**Proceedings of the Conference on
Promoting Undergraduate Research in Mathematics**

Research Experiences for Undergraduates Inverse Problems For Electrical Networks

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1. Overview

Our undergraduate research program at the University of Washington typically includes thirteen undergraduates two or three TAs and one full time faculty member. It lasts for eight weeks and begins in late June to coincide with our academic schedule. Most of the research is conducted on inverse problems concerning discrete electrical networks and continuous electrical bodies. In addition to electrical network problems students work on related problems involving such things as the discrete Helmholtz (Schrodinger) network. This REU site has focused on such problems since 1988 and has had great success in developing an extensive archive of results in this area (see `http://www.math.washington.edu/~reu/`).

We have had a wide variety of students in our program. In the past three years 21 of the 24 students supported by NSF have come from outside the state of Washington. Students come from large research universities, universities of moderate size, and colleges with limited resources. All groups of students have benefited from the program.

The objective of the program is to quickly involve students in a research project in an active area of mathematical research. This is done by intense interaction among faculty, students, and TAs. Students are given reading material before the program begins. During the first week, the students are given a crash course on electrical inverse problems and related matters. After this introduction, we help them formulate and work on open problems. Many of the problems we suggest are natural continuations of problems that students have worked on over the years. Students then proceed to investigate these problems, meeting with a faculty or TA adviser every day. Students write up the results of their investigations as progress and final reports. Progress reports are given in weeks three and four and final reports in week eight. All reports are written, distributed to all students in the program, and presented orally. Students are given wide freedom in their choices; they often work on and solve several problems. An active social program is arranged so that students interact intensely with each other.

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The University of Washington supports the REU program in a variety of ways: computer facilities, dedicated classroom space, supplies, mailing, photocopying, student travel, and the usual administrative help for such a program. A VIGRE grant contributes support for TAs and some additional students. The Department has awarded a special fellowship, the Bob and Elaine Phelps Fellowship to support a student in the program. The University of Washington Summer Program pays partial salary for a faculty member. We are applying for a grant from Boeing to stabilize the support of TAs and UW students.

2. Student Activities

The main objective of this eight-week program is to involve students quickly and intensely in mathematical research. Except for the first week, students spend full time doing research on unsolved problems. We encourage students to formulate and work on problems as soon as possible. We have a brainstorming session at the end of the first week to come up with a list of problems. This year (2006) students came up with twenty-two research topics. When we first began to involve undergraduates in research, we expected that it would be a long time before they could begin their work. We have have been pleased to find that they actually need only a few days. In 2006, everyone was working on a problem at the end of the first week.

To accomplish this we send them reading material in advance. We send the students a list of references late in April and expect the students to do some reading before the program begins. We also send them a copy of the book [1] as well as selected student papers. A complete collection of student papers is archived on the website `http://www.math.washington.edu/~reu/` . The student papers are good indicators of current directions an usually stimulate a lot of interest.

In the first week we give five hours of lecture per day to bring the students up to speed. They are also given instruction by our TAs on the use of our computing facilities and document preparation. They start to learn LaTeX during this period since we require that all papers be written in LaTeX. In the readings and lectures they learn about the conductivity equation, the Dirichlet problem, and related problems for electrical networks. They learn about circular planar graphs, critical graphs, and medial graphs, response matrices, and the $\bigstar - \mathcal{K}$ transformation. They also learn about inverse problems in these areas. They learn about a variety of network functions such as edge and vertex conductivity functions, directed network conductivities, scattering problems for networks, and Schrodinger networks, and Gröbner bases. After this introduction, we help them formulate open problems. Many of the problems we suggest are natural continuations of problems that students have worked on over the years.

They then proceed to investigate these problems, meeting with a faculty adviser or TA every day. Students write up the results of their investigations and give several oral reports. Students often work on and solve several problems. Students can choose to work on their own or in teams. Most of them said they were very happy to be given the freedom to choose their own problems and work on their own time schedules. Several said that this was the most intense learning experience that they had ever had.

They are not given fixed assignments nor are they all expected to study the same material. However, we do suggest problems that we think they will find

interesting, and we suggest promising directions. Some students choose to extend work done by former students in the program. In many cases our former students did not completely solve the problems they worked on. Thus we have had students work on parts of the same problem over a period of several years. For example substantial work on non-planar networks has continued during all of the years of the program. We encourage them to be bold with conjectures and not to worry about success or failure. In our recent REU programs this approach has led to a wide variety of ideas for projects. Some worked out quickly to produce good results. Others seemed promising, but could not be completed in the short time of program. Succeeding students often continue previous work during the following year and produce more results. Some of the students have written joint papers with the principal investigator and the results of others has appeared in the monograph [1] co-authored by the REU Director. This monograph appeared in the year 2000 and is the definitive work on discrete inverse problems. It could not have appeared without the contributions of the students in the REU program. Recent progress has been rapid with students building on previous work and answering questions that have been around for many years.

The involve original work in an active area of mathematical research. The AMS meetings at Arcata in 1989 and Seattle in 1995 on inverse problems and the various special meetings of SIAM and IEEE on inverse problems attest to the current interest in inverse problems. There was a semester long program in inverse problems for Autumn, 2001 at MSRI at which there was a week-long PASI on inverse problems aimed at post-docs and graduate students. PIMS has an active collaborative group on Inverse Problems that includes many University of Washington faculty. The director of the program is active in this area, is in constant contact with other workers, and knows the current state of research. The virtue of discrete problems in inverse conductivity is that they can be approached by undergraduate students with a good understanding of calculus, differential equations, linear algebra and elementary physics.

There are guest lectures from local faculty and these faculty are available to consult on questions about graph theory, inverse problems, combinatorial algebra, partial differential equations, numerical analysis and optimization. At the end of the term students submit papers describing their work. These papers are posted on the website `http://www.math.washington.edu/~reu/` Some continue to work on their papers after the formal termination of the program. This is encouraged and the proposer will continue to work with these students, correspond with them, and help them further their careers.

The students are told early on that they will be expected to write an exposition of their work and it is suggested that this writing should begin as soon as possible as this is frequently a lengthy and difficult process. We have encouraged the students to show us written work as soon as the second or third week. We start to help them right away in the formulation and of their ideas and solution of their problems. We are continuing to discover what students are capable of doing. The students seem to be more sophisticated and original each year.

We have learned a lot from the student responses to the evaluation forms that we distributed. The students have found that a solid understanding of linear algebra is essential. It is easy for us to overlook the fact that such an elementary subject is so important. We have become more aware of this requirement and have tried to

assist the students in learning all the linear algebra that they need. The students said that the paper writing is one of the most difficult parts of the program. We agree with that assessment and we assist them in learning how to write by carefully reviewing their work with them. We ask them to be very critical of their own work and to strive to make it as clear and unambiguous as possible.

Finally we tell them that this program is intended to give them the freedom to experiment. We want them to find a problem, learn all they can about it, attempt to solve it and write up the results. We stress the importance of determination and that they must keep trying even in the face of repeated failures. We tell anecdotes about former students and their work. We point out that often what seems to be a crazy idea turns out in fact to be quite fruitful. We also tell them that sometimes the discoverer never sees the real importance of his or her ideas. What they should be getting from this program is an idea of what it is like to do mathematical research and whether it suits them or not. In that sense the program should be a success for everyone.

A room is reserved for the morning meetings. This room is available to the students all day for unscheduled discussions. The MSCC lab and the Department's computer rooms are open from 7 a.m. to 11 p.m. on weekdays and from 8 a.m. to 10 p.m. on weekends and holidays. There are terminals in the dorms connected to the campus network. Introductory computing classes will be scheduled early in the session to familiarize the students with the computing facilities. Space in a nearby student dormitory will be reserved for the REU students. There is a convenient and comfortable student union which is near the computer lab and the classroom, where our students can gather to socialize. We also invite the students to a local beach club to swim and relax and we have several picnics, regular Frisbee and softball games, hikes, sight-seeing activities, and other parties. There are social events nearly every day. This year's social schedule is archived at `http://www.math.washington.edu/~reu/schedule/current`. Our students have worked well together and have maintained contact with us and each other after the formal termination of the program.

3. Student Recruitment and Selection

We bring in students from a variety of colleges and university, maintaining the mix of small, medium, and large universities.

The quality of the students in the program has been high. Two of the students have been given honorable mention in the competition for the Alice T. Schafer award. There are seven recent winners of the Goldwater Scholarship. In 2006, one student was awarded both a Rhodes and a Marshall. Most of the students have gone on to graduate school in mathematics or a related discipline and many of them have been awarded fellowships. A number of these students have winners in the International Mathematical Contest in Modeling. The University of Washington has had four winning teams in the last three years (two problems are offered each year). They have won all of the prizes (MAA, SIAM, and INFORMS). The three-person teams were made up entirely from alumni of our REU program. The REU problem taught them how to formulate, solve, and write up solutions to problems. In addition, three winning teams from other universities have included REU alumni.

Several alumni of the REU program are now faculty members at highly regarded universities. Many of these students have said they would not have considered

graduate school if it had not been for the REU program and some have switched from another discipline to mathematics because of their experience in the program.

We have a website with information about the program and selected papers for interested parties to read. This year's web address is `http://www.math.washington.edu/~morrow/reu06/reu.html`. We have made a selected list of universities and colleges in the United States and either by mail or by personal contacts we make the local undergraduate advisers aware of the program. This list includes institutions at which research opportunities are limited and which are located throughout the United States. We distribute flyers and application forms to faculty and students and follow up to attempt to get the best applicants. Since our program has been in operation a number of years we have benefited from the positive publicity given it by the participants of the program. An article about our program appeared in *M*ath Horizons in 2004.

In addition to the usual applications from students on the West Coast, we have many requests from Eastern, Southern and Midwestern students, who probably learned about our program from a faculty member, web site, mailing, or former student. We have had students from more than sixty universities and colleges.In the most recent three years twelve women have participated in the program and we have had one woman TA.

Our program is suitable for students with the following background:

- Differential equations at the level of *Boyce and DiPrima* (typically a sophomore course).
- Linear algebra at the sophomore or junior level including some discussion of numerically solving linear equations
- Advanced calculus, especially Green's theorem
- First year physics (mechanics, electricity and magnetism)
- C, Mathematica, Maple, or Matlab

We ask prospective students to describe their mathematical education, list any special awards and write a short essay describing their interest in the program. We ask for two letters of recommendation. Students will be selected in early April.

4. Project Evaluation

The REU program at the University of Washington has been very successful at introducing students to mathematical research and instilling in them an enthusiasm for research as a career. The faculty and TAs encourage the students to continue the research that they have begun by maintaining contact with the students and helping them write senior theses on the topics that they are interested in. We encourage them to apply to graduate school and counsel them on appropriate schools and write letters of recommendation. As mentioned earlier, graduates of the program include NSF fellows and tenure track faculty at major institutions. We also count among our graduates, medical doctors, Wall Street financial mathematicians, and software researchers. We have encouraged our students to apply for teaching assistantships as undergraduates and many have been accepted as TAs. Some become TAs for our REU program. We are gratified to find that our students are frequent recipients of national and international awards, such as NSF Fellowships, Rhodes Scholarships, Sloan Fellowships, and Mathematical Contest in Modeling awards.

Each year students fill out a questionnaire with information that allows us to keep track of them. They also make comments and suggestions for ways to improve

the REU. On the last day of the program we ask the students to meet as a group, select a "scribe" and make a report giving us their advice and commenting on their experiences. We have established a website that allows them to keep up with their fellow students and to see progress on problems. We also ask faculty from other institutions to visit our program and write an evaluation.

BIBLIOGRAPHY

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