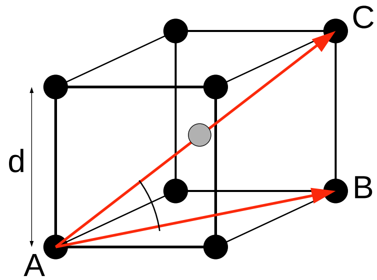
Dot (scalar) products and cross (vector) products are used widely in physics. You can review general properties of vectors as needed in chapter 1 of University Physics (UP), your General Physics textbook. Kinematics of 1-D and 2-D motion are found in chapters 2 and 3 of UP.

Problems for Friday, September 2:

(1) You are watching people practicing archery when you wonder how fast an arrow is shot from a bow. With a flash of insight you remember your physics and see how you can easily estimate what you want to know by a simple measurement. You ask one of the archers to pull back her bow string as far as possible and shoot an arrow horizontally. The arrow strikes the ground at an angle of about 80 degrees from the vertical at 100 ft from the archer. So how fast is the arrow launched?

(2) Use the properties of the scalar or dot product of vectors to answer the following:



Some metallic elements have their atoms arranged on a body-centered cubic structure shown at left, with one atom at each corner of a cube and one more atom (gray) at the center of the cube. The length of a side is d.

Consider the vectors connecting atoms A and B (a face diagonal) and atoms A and C (usually called the body diagonal).

Determine the angle between these two vectors

(3) Consider three arbitrary vectors: $\vec{A}$, $\vec{B}$, and $\vec{C}$. Write out in terms of the x, y, and z components of these vectors the quantity $(\vec{A} \times \vec{B}) \cdot \vec{C}$.
Then explain in a systematic and step-by-step way how to re-arrange the terms so that your new expression can be easily recognized as $\vec{A} \cdot (\vec{B} \times \vec{C})$.

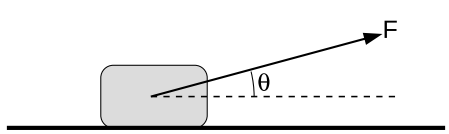
See Chapters 4 and 5 of University Physics for a review of Newton's laws,freebody diagrams and dynamics. See summaries of chapters 6-8 for a reminder the work-kinetic energy theorem, conservative forces, and conservation of energy and conservation of momentum principles.

Computational task: Begin learning the rudiments of Matlab in MWAH 395. Matlab is installed on the computers there. Read through chapter 1 of *Getting Started with MATLAB*. Read and work through tutorial lessons 2.1-4, 2.6 (*Working with arrays and matrices*), and 2.10 (*Working with files*) in chapter 2. Try working (at least) the odd-numbered exercises for each lesson to re-inforce your understanding. You don't have to turn these in. Note: MWAH 395 is reserved Tu and Th from 2-5 pm for Instrumentation lab.

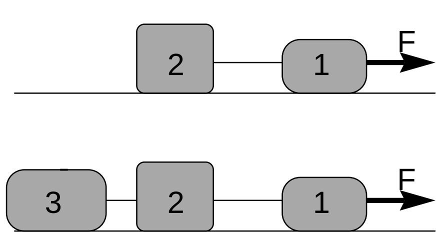
Problems for Friday, Sept. 16:

(4) Kinematics of uniform circular motion: A particle is moving in uniform circular motion about the origin. Its position can be described by the vector $\vec{r} = R\cos\omega t\,\hat{i} + R\sin\omega t\,\hat{j}$, with $\omega > 0$ as the angular speed of the object. Use this $\vec{r}$ to answer the following:
(a) Where is this particle at $t = 0$? Is this particle moving clockwise or counter-clockwise as seen from above? (Justify your answer.)
(b) Prove that $\vec{v}$ has a magnitude of ωR.
(c) Prove that $\vec{v}$ is always perpendicular to $\vec{r}$.
(d) Find the acceleration vector, $\vec{a}$. What is its magnitude? Indicate its direction in a diagram, along with $\vec{r}$ and $\vec{v}$, when $t = \frac{5\pi}{4\omega}$.

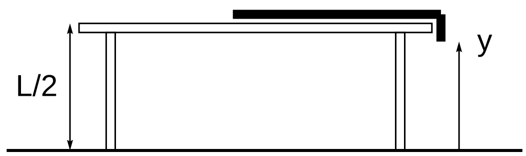
(5) A block mass $m = 5.0\,\text{kg}$ is on a surface with coefficients of friction $\mu_s = .30$ and $\mu_k = 0.25$. The angle of the pulling force can be varied. (a) What angle θ requires the smallest force to start the block in motion? Explain qualitatively why is there a minimum F at some angle. (b) What is this minimum force? (c) What is the resulting acceleration of the block once it starts to slide, assuming the pulling angle doesn't change?



(6) Two blocks are connected by a light string. A force F is pulling to the right as shown; there is no friction between the blocks and the supporting surface. When a third block is added as shown and the applied force remains the same, does the tension in the string connecting blocks 1 and 2 change? If so, does it increase or decrease? Justify your answer with arguments firmly rooted in Newton's laws. Do the relative sizes (masses) of the blocks matter? If so, how?



(7) A metal chain made of many small identical links with total mass M and length L lays straight across a level, low-friction table whose height is $L/2$. Initially the chain is held at rest with 1/8 of it hanging over the end of the table. (a) Find the chain's initial acceleration when released. (b) Find a formula for the acceleration of the chain as it pulls itself off the table in terms of the distance the chain is from the floor, y. (c) How fast is the chain moving when its lower end first reaches the floor? (Best approached from energy considerations, not dynamics and acceleration, since forces are not constant here.)



Problems due Friday, September 23:

Reading: Read chapter 1 and through p. 28 in chapter 2 in *Vibrations and Waves.*

Do problems 1.3, 1.8(c), 1.10, 1.11 from Vibrations and Waves.
There is a typo (at least in some printings) in problem 1.10: the differential equation should read

$$\frac{d^2y}{dx^2} = -k^2y \ .$$

Matlab Exercise MX-1: Read sec. 3.1,2,4,8 and 4.1 in *Getting Started with Matlab.* Section 6.1 has a quick intro to plotting with the `plot` command that's better than the section from chapter 3.

Write, test, and submit by e-mail a Matlab script file that:
(a) creates an array/vector named t that contains values from 0 to 20 in increments of 0.1;
(b) defines a quantity named omega, (ω) and sets it equal to 2.0;
(c) calculates two functions of t: (1) $e^{-t/5}$ and (2) the real part of $e^{j\omega t} + e^{-j\omega t}$ at each of the time values in t;
(d) plots on one graph the two functions vs. time.
Investigate in the Matlab help system the functions `real` and `imag`, as needed for part (c). Please name your script file with your UMD username embedded, as in smith456_mx1.m

Test #1 Monday, October 3.

Reading: Read the rest of chapter 2 and chapter 3 through page 57 in Vibrations and Waves.

For Friday, September 30: Do problems:

- 2.1 b,c (be sure to explicitly find the values of A and α for each case.)
- 2.3 (do this analytically on paper - not with Matlab);
- 2.6 Use Matlab and lissajous.m to do this, **but don't hand in**. Take $\omega = 1$.
- 3.3
- AND

Matlab Exercise MX-2: Consider an object, mass m, thrown straight up into the air from an initial height y_0 with an initial velocity v_0. It is subject to gravity (use $g = 10.0\,\text{m/s}^2$ *and* a drag force that depends on the velocity of the object, $\vec{f} = -b\vec{v}$.
(a) Draw a freebody diagram for this object while it is flight and use that to write down Newton's 2nd law for it in the form $a = F/m$, but spelling out the details of what the F is.
(b) Then complete, using base_mx2.m as a starting point, a Matlab script to calculate and store in arrays the 1-D motion (position, velocity, and acceleration) of the object thrown straight up in the air. Calculate the motion as a function of time, from $t = 0$ to $t = 10\,\text{sec}$ for a 1 kg object whose initial position is 100 m and initial velocity is 25 m/s and has a drag coefficient of $b = 0.4\,\text{Ns/m}$.
Complete the script using the simple numerical algorithm described in class to numerically integrate Newton's 2nd law:

$$v_{i+1} = v_i + a_i\Delta t$$

$$x_{i+1} = x_i + v_i\Delta t + \frac{1}{2}a_i(\Delta t)^2.$$

The script does the repeated iterations in the algorithm above with a `for` loop. *Getting Started with Matlab* describes `for` loops and other flow control structures used to repeat a series of steps in a calculation in sec. 4.3.4. You are being asked to fill in just a few lines in the basic script, but it is important that you take the time to understand what the script does, line-by-line.

(c) Finish the script to have it plot position and velocity as functions of time and compare these to the drag-free motion. Most of this is already in the base script. Name your script username_mx2.m.

(d) Once your script/program is working, use it to compile a table of the final position and velocity values for timesteps Δt ($= dt$ in the script) ranging from 1.0 sec down to 1.0×10^{-6} sec. Do this by hand – changing dt manually (try decade steps – factors of 10) and running the script again. For what size time step are final values consistent to 3 significant figures - i.e. smaller time steps produce final positions and velocities that differ from the next larger timestep by no more than about 0.1%?

Tabulate these results, print out plots of position and velocity for a time step that is small enough to give reliable results. Email your finished script.

Reading: Finish chapter 3 in Vibrations and Waves

For Tuesday, Oct. 11: Do problems 3.5, 3.7, 3.10, 3.11, and:

Matlab exercise MX-3: Adapt your previous script file (mx2) for numerically modeling a linear drag force on a vertically moving object to instead model a mass on a spring, also subject to gravity, but no drag force. Use a spring constant $k = 10\,\mathrm{N/m}$, and a 1 kg mass. Study the motion over the time interval $t = [0, 10]\,\mathrm{s}$. Use an initial position of $x_o = 1\,\mathrm{m}$ and $v_o = -2.5\,\mathrm{m/s}$. Explore the influence of the size of the timestep (from $0.1\,\mathrm{s}$ down to $1 \times 10^{-4}\,\mathrm{s}$) on $x(t)$ and $v(t)$. Describe what is wrong (unphysical) with the plots, particularly at larger values of Δt. Print out plots of $x(t)$and $v(t)$ for $\Delta t = 0.01\,\mathrm{s}$.

Matlab exercise MX-4: Because Matlab was designed to handle arrays of numbers efficiently, Matlab is a good way to deal with vectors encountered in physics. The snippet of code below is a Matlab function file named gravF12. You give it four parameters, two masses and their vector positions, and it returns the vector containing the gravitational force components.

```
function [ F12 ] = gravF12( m1, r1, m2, r2 )
% gravF12 computes vector gravitational force
% m1 = mass1, r1 = vector position of 1,
% m2=mass2 r2 = vector position of m2
% all in SI units
G=6.67E-11; % N m^2 / kg^2
r=r1-r2;
rmag=norm(r);
rhat = r/rmag;
F12 = -G*(m1*m2/rmag^2)*rhat;
end
```

Function files are described in Pratap's book in sec. 2.5 and 4.2. A simple example script of how this function might be used or "called" when calculating the gravitational force between Earth and its moon is:

```
re = [ 0, 0, 0 ]; me = 6.0E24; % Earth is put at origin
rm = [ 272E8 , 272E8, 0]; mm = me/81; % Moon has x,y components to its position
Fg = gravF12(me,re,mm,rm)
```

Figure out what each line in the function file does and provide a comment explaining what it does. Think about translating each line back into "physics-y" looking math instead of programming code. (Use Matlab's help system to learn about any new or unknown Matlab functions used: e.g. " `help xxx`" to learn what `xxx` does.
Draw a diagram illustrating all the variables used in the function.
Is the force vector it returns the force exerted on mass 2 by mass 1 or the force exerted on mass 1 by mass 2?

Reading: Read through p.88 in Vibrations and Waves

For Tuesday, October 18: Do problems 3.14, 3.17, 3.19.
Two comments on 3.17: (1) The figure is misleading. At equilibrium the fluid reaches the height h on each side. Therefore, when it is displaced from equilibrium so that it rises an amount y on the left, it is *below* h on the right (by an amount you need to and can easily figure out), not still at h as the figure suggests. One way to figure out the potential energy requested in part (a) is to consider how much work you would need to do pushing down on the right side to push the fluid up to y on the left. (2) In part (c), the approximation $y \ll h$ is assumed in calculating the kinetic energy of the vertical columns, which simplifies those contributions.

Matlab exercise MX-5: Use Matlab to plot the analytic solutions for $x(t)$ for a damped harmonic oscillator under three conditions:
(a) lightly damped (with $\gamma/2 = .1\omega_o$);
(b) critically damped ($\gamma/2 = \omega_o$);
(c) overdamped: ($\gamma/2 = 2\omega_o$).
Use $k = 10\,\text{N/m}$, $m = 0.2\,\text{kg}$. Model the system with initial conditions of $x_o = 0.10\,\text{m}$ and $v_o = 1.0\,\text{m/s}$. Choose a time range that is appropriate for best illustrating the different behaviors of the solutions. You will need to work out in advance on paper what the constants (e.g. A_o, α, etc.) are for each appropriate solution from the specified initial conditions. Use subplots to put all three $x(t)$ graphs in a single figure. Make the horizontal axis time - *not* just an array index. Print out your plots.

Matlab exercise MX-6: Improving the integration algorithm: Our method of integrating Newton's 2nd law to find $x(t)$ and $v(t)$ assumes the acceleration is constant throughout the timestep and uses the value of a at the start of the timestep. Our expression for x already recognizes that v changes during the timestep; now we want to make a better estimate of a during the timestep while still using constant acceleration formulas.

We begin at x_i with velocity v_i. We want to know where we will be and how fast we will be moving one timestep Δt later. The force at the start of the timestep is $F(x_i, v_i)$. Instead of this value, we want to make a reasonable guess as to what the force is at the mid-point of the timestep – call this $F_{i+\frac{1}{2}}$. We do this in the belief that this will better represent the average force acting during the timestep. To calculate this we need to know x and v in the middle of the timestep. We can already estimate Δv and Δx for the entire timestep using our basic algorithm:

$$\Delta v \approx a_i \Delta t \qquad\qquad \Delta x \approx (v_i + \frac{1}{2} a_i \Delta t)\Delta t$$

Therefore, we can estimate that at the midpoint of the timestep

$$v_{i+\frac{1}{2}} \approx v_i + \frac{\Delta v}{2} \quad \text{and} \quad x_{i+\frac{1}{2}} \approx x_i + \frac{\Delta x}{2}.$$

Then we can get an improved estimate of the average force (and acceleration) during the time step:

$$F_{i+\frac{1}{2}} = F(x_{i+\frac{1}{2}}, v_{i+\frac{1}{2}}) \quad \text{and} \quad a_{i+\frac{1}{2}} = \frac{F_{i+\frac{1}{2}}}{m}$$

Given this better (we hope!) estimate of the average acceleration during the timestep, we can finally compute our next v and x:

$$v_{i+1} = v_i + a_{i+\frac{1}{2}} \Delta t$$

$$x_{i+1} = x_i + v_i \Delta t + \frac{1}{2} a_{i+\frac{1}{2}} (\Delta t)^2$$

This algorithm requires more calculations by the computer for each timestep. If it is a substantial improvement, it will do the job more accurately, preferably using larger (and fewer) timesteps.

Implement this improved algorithm, building on your previous script for MX-3. Comment your code to make it understandable to someone reading it. Use it to explore the same mass on a spring as Matlab exercise 3, again using a range of timestep sizes. Print out plots of $x(t)$ and $v(t)$, for $\Delta t = 0.01\,\mathrm{s}$. Compare and contrast the performance of the two algorithms.

Reading: Read through the rest of chapter 4 in Vibrations and Waves

For Wednesday, October 26: Do problems 4.3, 4.5, 4.11, and 4.13.

Matlab exercise MX7: If Earth's moon were at its current distance from Earth, but both bodies were initially at rest, how long would it take for the moon to fall to Earth? How fast would it be going relative to Earth? (Ignore the presence of the sun, etc. Treat Earth + Moon as an isolated system.) (a) Answer these questions using the numerical methods developed so far to iterate the motion of the objects through time with Newton's 2nd law. To make things simple, choose initial conditions that put Earth at the origin and the moon on the x axis at the appropriate distance away at $t = 0$, with both initially at rest.

Instead of a `for` loop, you might want to implement a `while` loop in Matlab. Implement the improved scheme from exercise MX6 to advance the positions and velocities in small time steps dt so long as (e.g. `while`) the center-to-center distance between Earth and Moon is greater than Earth's radius + Moon's radius). Comment your code thoroughly.

(b) Compare your numerical final relative velocity when Earth and Moon first touch to the values you can predict using energy and momentum conservation ideas.

Reading: Read chap. 5 in *Vibrations and Waves*.

Test #2: Monday, Nov. 7???

For Friday, Nov. 4: Do problems 5.9, 5.14.

MX8. Assume the moon is in a circular orbit about the earth. (Forget that we're orbiting the sun.) (a) (Outside of Matlab) Given the Earth-Moon distance and the gravitational force law, calculate the moon's orbital speed. Calculate the orbital period. (b) Building on MX7, treating Earth as at the origin initially, assign the moon an initial position on the x axis at the appropriate distance and give it an initial velocity as found in (a) in the $+y$ direction. Produce a plot showing the motion of Earth and the moon (in the x-y plane) for about 3 lunar orbits. Print out a plot that seems reliable. Comment on the results. (c) Fix things up by giving Earth an appropriate initial velocity and re-run the code and produce a plot of the orbital motion. Compare to (b). (What does 'appropriate initial velocity' mean and how do you find this?) [Note: both Earth's and moon's positions and velocities need to be vectors now. If your MX7 code did a strictly 1-D calculation, you'll need to upgrade.]

Reading: Read chap. 6 in *Vibrations and Waves* and start chapter 7.

For Wednesday, Nov. 16: Do problems 6.1, 6.6, 6.7, 6.12

Reading: Read chap. 7 in *Vibrations and Waves.*

For Wednesday, Nov. 23: Do problems 7.2, 7.5, 7.11 (making plots in Matlab for this is OK, but you still need to think about and interpret what it all means).

Find the general expression for the Fourier coefficients B_n for a "square-wave" shape:

$$\begin{aligned} y(x) &= 0 \qquad 0 < x < L/4 \\ &= C \qquad L/4 < x < 3L/4 \\ &= 0 \qquad 3L/4 < x < L. \end{aligned}$$

Simplify your result for B_n as much as possible. Then evaluate the first 8 B_n explicitly. See if you can discern a general pattern.

Reading: Read chap. 8. in *Vibrations and Waves.* See also some reading resources mentioned on the course web page. We'll focus mostly on two-slit interference and diffraction effects.

For Friday, Dec. 2:
Do problems 7.18, 7.23, 7.25.
Two loudspeakers are pointed parallel to the y-axis, a distance $b = 4.0$ meters apart on the x axis, located at $x = \pm 2.0\,\mathrm{m}$. They emit sound of the same wavelength $\lambda = 1.0\,\mathrm{m}$ and are driven in phase. A person stands directly in front of a distance $D = 12.0$ meters directly in front of one speaker. (a) What's the path difference for the sound reaching the person from the two speakers? Is this location characterized by mostly constructive or mostly destructive interference? (b) If the person walks gradually directly backwards parallel to the y-axis, away from the speakers, where will she first experience a maximum in loudness? (c) If she instead walks parallel to the x axis from her original position, headed toward (and perhaps crossing) the y-axis, where will she sense her first two minima in loudness?

Due Dec. 9. This can be a group project by up to 4 persons, submitting one (well-prepared) summary/report. These descriptions are deliberately vague, with some initiative and creativity expected in carrying them out. (20 points)

Matlab Exercise MX9.
Extend the numerical solution of Newton's 2nd law to three gravitationally interacting bodies - sun, earth, and moon. (a) Establish appropriate estimates of reasonable initial conditions. (b) Numerically calculate and plot out their motion over a couple years. (c) Invent an alternate three-body solar system of your own invention and model it numerically.

OR

Experimental exercise EX1.
Use the motion sensor and mass spring system in MWAH 395 to record the decay of a freely oscillating mass on a spring. From the recorded oscillations, figure the spring constant k of the spring (take into the account the mass of the spring - see p 60-61 in French), the drag constant γ and the Q of the system. Explain clearly and completely how you do this, documenting your method and calculations. Does the decay data really support the model of a linear drag force, $-bv$ as opposed to a drag force proportional to $-cv^2$? (Suggestion, if drag is linear, any value of γ or b extracted from the observations should be independent of initial amplitude of motion, or be the same if determined over different segments of the same decay.

Uncollected problems for chapter 8.

What is the angle the first nodal line (destructive interference) makes with the y axis in the problem of two loudspeakers on the previous HW set? Assume the listener is far from the speakers.

A double-slit experiment is to be set up so that the bright fringes appear 1.27 cm apart on a screen 2.13 m away from the two slits. The light source was wavelength 500 nm. What should be the separation between the two slits?

Youngs double-slit experiment is performed immersed in water ($n = 1.333$). The light source is a He-Ne laser, $\lambda = 633.$ nm in vacuum. (a) What is the wavelength of this light in water? (b) What is the angle for the third order maximum for two slits separated by 0.100 mm.

French: 8.16b, 8.17

AC Circuits and impedances (a) Find the complex frequency-dependent impedance of a resistor R in parallel with a capacitor C. Express it in polar form. (b) Evaluate the magnitude and phase of the impedance for $R = 500\,\Omega$ and $C = 4.7\,\mu$F at a frequency of 3000 Hz. A voltage source is hooked up driving the parallel RC combination with a 2.0 V amplitude at 3000 Hz. Find the amplitude and the phase (relative to the driving voltage) of the total current.