Math 5330 Homework 6
Spring 2009

1. Find a 100-digit probable prime. I will give an extra point if your prime is different from those of the rest of the class.

2. Let $m = 10^{100} + 37$. Use Fermat's Little Theorem to prove that m is not prime.

3. Use Fermat's Little theorem to verify that 561 is a Carmichael number. Hint: show that for any a, $a^{561} \equiv a \pmod{3}$, $a^{561} \equiv a \pmod{11}$, $a^{561} \equiv a \pmod{17}$. Conclude (with reasons) that $a^{561} \equiv a \pmod{561}$.

4 If $n > 2$, prove that $\varphi(n)$ is even. (Do this without using the formula for $\varphi(n)$.)

5. a. Find the smallest base 2-pseudoprime $n > 1000$.
   b. Give a paper, pen and calculator demonstration that $2^{n-1} \equiv 1 \pmod{n}$.
   c. Prove (as in problem 3) that n is a Carmichael number.

6. Prove that if p is a prime, then $\varphi(p^n) = p^{n-1}(p - 1)$. (Do this without using the formula for $\varphi(n)$.)

For extra credit, you can try these problems from the book:
page 58 numbers 21, 22 and 24.