1. Let \( A = \begin{pmatrix} 1 & 1 & 1 & 2 & 1 \\ 2 & 2 & 2 & 4 & 2 \\ 3 & 3 & 4 & 7 & 5 \end{pmatrix} \).

(a) Find all solutions to \( Ax = \begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix} \). Put your answer in vector parametric form.

(b) Find a basis for the null space of \( A \).

(c) Find a basis for the column space of \( A \).

(d) Find a basis for the row space of \( A \).

(e) Find a basis for the column space of \( A \) which is different for that found in part (c). This basis should be “nice,” meaning that it should be immediately obvious from this basis if some general vector is in the column space.

(f) Write the fourth column of \( A \) as a linear combination of earlier columns of \( A \).

(g) Find a dependence relation among the rows of \( A \).

2. Find the inverse of \( A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix} \).

3. In this question, \( A, B \) and \( X \) are \( 2 \times 2 \) matrices.

(a) Solve \( X \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 4 & 8 \end{pmatrix} \) for \( X \). (Note: \( \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \) is invertible.)

(b) Find all solution \( X \) for \( X \begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 4 & 8 \end{pmatrix} \). Give one explicit solution

(c) Show that \( \begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix} X = \begin{pmatrix} 2 & 4 \\ 4 & 8 \end{pmatrix} \) has no solutions. (Here, \( X \) is on the right instead of the left.)

4. A square matrix \( R \) is called a reflection matrix if \( R^2 = I \). Here are some examples: \( \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \). Find a \( 2 \times 2 \) reflection matrix with all entries nonzero. Can you find all \( 2 \times 2 \) reflection matrices? That is, find necessary and sufficient conditions on \( a, b, c, d \) so that \( \begin{pmatrix} a & b \\ c & d \end{pmatrix}^2 = I \).