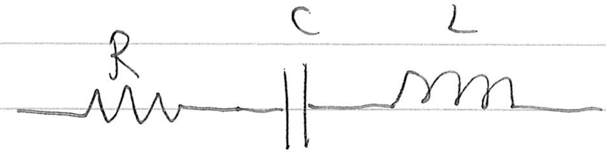
(1) (a)



$Z = R + \frac{1}{i\omega C} + i\omega L$ series combination

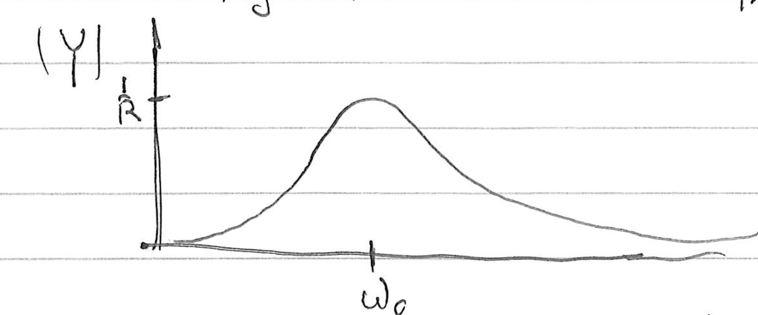
$Z = R + i\left(\omega L - \frac{1}{\omega C}\right)$

$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

$|Y(\omega)| = \frac{1}{|Z|} = \frac{1}{\sqrt{R^2 + \omega^2 L^2 \left(1 - \frac{\omega_0^2}{\omega^2}\right)}}$ & $\omega_0^2 = \frac{1}{LC}$

$Y(\omega) = \frac{1}{\sqrt{R^2 + (\omega L)^2 \left[1 - \omega_0^2/\omega^2\right]}}$

Y max at $\omega = \omega_0 \left(= 1/\sqrt{LC}\right)$
where only R matters $Y_{max} = \frac{1}{R}$

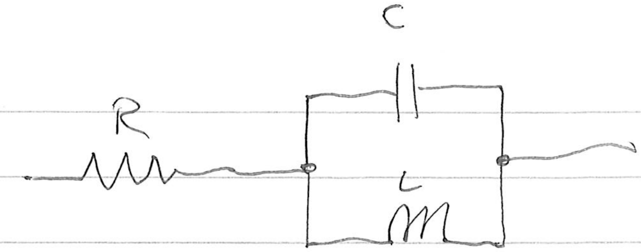


At low freq's capacitor $|Z_c| = \frac{1}{\omega C}$ becomes large, making Y small
At high freq's inductor $|Z_L| = \omega L$ becomes large, making Y small

$Y_{max} = \frac{1}{R}$ max. current $= I = Y_{max} V_{in}$

so $I_{max} = \frac{V_0}{R} e^{i\omega t}$

① (b)



Treat C & L in parallel first

$$Z_{LC} = Z_{par} = \left(\frac{1}{Z_C} + \frac{1}{Z_L}\right)^{-1}$$

$$= \left(i\omega C + \frac{1}{i\omega L}\right)^{-1}$$

$$= (i\omega C)^{-1}\left[1 + \frac{1}{i^2\omega^2 LC}\right]^{-1}$$

$$Z_{LC} = \frac{-i}{\omega C}\left[1 - \frac{\omega_0^2}{\omega_2}\right]^{-1} \qquad \omega_0^2 = \frac{1}{LC} \text{ again}$$

$$= \frac{-i}{\omega C}\,\frac{1}{1-\omega_0^2/\omega^2}$$

Total $Z = R + Z_{LC}$

$$= R - i\left(\frac{1}{\omega C}\,\frac{1}{(1-\omega_0^2/\omega^2)}\right)$$

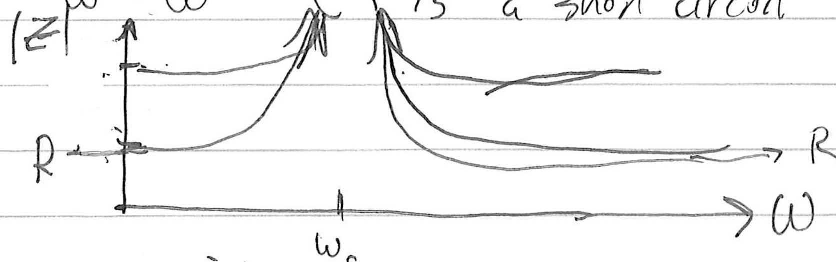
$$|Z| = R^2 + \frac{1}{\omega^2 C^2}\,\frac{1}{(1-\omega_0^2/\omega^2)^2}$$

As $\omega \to \omega_0$ the 2nd term $\to \infty$ $\quad Z \to \infty$

As $\omega \to 0$ $\quad L$ is a short circuit $\quad Z \to R$

As $\omega \to \infty$ $\quad C$ is a short circuit $\quad Z \to R$

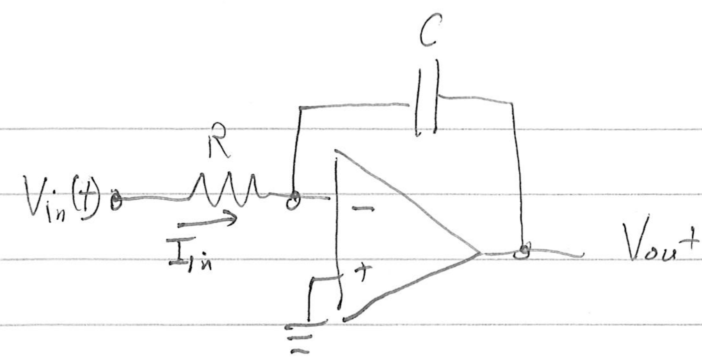
So



$I_{max} = \frac{V_0 e^{i\omega t}}{R}$ at very low or high $\begin{pmatrix}\omega \ll \omega_0 \\ \omega \gg \omega_0\end{pmatrix}$ freq

$I_{min} = 0$ at $\omega = \omega_0 = 1/\sqrt{LC}$

(2)



G.R, $V_+ = V_-$

Since the non-inverting pin (+) is connected to ground, the op-amp will adjust V_{out}, trying to make the − pin also ground.

So $I_{in}(t) = \dfrac{V_{in}(t) - 0}{R} = \dfrac{V_{in}(t)}{R}$

G.R. inputs draw no I

Since no current goes into the − pin those charges flowing thru R must go to the capacitor and with

$$Q = CV_c$$

$$I = \frac{dQ}{dt} = C\frac{dV_c}{dt}$$

The voltage ~~drop~~ across the capacitor $= V_c = -V_{out}$

So $$I = -C\frac{dV_{out}}{dt}$$

$$\frac{V_{in}}{R} = -C\frac{dV_{out}}{dt} \Rightarrow \frac{dV_{out}}{dt} = \frac{-1}{RC}V_{in}(t)$$

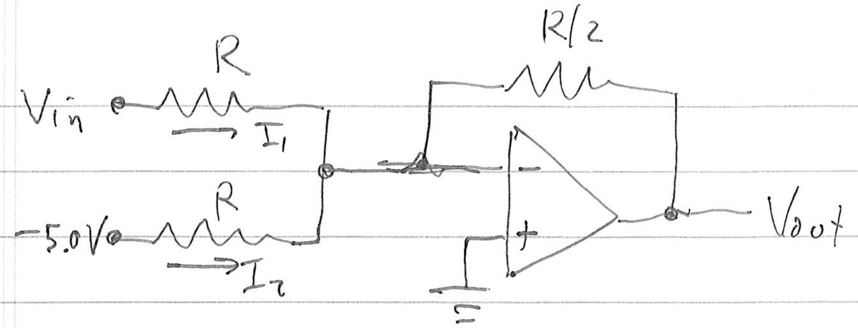
Integrating $$\int_{t=0}^{t} \frac{dV_{out}}{dt'}\,dt' = -\frac{1}{RC}\int V_{in}(t)\,dt$$

$$V_{out}(t) - \cancel{V_{out}(0)} = \frac{-1}{RC}\int_0^t V_{in}(t)\,dt$$

Assume $V_{out} = 0$ initially

$$V_{out}(t) = \frac{-1}{RC}\int_0^t V_{in}(t')\,dt'$$

(3)



G.R.: $V_- = V_+ \Rightarrow V_- = 0$ The - pin acts like "ground"

Then the currents through the input resistors are

$$I_1 = \frac{V_{in}}{R} \qquad I_2 = \frac{-5.0V}{R}$$

And these two I's combine or sum at the - input

G.R. but must flow around thru R/2, not into the V_- pin

So $V_{out} = -(I_1 + I_2)\frac{R}{2}$ (= voltage drop across $\frac{R}{2}$)

$$= -\left(\frac{V_{in}}{R} + \frac{(-5.00V)}{R}\right) \cdot \frac{R}{2}$$

$$V_{out} = -\left(\frac{5.00V}{2} - \frac{V_{in}}{2}\right) = \frac{V_{in}}{2} - 2.50V$$

If $V_{in} = 0$ $V_{out} = -2.50V$

$V_{in} = -5V$ $V_{out} = -\frac{10V}{2} = -5.0V$

$V_{in} = +5V$ $V_{out} = \frac{5-5V}{2} = 0V.$