1. Which of the following are subspaces? In each case, justify your answer.

   (a) $U$ = the set of all upper triangular matrices in $M_{2\times2}$.
   (b) $V$ = the set of all matrices in $M_{2\times2}$ with at least one entry equal to zero.
   (c) $W$ = the set of those polynomials in $P_3$ with coefficients of $t$ and $t^2$ equal to each other. That is, $2t^3 - t^2 - t + 5$ is in $W$ but $4t^3 + 3t^2 + 2t + 1$ is not.
   (d) $X$, the subset of $P_3$ with $p(1) \neq p(-1)$.

2. Find bases for each of the following subspaces.

   (a) $U$ = the set of all points on the plane $x - y - 2z = 0$.
   (b) $V$ = the set of all $2 \times 2$ matrices $A$ with $A = A^t$.
   (c) $W$ = the set of all polynomials in $P_3$ with $p(1) = p(-1)$.

3. Let $A$ be the matrix

   \[
   \begin{pmatrix}
   1 & 1 & 1 & 3 & 2 \\
   3 & 3 & 5 & 5 & 12 \\
   5 & 5 & -1 & 27 & -8
   \end{pmatrix}
   \]

   (a) Find a basis for the Null Space of $A$.
   (b) Find a basis for the Column Space of $A$.
   (c) Find a basis for the Row Space of $A$.
   (d) By row reducing $A^t$, find a “nice” basis for the Column Space of $A$.
   (e) Write the fourth column of $A$ as a linear combinations of earlier columns.
   (f) Find another dependence relation among the columns of $A$ (other than column 1 = column 2).
   (g) Find a dependence relation among the rows of $A$.

4. Suppose $\{u, v, w\}$ is a linearly independent set.

   (a) Show that $\{u + v, v + 2w, u + 2v + 2w\}$ is linearly dependent.
   (b) Carefully show that $4u + 6v + 4w$ is in Span$\{u + v, v + 2w, u + 2v + 2w\}$.
   (c) Carefully show that $6u + 4v + 6w$ is not in Span$\{u + v, v + 2w, u + 2v + 2w\}$. Explain where the independence of $\{u, v, w\}$ is needed.

5. Extend the set $\left\{ \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \right\}$ to a basis for $M_{2\times2}$. Explain your reasoning.
6. The matrix $A$ in problem 3 was just one example of a $3 \times 5$ matrix; there are lots more $3 \times 5$ matrices around. Fill in the blanks: For every $3 \times 5$ matrix $B$,

$$\_\_\_\_\_\_\_\_\_\_ \leq \dim(\text{Null}(B)) \leq \_\_\_\_\_\_\_\_\_\_. $$

Justify your answer.

7. Suppose $A$ is a $3 \times 3$ matrix and \[
\begin{pmatrix}
1 \\
1 \\
1
\end{pmatrix}
\text{ and } \begin{pmatrix}
1 \\
2 \\
1
\end{pmatrix}
\] are both in $\text{Nul}(A)$.

(a) Show that \[
\begin{pmatrix}
2 \\
3 \\
2
\end{pmatrix}
\] is in $\text{Nul}(A)$.

(b) What can you say about the rank and nullity of $A$?