Math 5372 Exam 2 Review
Spring 2009

1. Let $R = 3Z \oplus 4Z$, let $A = 9Z \oplus 4Z$.
   a. Show that A is an ideal of R.
   b. Determine whether A is a prime ideal of R.
   c. Determine whether A is a maximal ideal of R.
   d. Write out the addition and multiplication tables for R/A. Is R/A a field? An integral domain? Why does this not contradict the theorems on page 259 of the book?

2. Show that $R[x]/\langle x - 5\rangle \cong R$.

3. Show that $Z[\sqrt{n}]/\langle\sqrt{n}\rangle \cong Z_n$ if n is not a square. Hint: define $\varphi: Z[\sqrt{n}] \to Z_n$ by $\varphi(a + b\sqrt{n}) = a \bmod n$.

4. Show that every field is a principal ideal domain.

5. Factor $x^{12} - 1$ into irreducible polynomials over Q. Justify that each term is, in fact, irreducible.

6. Find all irreducible polynomials of degree 2 and 3 over Z_2. Use this to show that $x^6 + x^3 + 1$ is irreducible over Z_2.

7. a. Show that $\langle 2\rangle$ is a maximal ideal in Z but not in Z[i].
   b. Show that $\langle 3\rangle$ is a maximal ideal in Z AND in Z[i].

8. a. Show that $\langle\sqrt{5}\rangle$ is a maximal ideal of $Z[\sqrt{5}]$.
   b. Show that $\langle\sqrt{6}\rangle$ is not a maximal ideal of $Z[\sqrt{6}]$.