Week #9  Computational Methods  Fall 2017  

For Monday (10/23):

**Hand in:** Exercise 5.16 with use of nonadaptive RK4; estimate the accuracy of your result.

**Read:** Sections 5.10-12.

**Consider:** Write the equation of motion for a damped oscillator driven by an oscillatory force and construct the analytic solution.

For Wednesday (10/25):

**Hand in:** Derive the Numerov algorithm

\[
y_{i+1} = \left[ 2 \left( 1 + \frac{5}{12} h^2 g_i \right) y_i - \left( 1 - \frac{1}{12} h^2 g_{i-1} \right) y_{i-1} \right] / \left( 1 - \frac{1}{12} h^2 g_{i+1} \right)
\]

for the solution of second-order equations, such as the Schrödinger equation, which are of the form

\[
\frac{d^2 y}{dx^2} = g(x) y(x).
\]

Here \( y_i \equiv y(x_i) \), \( g_i \equiv g(x_i) \), and \( x_i \equiv x_0 + ih \).

**Hint:** Use Taylor expansions to show that

\[
y_{i+1} - 2y_i + y_{i-1} = h^2 y''(x_i) + \frac{1}{12} h^4 y^{(iv)}(x_i) + \mathcal{O}(h^6)
\]

and write

\[
y^{(iv)}(x_i) = \frac{d^2}{dx^2} \left[ g(x) y(x) \right]_{x=x_i} = \frac{g_{i+1} y_{i+1} - 2g_i y_i + g_{i-1} y_{i-1}}{h^2} + \mathcal{O}(h^2).
\]

**Read:** Section 5.14.

**Consider:** To use the Numerov algorithm for the solution of a boundary-value problem by the shooting method, what takes the place of guessing \( y'(x_0) \)?

Use the modified Euler method (with \( h = 0.25 \)) for the nonlinear boundary-value problem:

\[
\frac{d^2 y}{dx^2} = \frac{1}{2} y^3, \quad y(1) = -\frac{2}{3}, \quad y(2) = -1.
\]

Compare the result with the exact solution \( y = 2/(x - 4) \).

For Friday (10/27): No class. Fall break!

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**Project #4** (due: 11/3)

Exercise 5.6, but with use of the modified Euler and nonadaptive RK4 methods. Each differential-equation method should be implemented and tested as its own separate, general-purpose procedure and then used to solve the “projectile” problem. In addition to finding the velocity as a function of time, determine the position. [**Hint:** Be careful about the choice of coordinate axis relative to the form of the equation of motion given in the exercise statement. Does it point up or down?]