1. Find the characteristic polynomial, the eigenvalues, and the eigenspaces of each of the following matrices.

\[(a) \quad A = \begin{pmatrix} 0 & 8 & 1 \\ 1 & 0 & 0 \\ 0 & 4 & -1 \end{pmatrix}, \quad (b) \quad B = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad (c) \quad C = \begin{pmatrix} 0 & 0 & -2 \\ 1 & 0 & 3 \\ 0 & 1 & 0 \end{pmatrix} \]

\[(d) \quad D = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & -3 \\ 0 & 1 & 3 \end{pmatrix}, \quad (e) \quad E = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 3 \end{pmatrix} \]

2. Evaluate \( \lim_{n \to \infty} A^n \) where \( A = \begin{pmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{pmatrix} \).

3. If \( A \) is a real symmetric \( 2 \times 2 \) matrix, prove that there is a real matrix \( P \) for which \( P^{-1}AP \) is a diagonal matrix.

4. If \( A \) is a \( 2 \times 2 \) matrix with complex eigenvalues \( a \pm bi \) show that there is a real matrix \( P \) for which \( P^{-1}AP = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \).

5. Let \( T : P^3 \to P^3 \) be defined by \( T(p(x)) = xp(x+1) - (x+1)p(x) - xp'(x) \).

   (a) Find the characteristic polynomial of \( T \).
   (b) Find a basis for each eigenspace of \( T \).

6. Let \( T : F^{2\times2} \to F^{2\times2} \) be defined by \( T(A) = A^t \), the transpose of \( A \).

   (a) Find the characteristic polynomial of \( T \).
   (b) Find a basis for each eigenspace of \( T \).

   For extra credit:

7. Repeat question 5, but on \( P^n \) instead of \( P^3 \).

8. Repeat question 6, but on \( F^{n\times n} \) instead of \( F^{2\times2} \).

9. If \( A \) is a real \( 2 \times 2 \) matrix with real, irrational eigenvalues, show that something similar to (4) can be done using a matrix \( P \) with only rational entries.