9.40  

**Fabry–Pérot Interferometer**  

\[ r = 0.8944 \text{ for mirrors} \]
\[ R = r^2 = 0.8016 \]

\[ \Sigma = \Sigma_t \]

Coefficient of finesse

\[ \frac{I_t}{I_{t\text{max}}} = \frac{1}{1 + F \sin^2 \frac{\theta}{2}} \]

\[ F = \left( \frac{2r}{1 - r^2} \right)^2 \]

\[ d_s = \frac{2\pi}{d} \]

(ignoring phase changes due to reflection)

\[ \frac{I_{\text{min}}}{I_{\text{max}}} = \frac{1}{1 + F} \]

\[ 1 - A(s) = 1 \]

\[ \frac{I}{I_{\text{max}}} = \frac{1}{2} = \frac{1}{1 + F \sin^2 \frac{\theta}{2}} \]

Finesse = \frac{\text{Spacing of peaks}}{\text{Width of peaks}} = \frac{2\pi}{(2\pi/2)} = \frac{2\pi}{(2\pi/2)} = \frac{2\pi}{(2\pi/2)} = \frac{4}{\sqrt{F}}

(Hecht)

(9.64)

For mirrors here \( R = 0.8016 \)

so \( F = \frac{4R}{(1 - R)^2} = 0.8944 = 80 \)  

(Coefficient of finesse)

Width \( \delta = \frac{4}{\sqrt{F}} = 0.447 \)

Finesse \( \frac{\delta}{\lambda} = \frac{2\pi}{\lambda} = 14 \)  

Contrast = \frac{(I_{\text{max}})_{\text{max}}}{(I_{\text{max}})_{\text{min}}} = \frac{1/(1 + F)}{(1 + F)} = 1 + F = 81 \]  

\[ \frac{1}{1 + F} \]  

\[ \frac{1}{1 + F} \]  

\[ \frac{1}{1 + F} \]  

\[ \frac{1}{1 + F} \]
Normal incidence rays so \( \cos \theta = \cos \theta = 1 \)

\( R_1 = \) reflection from coating/air interface
\( R_2 = \) reflection from coating/glass interface

\[ d_f = \frac{\lambda}{4n_f} = \frac{540 \text{nm}}{4 \times 1.24} = 109 \text{ nm} \]

To achieve destructive interference, ray \( R_2 \) needs to be \( \frac{\lambda}{2} \) out of phase with \( R_1 \), reflected from 1st interface. So at normal incidence with \( \cos \theta = 1 \)

\[ \Delta \lambda = 2d_f \cos \theta = \frac{\lambda}{2} \]

\[ d_f = \frac{\lambda}{4n_f} = \frac{\lambda}{4} \]

\( n_f \) still not fixed

In addition, we want the reflection to be of the same strength. Since reflection coefficients depend on \( n_f \) & \( n_g \) through Fresnel equations, the optimal \( n_f \) is given as \( \sqrt{\frac{\text{air}}{n_g}} \) (check 9.10 in section 9.7.2)

\[ n_f = \sqrt{1.0 \times 1.54} = 1.24 \]

Coating thickness \( d_f = \frac{\lambda}{4n_f} = \frac{540 \text{nm}}{4 \times 1.24} = 109 \text{ nm} \)
10.6 Single slit

\[ I = I(0) \frac{\sin^2 \beta}{\beta^2}, \quad \text{with} \quad \beta = \frac{kb \sin \theta}{2}, \quad \frac{\pi b \sin \theta}{\lambda} = m \pi, \quad m = 1, 2, \ldots \]

Minima when \( \sin \beta = 0 \) (except \( \beta = 0 \))

\[ \sin \left( \frac{\pi b \sin \theta}{\lambda} \right) = 0 \]

\[ \frac{\pi b \sin \theta}{\lambda} = m \pi \quad m = 1, 2, \ldots \]

\[ \sin \theta_m = \frac{m \lambda}{\pi b} = \frac{m \lambda}{b} \]

So 1st minimum at \( \sin \theta_1 = \frac{\lambda}{b} \)

\[ y_{\min} = L \tan \theta_1 \approx L \sin \theta_1 \quad \text{provided} \quad b > \lambda \]

So \( y_{\min} \approx L \frac{\lambda}{b} \)

(b) Lens close to aperture takes plane wave-like diffracted beams and focusses them at a distance \( f_2 \) from lens. Think about what a converging lens does to plane waves, Fig 10.3

So screen at \( f_2 \) produces min.

Now \( y_{\min} = f_2 \sin \theta_1 = f_2 \frac{\lambda}{b} \)

(\( \sim f_2 \theta_1 \))
\( l = 1152.2 \text{ nm (IR)} \) single slit

10th minimum is at \( \theta = 6.2^\circ \)

From problem 10.c

\[
I(l) = \frac{\sin^2 \beta}{\beta^2} \quad \beta = \frac{k \lambda \sin \theta}{d} = \frac{16 \times 1.16 \mu m}{d}
\]

\( I = 0 \Rightarrow \sin \beta = 0 \) \( \beta = \pm n \pi ( \text{odd} ) \)

\[
\frac{\pi 16 \sin \theta m}{d} = n \pi \quad m = 10 \text{ kard}
\]

\[
b = \frac{md}{\sin \theta m}
\]

\[
b = \frac{10 \times 1152.2 \mu m}{\sin 6.2^\circ}
\]

\[
= \frac{1.067 \times 10^5 \mu m}{1.067 \mu m}
\]

\[
= 106.7 \mu m = 0.107 \text{ mm}
\]

In water: \( \lambda_0 \rightarrow \frac{\lambda_0}{n} \quad n = 1.33 \)

\[
\sin \theta_n = \frac{md}{b} = \frac{m \lambda_0}{nb}
\]

\[
\theta_n = \sin^{-1} \left( \frac{m \lambda_0}{nb} \right) = \sin^{-1} \left( \frac{10 \times 1152.2 \mu m}{1.33 \times 1.067 \times 10^5 \mu m} \right)
\]

So \( \theta_10 = 4.6^\circ \) in water

acts like a wider slot \( b \rightarrow nb \)

so narrow diffraction pattern
For outgoing rays to be in phase each successive reflection needs to yield a phase change of 2\pi m (normal incidence)

Path length for 1st ray \( l_1 = d \)
Path length for 2nd ray \( l_2 = 3d \)
Path length for 3rd ray \( l_3 = 5d \)

So \( \frac{2\pi}{\lambda} (l_2 - l_1) = m(2\pi) \)
\[ 2d = m\lambda \]
and a change in \( d \) must add exactly \( 1d \) to achieve constructive interference as MM moving mirror is moved. Round trip added distance is 2(\( d \)) = \( \lambda \)

So 1 full finite pair (bright-bright or dark-dark) means MM has moved \( d \)

1 full revolution of mirror yields 80 fringes (repeated on 2 trials) \( \Rightarrow 2\Delta d = 80\lambda \)

So \( \frac{\Delta d}{\text{rev}} = 40\lambda = 40 \times 632.8 \text{ nm} = 2.53 \times 10^{-4} \text{ m} \)
So \( \frac{\Delta d}{\text{rev}} = 2.53 \text{ \mu m} = 0.0253 \text{ mm per revolution} \)
This suggests 1 division on the barrel is $\pm 0.001$ mm of mirror travel. (25 divisions) or 1 rev = $0.001'' = 25.4 \mu m$. Probably can't estimate true $\#$ of fringes to better than $\pm \frac{1}{2}$ fringe so estimated uncertainty $\Rightarrow 80 \pm \frac{1}{2}$ fringes

\[
\frac{\Delta d}{\text{rev}} = \left(40 \pm \frac{1}{4}\right) \\mu m/	ext{rev}
\]

\[
= \left(2.53 \pm 0.16\right) \mu m \text{ per revolution}
\]

So \[
\frac{\Delta d}{\text{rev}} = 25.2 \pm 2 \mu m \text{ per rev. (consistent with } 0.001'' \text{ per revolution)}
\]

\[
= 25.4 \mu m
\]