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The Duluth Undergraduate Research Program 1977-2006

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Introduction

In this article I describe various aspects of my program such as funding, problem selection, recruitment, structure, follow through, and results. It is an update of the one that appeared in the Proceedings of the Conference on Summer Undergraduate Mathematics Research Programs
(see www.ams.org/employment/REUproceedings.html).

Funding

Over the years primary support for my programs has come from the National Science Foundation and the National Security Agency. My department and college have also contributed substantially. In 2006, participants received a stipend of $2250, a travel allowance, housing and a subsistence allowance of $1250.

Problems

Obviously, the selection of appropriate problems is of fundamental importance to a successful research program. I search for problems that meet the following criteria: not much background reading is required; partial results are probable; recently posed; new results will likely be publishable.

Graph theory, combinatorics and number theory provide the source of most of my problems. I find problems by perusing recently published journals, math arXiv, attending conferences, and writing people. As a rule of thumb, I begin the summer with twice the number of problems as I have students. I occasionally have a student continue work begun in a previous program by someone else. Although each student has his or her own problem, I encourage students to discuss their problems with fellow program participants. Matching students with problems is a critical task. The skill with which this is done is a major factor in the success of a program. Undergraduate students, even the most talented ones, have a tendency to become frustrated and want to give up too soon. Here I serve as a counselor

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and cheerleader, offering an idea, a reference or a pep talk. Of course, it sometimes happens - about half the time in fact - that a problem is inappropriate. Sometimes a problem is too easy or too hard; sometimes we discover it is already solved by someone else. In these cases I simply assign a new one.

Although I am the only faculty person involved in the REU, two graduate students who were former participants in the program return to act as research advisors to new participants. I choose advisors who have the personality and talent to interact well with others. Another important source of support is the fifteen or so former participants who visit from one to three weeks.

Recruitment

Recruiting well-qualified students has not been difficult. Applicants are generated by a mailing of announcements to a large number of mathematics departments nationwide and a mailing of an announcement and descriptive letter to many students who place in the top 100 of the Putnam Competition. Word-of-mouth advertising by former participants has resulted many outstanding applicants. Detailed information and an application form is available from my web site: www.d.umn.edu/~jgallian. Over the past two years the program web site has been visited about 8,500 time per year. In recent years the program has had 8-10 participants selected from approximately 80 applicants.

Selection of participants is based on letters of recommendation, response to questions on the application form, performance in high school mathematical competitions and the Putnam competitions, previous research experience, reputation of the home school, and course work. I give extra weigh to those applicants who have a particular interest in combinatorics. This can be demonstrated by the courses they have taken, participation in the Budapest Semesters in Mathematics Program (which emphasizes combinatorics), past research projects, or independent study the applicants have done. Students who have participated in mathematics summer programs such as the Hampshire program, the Ross program or the Boston University Promys program are usually well prepared in combinatorics.

It takes more than problem solving skills to do original research. Social skills are also needed to get along well with others in the program. Good "group chemistry" contributes much to the success of the program. Desire to succeed, enthusiasm, outstanding work ethic, and ability to work well with others are as important as raw talent. I try to select people who would be fun to spend the summer with.

Structure

My programs are loosely structured. Each student is given his or her own problem together with an article or two as resource material. Each week the participants give talks on their progress during the previous week to the group. This gives me, the research advisers and visitors an opportunity assess progress, raise questions, make suggestions and identify difficulties. Preparing their talks helps the students organize their work. It also serves as good preparation for presenting their results at a conference after the program is over. Occasionally we have a visitor present a colloquium. Typically, we have lunch as a group three times a week. I meet with students individually upon request and when I feel such a meeting might be

beneficial. I have weekly meetings with the advisors to discuss the progress of each participant and plan ahead.

The housing arrangements are a critical part of the program structure. UMD has attractive, furnished three-bedroom apartments with living rooms, kitchens and bathrooms. I reserve six of these that are contiguous (in fact, the students have a wing of the building to themselves). The students move freely among the six apartments so that the housing is like one large apartment complex. Since there are no instructional lectures and few formal meetings, the students spend most of their time in the apartments. The research assistants, returning participants, and visitors live with the new participants or in adjacent apartments. This mixture of old and new people works well. The living arrangements naturally foster interaction and collaboration. I frequently drop by the apartments to see how things are going, to answer questions, and to provide encouragement.

"Field trips" are a component of the program. It is important that the students enjoy their summer. Together, we go alpine sliding, rock climbing, white water rafting, visit beautiful parks in the area, bike, play softball and basketball, and walk along the shore of Lake Superior. On weekends, the participants have access to two university vehicles paid for by the UMD Math department. This makes it convenient for them to see movies, shop and eat out. Occasionally we have lunch at a restaurant. The field trips and group lunches foster a sense of camaraderie among the students, advisers, visitors and me.

Ideally, by the seventh week of the program the students are writing up their work. Papers are written in a style suitable for submission to a research journal. All manuscripts are read by me, the two research advisers, and one or more visitors. The readers make suggestions and comments and eventually I come to an agreement with the students on versions to be submitted for publication. Typically this process is completed by the end of the summer program or shortly thereafter, although there have been instances where it has taken several years to get a paper in publishable form. (Once people leave Duluth there is a natural tendency to concentrate on other matters.)

Follow Through

Except for finding a sufficient number of appropriate research problems, the follow through on manuscript preparation is my most difficult job. When the students leave Duluth I often have, at best, a first draft of their work. It typically takes many letters and phone conversations before the manuscripts are ready to be submitted to journals. Then, many months later, there are the inevitable referees' reports recommending revisions, necessitating another round of letters, phone calls and rewriting. By the time the referees' reports come, the students are busy with other things and are not always eager to follow through.

I view publication of the work done in my REU as a beginning rather than an ending in itself. Indeed, many participants from my program have continued to publish as an undergraduate or graduate student.

I strongly encourage participants to present their work at the annual joint meetings of the American Mathematical Society and the Mathematical Association of America. Rather than participate in the student poster session or the special session for research by undergraduates, I prefer that the Duluth students present their

work in the topic-specific contributed paper sessions. In sessions devoted to combinatorics or number theory the audience is interested in the topic and consequently the student will likely meet and have the opportunity to network with others who work in the field. Moreover, I feel that at a conference the undergraduates should be treated like professionals and not be segregated according to experience. Presenting their work at the joint meetings has proved to be a valuable experience for the students. They attend talks, meet people and have people ask about their work. Funding for this typically is provided by the home institution of the participants.

Follow through also includes writing letters of recommendation for fellowships and admission to graduate school and nominating participants for awards. In some cases I have written letters for former participants seeking employment after finishing the Ph. D. degree. I have even served as an external reviewer for tenure and promotion decisions for several participants from my program.

Project Evaluation

The effectiveness of the program is evaluated by a variety of means such as publications in well regarded professional level journals, talks given at conferences, number of students who enter graduate school, quality of graduate schools attended by the participants, number of students who receive Hertz, NSF, or DoD Fellowships, the extend to which participants return to visit future programs, and the long term relationships established among the participants themselves and with me. I track all program participants (see http:www.d.umn.edu/˜jgallian) and maintain regular contact with many of them throughout their years of graduate school and after.

Broader Impacts

The development of human resources is the explicit purpose of the program. Taking classes and participating in math competitions provide students with little or no experience with the research process. It is rare for even the best senior thesis to measure up to the standards required for publication by mainstream research journals. In contrast, the Duluth program has an extraordinary record of professional-level research done by undergraduates. Besides learning first-hand the nature of mathematical research and becoming part of the mathematics community, outcomes of participation in the program include: increased self confidence and self esteem; motivation to pursue a Ph.D. degree; enhanced chances of being admitted to a first-rate graduate school and receiving a fellowship; and the development of a network of people who will likely be important members of the mathematics community. The most important contribution the Duluth program makes to the development of human resources is the training of future generations of mathematicians who will foster undergraduate research when they become professionals. Indeed, four participants from the program have been directors of their own REU programs, a fifth is part of a four-school collaboration to involve undergraduates in research, a sixth served as a faculty adviser in the Williams College REU, and two more have been advisers in MIT's Summer Program in Undergraduate Research.

Results

Although the program participants are undergraduates, the level of the research they have done is unequivocally professional. Indeed, more than 100 papers written in the program have been published in professional-level refereed journals. Among them are 44 in the journal *Discrete Mathematics*, 10 in the *Journal of Combinatorial Theory* and 8 in the *Electronic Journal of Combinatorial Theory.* See www.d.umn.edu/~jgallian for a complete list.

Through 2006, 143 students have participated in the program. Of the 129 participants in the program who have received their Bachelor's degrees as of 2006, 116 have gone to graduate school. Of these, 83 have gone to MIT, Harvard, Berkeley, Chicago, Princeton, or Stanford and 87 have won graduate fellowships (Hertz, DoD, NSF). Sixty-four participants now have the Ph.D. degree. Two participants have received a Clay Mathematics Institute Long-term Prize Fellowship (Bhargava and Biss) and two have won the American Institute of Mathematics Five Year Fellowship (Ng and Develin).

Many of the women in my program have had extraordinary success. In the seventeen year history of the Association for Women in Mathematics (AWM) Schafer Prize ten women from the Duluth program have won the award (Ana Caraiana, Alexandra Ovetsky, Melody Chan, Melanie Wood, Ioana Dumitriu, Ruth Britto-Pacumio, Catherine O'Neil, Dana Pascovici, Zvezdelina Stankova and Elizabeth Wilmer) and eight have been named runner up (Margaret Doig, Elena Fuchs, Wei Ho, Karola Meszaros, Beth Robinson, Jessica Wachter, Susan Goldstine and Zvezdelina Stankova. With two exceptions, every winner of the MAA Elizabeth Putnam Prize (for outstanding performance by a woman in the Putnam Competition) has been in the Duluth program.

In the twelve year existence of the AMS/MAA/SIAM Morgan prize for research by an undergraduate the Duluth program has had seven winners (Manjul Bhargava, Daniel Biss, Joshua Green, Melanie Wood, Reid Barton, Jacob Fox and Daniel Kane) and four runner-ups (Kiran Kedlaya, Lenny Ng, Aaron Archer and Samit Dasgupta). With one exception, the research done at the Duluth REU was a major factor in the decision.

Conclusion

Although the term "paradigm shift" is frequently overused, it is appropriate to describe the transformation that research by undergraduates has undergone in the thirty years since I had my first summer research program. What was once considered as "not realistic" or even an oxymoron, is now ubiquitous. I am proud to have had the opportunity to contribute to this change in the mathematics culture.

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