Use Mathematical Induction to prove each of the following.

1. \[ 1 \cdot 2 + 2 \cdot 3 + \cdots + n(n + 1) = \frac{n(n + 1)(n + 2)}{3} \] for all \( n \geq 1 \).

2. \( 7^n - 1 \) is divisible by 6 for all \( n \geq 0 \).

3. \[ 1 + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{n^2} < 2 - \frac{1}{n} \] for all \( n \geq 1 \).

4. In calculus, we proved early on that \( \frac{d}{dx} x^n = nx^{n-1} \). If you look at that proof, it might strike you as awkward or unconvincing. Give a proof by Mathematical Induction (using the product rule in the form \( \frac{d}{dx} (x \ f(x)) = f(x) + x \ f'(x) \)).

5. Find all \( n \) for which \( n! \geq n^2 \). Use Mathematical Induction to prove your answer is correct.

6. By checking what happens when \( n = 1, 2, 3, 4 \), guess a formula for \[ \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{n(n + 1)} \]. You should get a nice simple answer in terms of \( n \). Prove your answer correct using Mathematical Induction.