The Putnam Competition from 1938-2006 *

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1. INTRODUCTION. The William Lowell Putnam Competition is held annually for the top undergraduate mathematics students in the United States and Canada. The first Putnam competition took place in 1938, but its genesis was a math competition held in 1933 between ten Harvard students and ten students from the United States Military Academy at West Point [2]. That competition was sponsored by Elizabeth Lowell Putnam in honor of her late husband William Lowell Putnam, who was a member of the Harvard class of 1882. That competition went so well that plans were made to have an annual competition in which all interested institutions could participate. This came about in 1938, when the first official Putnam competition was sponsored by the Mathematical Association of America. The examination was prepared and graded by members of the Harvard mathematics department and Harvard students were excluded the first year. There were both individual and team competitions. The questions were drawn from calculus, the theory of equations, differential equations, and geometry. (The problems are included at the end of this article.) Prizes in the first few years were \$500, \$300, and \$200 for the top three teams and \$50 each for the top five ranking individuals, who were designated as Putnam Fellows. By the year 1997 the prizes for the top five teams were \$25,000, \$20,000, \$15,000, \$10,000, and \$5,000, while Putnam Fellows received \$2,500 each. Moreover, each year one Putnam Fellow receives the William Lowell Putnam Fellowship for graduate study at Harvard.

The first competition had 163 individuals and 42 teams. The number of participants exceeded 1,000 for the first time in 1961, when 1,094 individuals and 165 teams took part. In 2006 there were 3640 students representing 508 institutions and 402 teams. The number of participants in the 2006 competition alone exceeds the total number of participants in the first seventeen competitions from 1938 through 1957. (The competitions were suspended from 1943-1945 because of World War II; in 1958 there were two competitions–one in the spring and one in the fall.) Coincidentally, in both 1980 and 1981 there were exactly 2,043 participants. Through 2006, there have been 107,452 participants. The record for the most number of schools to participate is 515, set in 2004. The 1946 contest, coming right after the war, had the lowest participation ever with just 67 contestants and 14 teams. Table 1 at the end of this article provides the list of the number of participants in each of the sixty-seven competitions through 2006.

In the first twenty-two competitions the number of questions varied from eleven to fourteen, but beginning with the 23rd competition in 1962, the exams have consisted of a three-hour morning session and a three-hour afternoon session, each having six questions worth ten points apiece. Institutions entering teams must designate the three team members before the competition is held. The team score is the sum of the ranks of the three team members. Thus, a team whose members finish in twenty-first, forty-ninth, and one hundred and second places has a score of 172. The lower a team's score, the higher its ranking. This method of team scoring places great weight on the lowest scoring member of the team since there is much bunching at lower scores. For example, in 1988 a team member with a score of ten ranked 1496, but a team member with a score of nine ranked 1686. In 2006 a score of one point generated 1266.5 team points, whereas a score of zero on that exam resulted in 2501 team points. Thus, even a one point difference in an individual's score can mean over a thousand points more for the team.

The fact that the team members are designated in advance and the method of summing the ranks for team scoring causes some peculiar results on occasion. In 1959, for instance, Harvard had four

*This is an updated version of an article published in the American Mathematical Monthly [5] in 2004.

Putnam Fellows but finished fourth in the team competition, and in 1966, 1970, 2005 and 2006 MIT had three Putnam Fellows but did not win the competition. There have been sixteen competitions in which the winning institution did not have a Putnam Fellow.

One might wonder about the most difficult Putnam problems over the years. Using data from 1974-2006, the only problem for which no one in the top 200 received a positive score was A6 on the 1979 exam. In 1999 for both B4 and B5 only a single person in the top 200 received a positive score. In each instance the score was two. These three problems are reproduced in the Appendix II.

2. TEAM PERFORMANCE. By a wide margin, Harvard has the best record in the Putnam competition. Through 2006, Harvard has won the team competition twenty-five times, while its closest rival, Caltech, has won the team title nine times. MIT is in third place with five titles. Tied for fourth place with four team titles each are Washington University and the University of Toronto. All four of Toronto's team titles occurred in the first six years of the competition. Toronto might have won all of the first six competitions except for the fact that it chose to disqualify itself in 1939 and 1941 because the Toronto mathematics department had prepared the questions. Starting with the fifth competition the questions have been prepared by a committee selected from different schools rather than having the department of the winning team of the previous competition prepare them. This meant that the winner of the previous year would not have to disqualify itself. Curiously, the Harvard team did not place in the top five in the first six competitions, but it has placed in the top five in fifty-two of the sixty-seven competitions held through 2006. During the first twenty competitions (1938-1959), the New York institutions Brooklyn College, Polytechnic Institute of Brooklyn, Columbia University, and City College of New York excelled in the team competition and in producing Putnam Fellows. Caltech's glory years were the six years 1971-1976 when they won the team competition five times. Excluding Harvard, only once has the same team won three years in a row. That was Caltech in 1971-1973. Between 1976 and 1986 Washington University won the team title four times and placed second four times. During that period Wash U had only two Putnam Fellows. Beginning about 1990 Duke University started to recruit the nation's best high school math students with the same fervor that they recruit the best high school basketball players. Since then Duke has emerged as Harvard's top rival by winning three times, finishing second twice, and taking third place six times. Interestingly, in this period Duke's Putnam team has performed as well as its men's basketball team! (Through 2006 the men's basketball team finished first three times and second three times, with one other appearence in the final four.) After finishing in the top five twenty-four times and in second place nine times prior to 2006, Princeton won its first team title in 2006. MIT took first place in both 2003 and 2004 and had three individuals place in the top 5 in both 2005 and 2006. The only state universities in the U. S. to win the team competition are Michigan State (three times), and the Universities of California at Davis (once) and at Berkeley (once). The highest place ever achieved by a liberal arts college was second by Oberlin College in 1972. That same year Swarthmore finished fourth. Harvard's longest winning streak was eight years (1985-1992), and its longest stretch without winning was fifteen years (1967-1981). The only tie for first place occurred in 1984 between the University of California at Davis and Washington University. Amazingly, in 1986, 1987, and 1990 every member of Harvard's team was a Putnam Fellow. A complete list of the top five schools and top five individuals each year can be found at http://www.maa.org/Awards/putnam.html. Table 3 lists every team that has placed fifth or higher in at least one competition along with the total number of Putnam Fellows from each of these institutions. The last four entries in the table list the institutions that have not placed in the top five in the team competition but have had at least two Putnam Fellows.

3. INDIVIDUAL ACCOLADES. As for producing Putnam Fellows, Harvard is again the overwhelming winner with ninty-five versus MIT's second place forty-five. On the other hand, between 2001 and 2006, MIT out did Harvard in Putnam Fellows fourteen to eight. Harvard has had four Putnam Fellows in the same competition on four occasions. Oddly, Harvard did not record its first Putnam Fellow until the sixth competition. Since then the longest period in which Harvard did not have a

Putnam Fellow is three years and that happened only once. Because of tie scores for fourth or fifth place, in thirteen competitions there have been six Putnam Fellows, while in 1959 a four-way tie for fifth place resulted in eight. Twelve of the fourteen competitions in which there were more than five Putnam Fellows have occurred since 1970. Through 2006, there have been 260 individuals who have been Putnam Fellows for a total of 351, counting multiplicity. Only seven people–Don Coppersmith, Arthur Rubin, Bjorn Poonen, Ravi Vakil, Gabriel Carroll, Reid Barton, and Daniel Kane. –have been Putnam Fellows four times. Fifteen people have been three-time winners: Andrew Gleason, Edward Kaplan, Donald J. Newman, James Herreshoff, Samuel Klein, Randall Dougherty, Eric Carlson, David Ash, Noam Elkies, David Moews, David Grabiner, Kiran Kedlaya, Lenny Ng, J. P. Grossman, and Ciprian Manolescu.[1] In Ash's fourth attempt at the Putnam he finished tied for sixth, just two points short of being a Putnam Fellow again. It should be noted that some of the three-time winners only took the exam three times. Through 2006 there have been forty-one people who have been Putnam Fellows exactly twice. It appears that there have never been two members of the same immediate family who have been Putnam Fellows. The closest are brothers Doug and Irwin Jungreis. Doug finished in the top five in 1985 and 1986 and Irwin finished in the second five in 1980 and 1982. Dylan Thurston, son of Fields Medalist William Thurston, finished in the second five in 1993. The first certain occurrence of a woman finishing in the Honorable Mention or higher categories was in 1948. In the announcement in the American Mathematical Monthly [7] she is listed as "M. Djorup (Miss), Ursinus College." Because many participants use the initials of their first and middle names (e.g., R. P. Feynman) it is possible that Djorup is not the first woman to achieve Honorable Mention or better status. The first woman Putnam Fellow was Ioana Dumitriu from New York University in 1996; the second was Melanie Wood from Duke in 2002; the third was Ana Caraiani from Princeton in 2003 and 2004. Since the ages of participants are not noted, there is no way to know who the youngest and oldest people to win the competition were. A candidate for the youngest is David Ash, who was a winner in 1981 at the age of 16 years and three months. Noam Elkies was a winner at the age of sixteen years, four months and Lenny Ng was also a Putnam Fellow at sixteen and eleven months. [2] A potential oldest winner is Samuel Klein, who was born in 1934 and won the competitions in 1953, 1959, and 1960. As a group, the five winners of the 2003 competition have amassed the greatest number of Putnam Fellow designations ever: Gabriel Carroll, Reid Barton and Daniel Kane won four time, Ana Caraiani won twice, and Ralph Furmaniak won once.

Unlike the early years of the Putnam competition, in the past twenty-five years or so many of those who have done exceptionally well in the Putnam competition have participated as high-school students in problem solving summer training camps in the United States and elsewhere in preparation for the annual International Mathematical Olympiad (IMO). Many of the international students who represented their countries in the IMO have come to the United States for their undergraduate degrees. The consequence is that the winners of Putnam competitions now come from many countries. The 2006 Putnam competition illustrates this well. All five winners were IMO gold medal recipients and 12 of the top 26 scorers in competition represented countries other than the United States or Canada in the IMO.

Over the sixty-seven competitions between 1938 and 2006 there have been only three perfect scores–one in 1987 and two in 1988. Although the top five scorers are always listed alphabetically, it is known that the 1987 perfect score was achieved by David Moews. What is amazing about this score is that the 1987 exam was a difficult one. The median score was one point and twenty-six points put one in the top two hundred (out of 2,170 participants). In 1987 the second highest score was 108, while the third highest score in 1988 was 119. The winners of the 1987 and 1988 competitions rank among the strongest groups of Putnam Fellows ever. Among them are Bjorn Poonen and Ravi Vakil, both four-time Putnam Fellows, David Moews and David Grabiner, both three-time Putnam Fellows, and Mike Reid, a two-time Putnam Fellow. In contrast to the 1988 scores, of the 1,260 contestants in the 1963 competition the highest score was sixty-two. That year, anyone with a score of twenty-eight was in the top 10%.

[1] The MAA should create action figures for all the people who were Putnam Fellows three or more times.

[2] In the version of this article published in the Monthly I had Elkies as the youngest winner that I knew of.

4. A PUTNAM WHO'S WHO. Over the years many distinguished mathematicians and scientists have participated in the Putnam. Among them are Fields Medalists John Milnor, David Mumford, Daniel Quillen, Paul Cohen, and John G. Thompson (Milnor, Mumford, and Quillen were Putnam Fellows; Cohen was in the second five; Thompson received Honorable Mention). Physics Nobel Laureates who have received Honorable Mention or better are Richard Feynman, a Putnam Fellow in 1939, Kenneth G. Wilson, a two-time Putnam Fellow, Steven Weinberg, and Murray Gell-Mann. The Nobel Prize winner in Economics John Nash (of "A Beautiful Mind" fame), to his great disappointment, finished in the second five of 147 individuals in 1947. Eric Lander, one of the principal leaders in the Human Genome Project, finished in the second five in 1976. Both Mumford and Lander are MacArthur Fellows. Distinguished computer scientist Donald Knuth received Honorable Mention in 1959. American Mathematical Society Presidents who did well in the Putnam are Irving Kaplansky (Putnam Fellow, 1938), Andrew Gleason (Putnam Fellow, 1940, 1941, 1942), Felix Browder (Putnam Fellow, 1946), and AMS and MAA President Ron Graham (Honorable Mention, 1958). Putnam Fellows in National Academy of Sciences include (this list may not be exhaustive) Elwyn Berlekamp, Felix Browder, Eugenio Calabi, Andrew Gleason, Melvin Hochster, Roger Howe, Irving Kaplansky, George W. Mackey, John W. Milnor, David Mumford, Daniel G. Quillen, Lawrence A. Shepp, Peter W. Shor, and Kenneth G. Wilson. Many others who have done well in the Putnam have won the prestigious research awards given by the American Mathematical Society. The 1956 Harvard team had both a future Nobel prize winner (Wilson) and a future Fields medalist (Mumford). Both were Putnam Fellows that year and Harvard's team finished first.

5. CONCLUSION. Table 4 provides the top five scores and the median score for each competition between 1967 and 2006.[3] Note that in five of those years the median score was zero and in six of them it was one! Between 1999 and 2006, only once was the medium score greater than 1. Also observe that in 1995 only one point separated the highest and fifth highest scores. In the period 1967–2006 the largest gap between the top score and the fifth highest score was thirty-five, while the largest gap between highest top score and the second highest was twenty-two. The largest median in the period was 19; the average median score is 5.3; the median of the median scores is 4. The greatest number of zero scores occurred in 2006, when 2279 out of 3640 participants registered scores of zero. The highest percentage of scores of zero occured in 2006 with 62.6% of the scores being zero. Table 5 gives the mean score, the percentage of the score of 0, and the score needed to finish in the top 500 in the period from 1987 to 2006.

Is there a lesson to be learned by examining the results of the Putnam competition? It seems that doing well on the Putnam exam correlates well with high achievement as a professional mathematician, but many of the best research mathematicians have not scored high on the Putnam and of course many have not even taken the exam.

Oh, by the way, the cadets of West Point beat Harvard that day in 1933. A cadet had the top individual score. Army's victory was reported in the newspapers and the Army team received a special letter of congratulations from the Army Chief of Staff, General Douglas MacArthur.

Reference [6], written by Putnam Fellows Kedlaya, Poonen, and Vakil, gives the problems with solutions and commentary from the Putnam competitions from 1985-2000. References [3] and [4] are articles that relate Putnam trivia. Reference [1] is an article that provides the views of the Putnam competition by a number of Putnam fellows. The web site http://www.d.umn.edu/~jgallian/putnamfel/PF.html provides information about Putnam Fellows.

[3]This was all the data that I could locate.

Table 1. Number of participants in the first sixty-seven competitions.

Year	*Number*	*Year*	*Number*	*Year*	*number*
1938	163	1962	1187	1984	2149
1939	200	1963	1260	1985	2079
1940	208	1964	1439	1986	2094
1941	146	1965	1596	1987	2170
1942	114	1966	1526	1988	2096
1946	67	1967	1592	1989	2392
1947	145	1968	1398	1990	2347
1948	120	1969	1501	1991	2325
1949	155	1970	1445	1992	2421
1950	223	1971	1596	1993	2356
1951	209	1972	1681	1994	2314
1952	295	1973	2053	1995	2468
1953	256	1974	2159	1996	2407
1954	231	1975	2203	1997	2510
1955	256	1976	2131	1998	2581
1956	291	1977	2138	1999	2900
1957	377	1978	2019	2000	2818
1958 S	430	1979	2141	2001	2954
1958 F	506	1980	2043	2002	3349
1959	633	1981	2043	2003	3615
1960	867	1982	2024	2004	3733
1961	1094	1983	2055	2005	3545
				2006	3640

Table 2. Number of teams 1975–2006.

Year	*Number*	*Year*	*Number*	*Year*	*Number*	*Year*	*Number*
1975	285	1983	256	1991	291	1999	346
1976	264	1984	264	1992	284	2000	322
1977	266	1985	264	1993	291	2001	336
1978	246	1986	270	1994	284	2002	376
1979	258	1987	277	1995	306	2003	401
1980	251	1988	257	1996	294	2004	411
1981	251	1989	288	1997	313	2005	395
1982	249	1990	289	1998	319	2006	402

Table 3. Winning teams in the first sixty-seven competitions.

Institution	*First Place*	*Second Place*	*Third Place*	*Fourth Place*	*Fifth Place*	*Putnam Fellows*
Harvard University	25	9	12	5	1	95
California Inst. Technology	9	3	5	5	5	21
Massachusetts Inst. Technology	5	9	8	8	6	45
University of Toronto	4	5	4	4	1	23
Washington University	4	4		1	2	6
Duke University	3	2	6			6
Brooklyn College	3	1	1			5
Michigan State University	3			2		5
University of Waterloo	2	3	6	2	4	8
Cornell	2	3	1	1	2	5
Polytechnic Inst. Brooklyn	2	1				3
Princeton University	1	9	4	7	4	20
University of Chicago	1	3	3	1	3	10
U. California, Berkeley	1	1	2	4	2	16
U. California, Davis	1	1		1		2
Queen's University	1		1	1		1
Case Western Reserve	1			2	1	4
Yale University		3	1	4	3	8
Columbia University		2	3			8
Rice University		1	1	1	1	3
U. Pennsylvania		1	1	1		3
City College New York		1		4		10
Dartmouth		1			1	2
U. British Columbia		1			1	1
Oberlin College		1				
Carnegie Mellon			2	1		3
Cooper Union			2			1
U. California, Los Angeles			1		1	2
Harvey Mudd College			1		1	
U. Maryland, College Park			1		1	
New York University			1			3
Miami University			1			
Mississippi Women's College			1			
Stanford University				3	2	

Table 3 (cont.). Winning teams in the first sixty-seven competitions.

Institution	*First Place*	*Second Place*	*Third Place*	*Fourth Place*	*Fifth Place*	*Putnam Fellows*
U. Michigan, Ann Arbor				1	2	
Kenyon College				1		2
Swarthmore				1		1
University of Manitoba				1		1
Illinois Inst. Technology				1		
McGill University				1		1
University of Kansas					1	
U. of Minnesota Minneapolis						3
Purdue University						2
U. Alberta						2
U. California, Santa Barbara						2

Table 4. Top five scores and median for the 1967–2006.

Year	*1*	*2*	*3*	*4*	*5*	*median*
1967	67	62	60	58	57	6
1968	93	92	89	85	85	10
1969	87	82	80	79	73	10
1970	116	107	104	97	96	4
1971	109	90	88	84	74	11
1972	85	79	66	63	59	4
1973	106	86	86	78	76	7
1974	77	70	62	61	57	6
1975	88	87	86	84	80	6
1976	74	70	68	64	61	2
1977	110	103	90	90	88	10
1978	90	77	74	73	71	11
1979	95	90	87	87	73	4
1980	73	72	69	68	66	3
1981	93	72	64	60	60	1
1982	98	90	88	85	82	2
1983	98	88	81	80	79	10
1984	111	89	81	80	80	10
1985	108	100	94	94	91	2
1986	90	89	86	82	81	19
1987	120	108	107	90	88	1
1988	120	120	119	112	110	16
1989	94	81	78	78	77	0
1990	93	92	87	77	77	2
1991	100	98	97	94	93	11
1992	105	100	95	95	92	2
1993	88	78	69	61	60	10
1994	102	101	99	88	87	3
1995	86	86	86	85	85	8
1996	98	89	80	80	76	3
1997	92	88	78	71	69	1
1998	108	106	103	100	98	10
1999	74	71	70	69	69	0
2000	96	93	92	92	90	0
2001	101	100	86	80	80	1
2002	116	108	106	96	96	3
2003	110	96	95	90	82	1
2004	109	101	99	89	89	0
2005	100	98	89	86	80	1
2006	101	99	98	92	92	0

Table 5. Mean, percent 0, Top 500 1997–2006.

Year	*Mean*	*pct. 0*	*Top 500*
1987	7.3	47.7	12
1998	14.8	30.3	28
1999	6.3	60.2	11
2000	5.3	57.7	11
2001	8.9	44.9	20
2002	11.0	34.7	24
2003	7.1	27.8	18
2004	8.4	53.6	22
2005	7.9	46.7	20
2006	6.2	62.6	14

6. APPENDIX I: EXAMINATION QUESTIONS FOR THE FIRST WILLIAM LOWELL PUTNAM MATHEMATICAL COMPETITION, APRIL 16, 1938.

MORNING SESSION: 9:00 to 12:00 NOON.

1. A solid is bounded by two bases in the horizontal planes $z = h/2$ and $z = -h/2$, and by such a surface that the area of every section in a horizontal plane is given by a formula of the sort Area $= a_0z^3 + a_1z^2 + a_2z + a_3$ (where as special cases some of the coefficients may be 0). Show that the volume is given by the formula $V = (1/6)h[B_1 + B_2 + 4M]$, where B_1 and B_2 are the areas of the bases, and M is the area of the middle horizontal section. Show that the formulas for the volume of a cone and a sphere can be included in this formula when $a_0 = 0$.

2. A can buoy is to be made of three pieces, namely, a cylinder and two equal cones, the altitude of each cone being equal to the altitude of the cylinder. For a given area of surface, what shape will have the greatest volume?

3. If a particle moves in a plane, we may express its coordinates x and y as functions of the time t. If $x = t^2 - t$ and $y = t^4 + t$, show that the curve has a point of inflection at $t = 0$, and that the velocity of the moving particle has a maximum at $t = 0$.

4. A lumberman wishes to cut down a tree whose trunk is cylindrical and whose material is uniform. He will cut a notch, the two sides of which will be planes intersectiong at a dihedral angle θ along a horizontal line through the axis of the cylinder. If θ is given, show that the least volume of material is cut when the plane bisecting the dihedral angle is horizontal.

5. Evaluate the limits:

(a) $\lim_{n\to\infty} \frac{n^2}{e^n}$ (b) $\lim_{x\to 0} \frac{1}{x} \int_0^x (t + \sin 2t)^{1/t} dt$

6. A swimmer stands at one corner of a square swimming pool and wishes to reach the diagonally opposite corner. If w is his walking speed and s is his swimming speed ($s < w$), find his path for the shortest time. [Consider two cases: (*a*) $w/s < \sqrt{2}$ and (*b*) $w/s > \sqrt{2}$].

7. TAKE EITHER (*a*) or (*b*).

(*a*) Show that the gravitational attraction exerted by a thin homogeneous spherical shell at an external point is the same as if the material of the shell were concentrated at its center.

(*b*) Determine all the straight lines which lie upon the surface $z = xy$, and draw a figure to illustrate your result.

AFTERNOON SESSION: 2:00-5:00 P.M.

8. TAKE EITHER (*a*) or (*b*).

(*a*) Let A_{ik} be the cofactor of a_{ik} in the determine

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix}.$$

Let D be the corresponding determinant with a_{ik} replaced by A_{ik}. Prove $D = d^3$.

(*b*) Let $P(y) = Ay^2 + By + C$ be a quadratic polynomial in y. If the roots of the quadratic equation $P(y) - y = 0$ are a and b ($a \neq b$), show that a and b are roots of the biquadratic equation $P[P(y)] - y = 0$. Hence write down a quadratic equation which will give the other two roots, c and d, of the biquadratic. Apply this result to solving the following biquadratic equation:

$$(y^2 - 3y + 2)^2 - 3(y^2 - 3y + 2) + 2 - y = 0.$$

9. Find all the solutions of the equation

$$yy'' - 2(y')^2 = 0$$

which pass through the point $x = 1, y = 1$.

10. A horizontal disc of diameter 3 inches is rotating at 4 revolutions per minute. A light is shining at a distant point in the plane of the disc. An insect is placed at the edge of the disc furthest from the light, facing the light. It at once starts crawling, and crawls so as always to face the light, at 1 inch per second. Set up the differential equation of motion, and find at what point the insect again reaches the edge of the disc.

11. Given the parabola $y^2 = 2mx$. What is the length of the shortest chord that is normal to the curve at one end?

12. From the center of a rectangular hyperbola a perpendicular is dropped upon a variable tangent. Find the locus of the foot of the perpendicular. Obtain the equation of the locus in polar coordinates, and sketch the curve.

13. Find the shortest distance between the plane $Ax + By + Cz + 1 = 0$ and the ellipsoid $x^2/a^2 + y^2/b^2 + x^2/c^2 = 1$. (For brevity, let

$$h = 1/\sqrt{A^2 + B^2 + C^2} \text{ and } m = \sqrt{a^2A^2 + b^2B^2 + c^2C^2}.)$$

State algebraically the condition that the plane shall lie outside the ellipsoid.

7. APPENDIX II: POSSIBLE MOST DIFFICULT PROBLEMS ON PUTNAM COMPEITION BETWEEN 1974-2006

1979 competition (no positive scores)

A-6 Let $0 \le p_i \le 1$ for $i = 1, 2, \ldots, n$. Show that

$$\sum_{i=1}^{n} \frac{1}{|x - p_i|} \le 8n(1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1})$$

for some x satisfying $0 \le x \le 1$.

1999 competition (only one positive score–2 points)

B-4 Let f be a real function with a continuous third derivative such that $f(x), f'(x), f''(x)$, $f'''(x)$ are positive for all x. Suppose that $f'''(x) \le f(x)$ for all x. Show that $f'(x) < 2f(x)$ for all x.

1999 competition (only one positive score–2 points)

B-5 For an integer $n \ge 3$, let $\theta = 2\pi/n$. Evaluate the determinant of the $n \times n$ matrix $I + A$, where I is the $n \times n$ identity matrix and $A = (a_{jk})$ has entries $a_{jk} = \cos(j\theta + k\theta)$ for all j, k.

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