**Proceedings of the Conference on
Promoting Undergraduate Research in Mathematics**

Wabash Summer Institute in Algebra (WSIA)

Mike Axtell, J.D. Phillips, and William Turner

1. Introduction

Since 2005, Wabash College has hosted the eight-week Wabash Summer Institute in Algebra (WSIA) in Crawfordsville, Indiana, through a grant from the National Science Foundation. WSIA provides an eight-week learning and research experience for twelve undergraduate students from across the country. The institute focuses on providing a common, cooperative experience for its participants, primarily in algebra, but also in ethics.

WSIA has several goals:

- to provide a meaningful mathematical research opportunity for undergraduates who are less likely to have a research-oriented experience than other student populations;
- to encourage participants to attend graduate school in the mathematical sciences and to help them develop some of the tools and confidence necessary to succeed there;
- to provide the support and framework for the participants to share their results and experiences with the larger mathematical community;
- to begin to provide the tools and knowledge necessary to become an independent, contributing member of the mathematical community;
- to create a highly diverse and supportive environment where the participants learn to work and live with a wide variety of individuals;
- to give students a significant experience studying the ethical dimensions of science and mathematics; and
- to attempt to accurately measure the success or failure of this program through a wide variety of evaluative processes with the assistance of an experienced agency.

As part of the institute, we hope to include up to three Wabash students each year. Because the funding for WSIA comes from an NSF grant, these students must be U.S. citizens or permanent residents. If no eligible Wabash student applies, and if international Wabash students show interest in the program, we may seek other funding from the college to allow an international Wabash student to participate.

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2. Research Topics

WSIA focuses on three areas of research in algebra: commutative ring theory, automated theorem proving and applications to loop theory, and computational linear algebra. The three research leaders cull research projects from topics related to their personal research.

Mike Axtell's research focuses on communitative ring theory and zero divisor graphs. He finds the research particularly suited to undergraduate study with comparatively little background needed outside a standard first semester abstract algebra course. It does not even require much experience in or knowledge of graph theory since the area currently uses very few non-basic graph theory ideas. After a brief introduction to graph theory and commutative ring theory, he guides students to investigate properties of the zero divisor graphs of various commutative rings, such as their diameter.

J.D. Phillips uses automated theorem provers such as Otter and Prover9 together with the related finite model builder Mace4 to investigate questions in loop theory and in other areas of algebra. This area, too, requires little knowledge past a standard first semester abstract algebra course. Under his direction, students have worked on the axiomizations of tri-medial quasigroups and digroups and have initiated the structural investigation of LC-loops.

William Turner's research is in symbolic computation, and in particular black box linear algebra, which takes an external view of a matrix, viewing it as a linear operator on a vector space. This view is particularly useful for highly structured matrices, and algorithms in the field use basic concepts such as recurrence relations and generating polynomials. Students who have only taken linear algebra can understand the topic, although some experience in abstract algebra, number theory, or programming can also prove useful. His research students have investigated using standard black box linear algebra finite field algorithms such as those using Wiedemann and Lanczos methods to compute the determinant of a matrix of rational numbers.

3. Program Outline

The eight-week program strives to guide participants from being students of mathematics toward becoming contributing members of the mathematical community. It does this through three relatively distinct phases.

3.1. Introductory Period. Participants find the first few weeks of the program fairly familiar. We dedicate the early weeks of the institute to lectures and problem-solving sessions conducted by the three research leaders and attended by all participants. In this respect, WSIA functions more like a graduate school or institute than a typical REU during the first few weeks. This serves to create a common body of knowledge among participants and foster a sense of shared experience and communal work ethic. Participants use these opening weeks to decide which area they wish to spend the rest of the summer pursuing.

During this phase, the three research leaders take turns working with all twelve participants to introduce them to their particular research projects. All three of us use styles similar to our normal classrooms; however, these styles vary from being lecture-oriented to discussion and problem solving sessions. Some of us focus heavily on students working on problems in groups to encourage the students to

forge a strong group identity. This rotation among the three of us means we expose the participants to all three of our styles—and their associated benefits—without any of us having to change our style into something we do not find comfortable.

This phase focuses on providing the participants with the necessary background knowledge and technical tools to participate in the research projects. This serves two purposes. First, it allows the participants to make an informed decision when we ask them which projects interest them. Perhaps more importantly, though, because all the participants have seen all of the research projects, we expect the entire group to be able to discuss all of the research projects to some extent. This gives the program some added unity rather than just comprising three distinct research projects.

At the end of this introductory period, we distribute the participants among the three projects. We ask the participants to rank their interest in the three projects. Using this information, we assign each participant to one of the three projects. In 2005, we assigned each participant to their first choice of projects, but this caused an imbalance in the number of participants in each project. In 2006, to avoid the problems resulting from this imbalance, we asked applicants for their preliminary research interests and used this information to help select participants. We used this preliminary interest statement only to select participants initially. After the introductory period, we asked the participants to rank the projects again, and we balanced the research projects by assigning participants so four or five participants worked on each project. The participants' choices helped inform our decision rather than dictating it. Most participants still received their first choice, and none received their last choice.

3.2. Ethics Component. In the week following the introductory period, David Neidorf conducts a workshop on ethics and research. Neidorf is the vice-president and dean of the college at Deep Springs College in California and the director of educational programs at Bioethics-In-Action, Inc. Previously, he was the director of the integrated studies program at Middlebury College.

The ethics workshop has three goals:

(1) to provide participants a significant and formative exposure to the interface between work in mathematics and ethics;
(2) to ensure that participants are thoughtfully aware of their responsibilities as both professionals and citizens, and are alert to the tensions and conflicts between these two roles; and
(3) to empower the skillful discussion and resolution of concrete ethical problems through the examination of ethical case studies according to varying schema of conceptual evaluation.

Because the range of concrete ethical issues concerning the professional practice of mathematics is not easy to predict, he pursues these goals by using a shared model of ethical issues that involves the work of technical specialists: current ethical controversies in the biotechnological manipulation of human abilities and characteristics.

During the five instructional days of the workshop, participants spend the afternoons starting on their research projects and the mornings devoted to the ethics component. These five mornings are divided roughly into three parts: The analysis of case studies and associated background reading, student presentation of

their independent analysis of new case studies, and group discussion of the student presentations.

Participants examine organizational case studies that raise ethical problems in three issue areas in biotechnology: Human Life-Span Extension, Enhancement of Human Performance, and Medical Modification Behavior. As they examine these issues, they identify ethical theories (e.g. utilitarianism, deontology, virtue ethics) and evaluative principles (e.g. autonomy, informed consent, distributive justice, freedom of belief, academic freedom).

We anticipate the ethics component of the REU will start participants down a path of a heightened sense of personal and professional ethical responsibility, and of increased ethical decision-making skills within their organizations. Since many participants will find themselves in leadership positions, this experience can play a significant role in their organizations as well as their personal lives.

3.3. Research Projects. Participants spend the last five or six weeks of the program working on their research projects. In the first week, participants divide their time between research and the ethics component, but they devote all of the remaining four or five weeks to their research project. During this time, participants work in small groups, supervised by a research leader. While each of us has his own style, these groups typically work very independently. We may only meet with the participants a few times each week, unless they run into difficulties and need to meet more frequently. Sometimes we meet with them multiple times in a day.

Research teams present weekly progress updates to the entire group. Since all twelve participants have seen the introductory materials for all of the projects, all participants should find these progress updates accessible.

Participants also prepare a paper and a talk. They present their talks at the Indiana University Undergraduate Research Conference held in Bloomington, Indiana, in late July, and we encourage them to submit their papers to undergraduate research journals or perhaps even professional research journals. We also encourage them to present their findings at the AMS/MAA Joint Mathematics Meetings Undergraduate Research Poster Session in January or at regional opportunities such as an MAA sectional meeting. Our NSF grant provides funding for half of the students to attend the Joint Meetings to present posters, and we encourage all participants to solicit funds from their home institutions to attend the meetings. So far, all participants who have wanted to attend the meetings have received enough funding through WSIA and their home institutions to attend.

4. Outside Speakers

Throughout the program, we invite mathematicians from neighboring institutions to come give a talk to the participants. Typically, the speaker will give a research talk, to which we invite the entire campus, not just the WSIA participants. Afterward, the speaker talks to the WSIA participants about graduate schools; an informal question-and-answer session follows this.

We have had a variety of speakers participate. Graduate programs and department chairs have talked about their particular graduate programs. Current graduate students, including a Wabash College graduate, have talked about their experiences in graduate school. A former WSIA participant talked about her experience applying to graduate schools. Some professors have had no direct involvement

in administering graduate programs. We have even had the president of the MAA give a talk about mathematics and biology.

5. Participant Recruitment and Selection

We recruit participants through a variety of methods. In addition to the abstract on the NSF's REU website, we distribute a flyer at the Joint Mathematics Meetings in January, send mass mailings to mathematics departments across the country, and maintain a local website[1]. We recruit participants most heavily from smaller institutions whose students may not have as many mathematical research opportunities.

The typical participant will be a rising senior or junior who has completed at least one semester of abstract algebra, although we may accept an otherwise exceptional applicant who has taken only linear algebra but who has other strengths, such as a good programming background. When reviewing applications, we look closely at the two letters of recommendation for evidence of the applicant's ability to work well in groups. After the imbalanced research groups in the first year, we have also looked at the applicant's non-binding research topic preference to help balance the research groups. We use this preference only in selecting participants initially and not in assigning participants to research groups.

In selecting participants, we also favor women and members of other groups that are underrepresented in mathematics. Besides all the philosophical reasons for doing so, we have discovered a very practical reason to give women preferential treatment. Because Wabash College is a liberal arts college for men, the college is not organized to handle many women students on campus. In fact, the women in WSIA are the only women students on campus during the summer. The college housing makes it impractical to have only one or two women living in a house. In order to house all participants most comfortably, and to not require a third house, which might be difficult to acquire, we must ensure nearly equal groups of men and women participants.

6. Social Activities

The social aspect of WSIA begins with the living arrangements. Wabash College houses the participants in two houses on the edge of the college's small campus. This communal living arrangement allows the participants to bond as a group as well as provides them with common spaces to work and socialize at any time.

We also have several organized social activities. Each of the three research leaders hosts a gathering at his home. Typically, all three of us and our families participate in all of these, helping provide food and entertainment. We host the first of these on the Sunday evening the participants arrive on campus, the day before the institute begins in earnest, to allow the participants to meet each other and us in a relaxed environment. Another professor hosts a gathering during the middle of the program, and the third hosts one about a week before the participants leave. In addition to meals at our homes, we also take the participants on a number of organized outings such as nature hikes, canoe trips, and attending minor league baseball games in nearby Indianapolis.

[1] http://www.wabash.edu/academics/math/wsia

The participants have also organized their own social activities. We have heard from the participants that they often prepare meals together. Sometimes this takes the form of one house (either the men's or the women's) preparing a meal for the other house, and then the other house reciprocates a few days later. Participants have traveled to Lafayette, Indianapolis, and Chicago for outings. Participants even organized a trip to the University of Illinois to visit the graduate program there.

7. Assessment

With the aide of Wabash College's Center of Inquiry in the Liberal Arts, we administer several evaluations to the participants throughout the program to gauge the change in the students' attitude toward mathematical research and in their self confidence in their ability to tackle new mathematical problems. These include a base-line evaluation at the beginning of the program, informal sessions throughout the program, and a longer written assessment at the end of the program.

We plan to remain in contact with the participants for five years after their participation in the program to attempt to measure the long-term impact of WSIA. In particular, we are interested in the rate at which WSIA participants enroll and succeed in graduate programs in the mathematical and other sciences. We hope to determine whether the participants continued to mature mathematically through graduate studies in the sciences and if WSIA had any impact on this maturation.

Department of Mathematics and Computer Science, Wabash College, P.O. Box 352, Crawfordsville, IN 47933

E-mail address: {axtellm,phillipj,turnerw}@wabash.edu