From the book: 3.1.4 and 3.1.6, 3.2.14 (Sections 3.1, 3.2). Also do the following:

1. Recall that if $v$ is a column vector of the right size and $A$ is an $m \times n$ matrix, then $A_k(v)$ is the matrix obtained by replacing the $k$'th column of $A$ by $v$. Define $T : F^3 \rightarrow F^3$ as follows. Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{pmatrix}$. Then $T(v) = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$, where $a = \det(A_1(v))$, $b = \det(A_2(v))$, $c = \det(A_3(v))$.
   (a) Find a basis for the kernel of $T$.
   (b) Find a basis for the range of $T$.
   (c) Find the matrix of the transformation.

2. Let $T : F^{2\times2} \rightarrow F^{2\times2}$ be defined by $T(A) = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} A - A \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$.
   (a) Find a basis for the kernel of $T$.
   (b) Find a basis for the range of $T$.
   (c) Find a simple formula for $T^2 = T \circ T$.

3. Let $T : P^3 \rightarrow R^{2\times2}$ be defined by $T(p(x)) = p(A)$, where $A = \begin{pmatrix} 2 & 4 \\ -1 & 0 \end{pmatrix}$. For example,
   $T(x^2 - 2x + 3) = A^2 - 2A + 3I = \begin{pmatrix} 0 & 8 \\ -2 & -4 \end{pmatrix} - \begin{pmatrix} 2 & 4 \\ -1 & 0 \end{pmatrix} + 3 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ -1 & -1 \end{pmatrix}$.
   (a) Find a basis for the kernel of $T$.
   (b) Find a basis for the range of $T$.

For extra credit:

4. The matrix in problem 1 had 0 for a determinant. Did this have a big influence on the problem? That is, if a matrix $A$ with nonzero determinant had been used, how much different in character would the problem have been?

5. More generally, if $A$ is $n \times n$ and $T : F^n \rightarrow F^n$ is defined analogously to what was done in problem 1, how does $T$ vary with the rank of $A$?

6. What are the range and kernel of $T$ in problem 3 if the domain is $P^n$ instead of $P^3$?