Math 5330 Homework 4
Spring 2009

Do from the book: Page 35 numbers 2, 9, 13a

1. Use the Euclidean algorithm to find the greatest common divisor of each pair of numbers. In each case, express the GCD as a linear combination of the two numbers.
   a. (45, 75) b. (102, 222)
   c. (666, 1414) d. (20785, 44350)

2. Find the greatest common divisor of 1234^{10} and 10^{1234}.

3. Let LCM(m, n) be the least common multiple of m and n. This means that LCM(m, n) is the smallest positive integer which is a multiple of both m and n. Prove that $\mathrm{LCM}(m, n) = \dfrac{mn}{\mathrm{GCD}(m, n)}$.

4. Find all integer solutions to each of the following equations.
   a. $33x + 121y = 1000$.
   b. $105x + 286y = 1002$.
   c. $2072x + 1813y = 2849$.

For extra credit:

1. Prove that if the successive remainders in Euclid's algorithm for gcd(m, n) are $r_0 = n, r_1, r_2, \cdots$, then $r_{i+2} < \frac{1}{2} r_i$ for all i.
   Use this to show that Euclid's algorithm takes at most $2 \log_2(n)$ steps.

2. Do 13b on page 35. (That is, show that with the version of Euclid's algorithm in that problem, there are at most $\log_2(n)$ steps.)