and to develop industrial projects at both the graduate and undergraduate levels for our majors. The industrial mathematics program at WPI has been extremely successful. More than 200 hundred students have completed industrial projects with 30 different companies. We work throughout the year to make new company contacts, maintain existing contacts, and to develop new project opportunities for our WPI and REU students.

REU Recruitment Each summer, we recruit between 10 and 12 undergraduates to take part in our intense, residential REU program which lasts eight and a half weeks. Interested students apply online at `http://www.wpi.edu/+CIMS/REU`. They fill out a standard application form with personal data, education history, courses taken, and are asked to rank a list of preferred project areas. Recommendation letters from faculty are also required. Very importantly, students must write a one-page essay describing his or her interest in participating in the WPI program. After review by the Principal Investigators, promising applications are selected and these students receive a phone call from the program coordinators in a joint information exchanging and interview process. Following NSF guidelines, consideration is given

to only U.S. citizens or permanent residents. However, exceptional applicants who do not satisfy these requirements are also considered, provided the availability of additional funding from our industrial partners; we have had one British student from Oxford University and a French student from the Université de Savoie. The final group of students gathered are all interested in applied and industrial mathematics and statistics, and have a diverse range of course backgrounds and interests since the industrial projects usually require a mix of probability, statistics, differential equations, numerical analysis, and optimization.

REU Program Structure The process of meeting a real-world problem, learning to ask good questions and doing the research needed to identify the key mathematical structure, and then refining and redefining the problem, is a crucial part of the industrial mathematics experience. Also, the project is not finished when the problem is "solved." It is important that the students communicate their solution to the company in a form that the company can understand and use. Communication skills, written and oral as well as listening skills, are crucial for a successful industrial mathematician. This is developed via (i) *daily meetings* with faculty advisors, (ii) *weekly presentations* to fellow students and faculty, and (iii) *regular meetings with industrial sponsors*.

The REU students work in teams of 2–6 and each group has at least one faculty advisor plus an industrial advisor. Each team is given an office with 2 or 3 networked computers. (In 2003, while the Deutsche Bank research team was testing their portfolio model, the students had a total of 7 computers working in parallel.) Teamwork is one of the skills required for a mathematician working in industry [1] and one responsibility of the faculty advisor is to observe and guide the team-building process. The students meet with the faculty advisor(s) every day over the course of the 8 and a half week project.

Each team makes periodic progress reports, in the form of written reports and oral presentations, for their fellow students, the faculty and industrial advisors. We also invite faculty from other departments and representatives from local companies to attend these *weekly presentations*. The students receive extensive feedback as the projects evolve through the eight weeks. The students gain valuable practice in presenting their work; the improvement in quality during the summer is quite impressive. At the end of the two months, a *Presentation Day* is organized for the students to formally present their final results for the faculty, invited university administrators, and industrial advisors. This is followed by a special, celebratory lunch.

Each team of participants must complete a *written report* based upon the research they have completed during the summer program. The purpose of this report is to describe the problem considered, the background literature read, the approach(es) taken, the results that have been obtained, and the questions motivated by the research. Participants begin writing parts of this report as early as the first week of the program so that the faculty advisor has an opportunity to assist the students in developing a proper style for writing mathematics.

In addition to the student presentations, we have invited *mathematicians working in industry* to meet the students and discuss their work. For example, Keith Hartt from Bogel Investment described his work as a quantitative analyst; Bruce

Kearnan, Senior Actuarial Associate and General Director at John Hancock Life Insurance described his work as an "actuarial historian"; Derek Kane, mathematician at DEKA Research and Development Corporation speaks often about responsible global scientific research.

There are also at least three special events scheduled during the program. The following are a some of the special events held in the past programs: (i) a visit to the facilities of *DEKA Research and Development Corporation* including fun rides on the iBOT and the Segway; (ii) a tour of *the Mathworks*, with a presentation by technical staff on engineering applications for Matlab as well as career paths for math majors; (iii) participation in the *Math Research Expo* organized by WPI and Boston University as part of the Focus on Mathematics partnership. In 2006, three of the REU students acted as judges in local math research expos in the middle schools. See `http://www.focusonmath.org/FOM/resources/math/mathexpo/`

In order that student-faculty interaction is not limited to the academic dimensions, a group recreational activity is planned for most weeks. Activities have included a lobster dinner in Mystic, Connecticut, Boston Red Sox games (tickets provided by John Hancock), trips to view the Fourth of July celebration in Boston Harbor, as well as barbecues at the faculty advisors' homes.

The industrial projects have become the foundation for several successful outreach programs; this vertical integration of our work will be collected and described in an upcoming article.

REU Projects Completed   Below, we list just a few of the projects completed in our REU program along with the industrial sponsor.
(a) *Modeling Fluid Flow in a Positive Displacement Pump*
    DEKA Research and Development Corporation, Machester, NH
(b) *Mathematical Model for an Electro-Pneumatic Pulsed Actuator*
    Applied Mathematics, Inc., Gales Ferry, CT
(c) *A Continuum Model for the Growth of Brain Tumors*
    IBM Corporation, Boston, MA
(d) *Statistics Procedures for Failure-mode Testing of Diagnostic Equipment*
    Veeder-Root, Simsbury, CT
(e) *Adaptive Risk Score Assignment Model for Underwriting Long-Term Care Insurance*
    John Hancock Life Insurance Company, Boston, MA
(f) *FEMLAB Electromagnetic-Thermal Model of Microwave Thermal Processing*
    Ferrite Corporation, Nashua, NH
(g) *Portfolio Optimization with Non-Smooth Constraints*
    Goldman-Sachs, New York, NY
(h) *Mathematical Model of the Self-Tapping Screw Insertion Process*
    BOSE Corporation, Framingham, MA

Where are they now?   Our WPI REU alumni can be found in academia, as graduate students and faculty, and others have gone on to successful careers in business and industry. From the feedback and correspondence that we get from our students [**2**], the program has made a positive impact on their lives.

From a WPI REU 2005 alum via email:

Subject: I got a job!
I was offered a job as an "engineering assistant" with the company Applied Research Associates at their lab on Tyndall Air Force Base near Panama City, FL. My position will mostly be mathematical modeling using MATLAB and FEMLAB (and some other random tasks, occasional lab assistancy).
. . .

Clearly, I have the REU to thank. The company actually discovered me through Monster.com because I had FEMLAB in my resume, and that was one of the main reasons they were interested in me. But beyond the "marketable skill" of knowing FEMLAB, the REU is what got me really interested in mathematical modeling–and working in a research environment–in the first place. So to my colleagues and our advisers and everyone else who helped us during those transformative two months last year, thank you!

References

[1] *The SIAM Report on Mathematics in Industry*, Society for Industrial and Applied Mathematics, Philadelphia, Pennsylvania. (Available from the Internet at `http://www.siam.org`.)

[2] Vernescu, B., Heinricher, A., *Research Experiences for Undergraduates in Industrial Mathematics and Statistics at WPI* in Proceedings of the Conference on Summer Undergraduate Mathematics Research Programs, edited by Joseph A. Gallian, 1999, Arlington, VA, pp. 213–219.

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