

- How many abelian groups are there of order $2^5 3^2 5^3$?
- Let G be a group and let H be a normal subgroup of G . Prove or give a counter example to the following: If H is abelian and G/H is abelian, then G is abelian.
- Let $G = \left\{ \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix} \mid a \neq 0, a, b \text{ are real} \right\}$. G is a group. Let $\alpha: G \rightarrow \mathbb{R}^*$ be defined by $\alpha \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix} = a$. Let $H = \left\{ \begin{pmatrix} 1 & c \\ 0 & 1 \end{pmatrix} \mid c \text{ is real} \right\}$.
 - Show that H is a normal subgroup of G .
 - What isomorphism of groups results from the First Isomorphism theorem using G , \mathbb{R}^* , and α ?
- Let $G = \mathbb{Z}_{40} \oplus \mathbb{Z}_{50} \oplus \mathbb{Z}_{90} \oplus \mathbb{Z}_{100}$.
 - Express G as a direct product of cyclic groups of prime power order.
 - Express G as a direct product in the form $\mathbb{Z}_{m_1} \oplus \mathbb{Z}_{m_2} \oplus \cdots \oplus \mathbb{Z}_{m_k}$ where m_1 is a multiple of m_2 is a multiple of \cdots is a multiple of m_k .
- How many homomorphisms are there from \mathbb{Z}_{40} onto \mathbb{Z}_{40} ? \mathbb{Z}_{40} into \mathbb{Z}_{40} ?
- $\mathbb{Z} \oplus \mathbb{Z}$ is both a ring and a group.
 - Which subgroups of the form $\langle (a, b) \rangle$ are subrings of $\mathbb{Z} \oplus \mathbb{Z}$?
 - What are the units of $\mathbb{Z} \oplus \mathbb{Z}$?
 - What are the zero divisors of $\mathbb{Z} \oplus \mathbb{Z}$?
- Let R be a commutative ring with unity.
 - If a is a zero divisor in R and b is any element of R such that $ab \neq 0$, prove that ab is a zero divisor.
 - Prove that if a is invertible in R , then a is not a zero divisor.
Hint: suppose $ax = 0$ for some x in R .
 - Let $R[i] = \{a + bi \mid a, b \text{ are in } R\}$. Prove that $a + bi$ is a zero divisor in $R[i]$ if $a^2 + b^2 = 0$ or $a^2 + b^2$ is a zero divisor in R .
Hint: $a^2 + b^2 = (a + bi)(a - bi)$.
- One of $\mathbb{Z}_5[i]$ and $\mathbb{Z}_7[i]$ is a field and the other is not. Determine which is which. Hint: use 6(c).