HW#3

① Noise

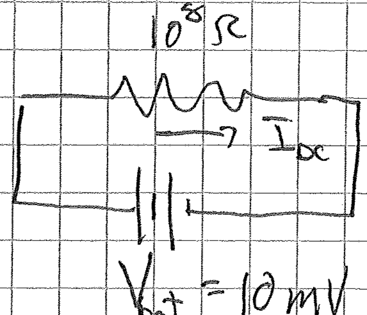
Johnson voltage noise V_J or $V_{TH} = \sqrt{4 k_B T R B}$ (rms)

$R = 100\ M\Omega = 10^8\ \Omega \left(\frac{V}{A}\right)$ $B = 150 \times 10^6\ Hz$ $T \cong 300K$

$$V_J = \sqrt{4 \cdot 1.38 \times 10^{-23} \frac{J}{K} \cdot 300K \cdot \left(10^8 \frac{V}{C/s}\right) \cdot 150 \times 10^6 \frac{cycles}{s}}$$

$$V_J = \sqrt{2.48 \times 10^{-4}\ V^2} \qquad \left(\frac{J}{C} = V\ !\right)$$

$$V_J = .016\ V_{(rms)} = 16\ mV$$



$$I_{DC} = \frac{V_{bat}}{R} = \frac{1.0 \times 10^{-2}\ V}{1.0 \times 10^8\ \Omega}$$

$$I_{DC} = 10^{-10}\ A \quad (0.1\ nA)$$

SHOT NOISE $I_{SHOT} = \sqrt{2 e I_{dc} B}$

$$= \sqrt{2 (1.60 \times 10^{-19} C)\left(1 \times 10^{-10} \frac{C}{s}\right) 150 \times 10^6 \frac{cycles}{s}}$$

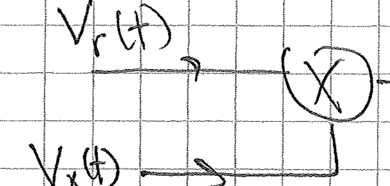
$$= 6.93 \times 10^{-11}\ A$$

$$V_{NOISE} = I_{SHOT} \cdot R = (.0693\ nA)(1 \times 10^8\ \Omega) = 6.9\ mV$$

So Johnson noise is ~16mV compared to about ~7mV (rms) due to current fluctuations from discreteness of charge.

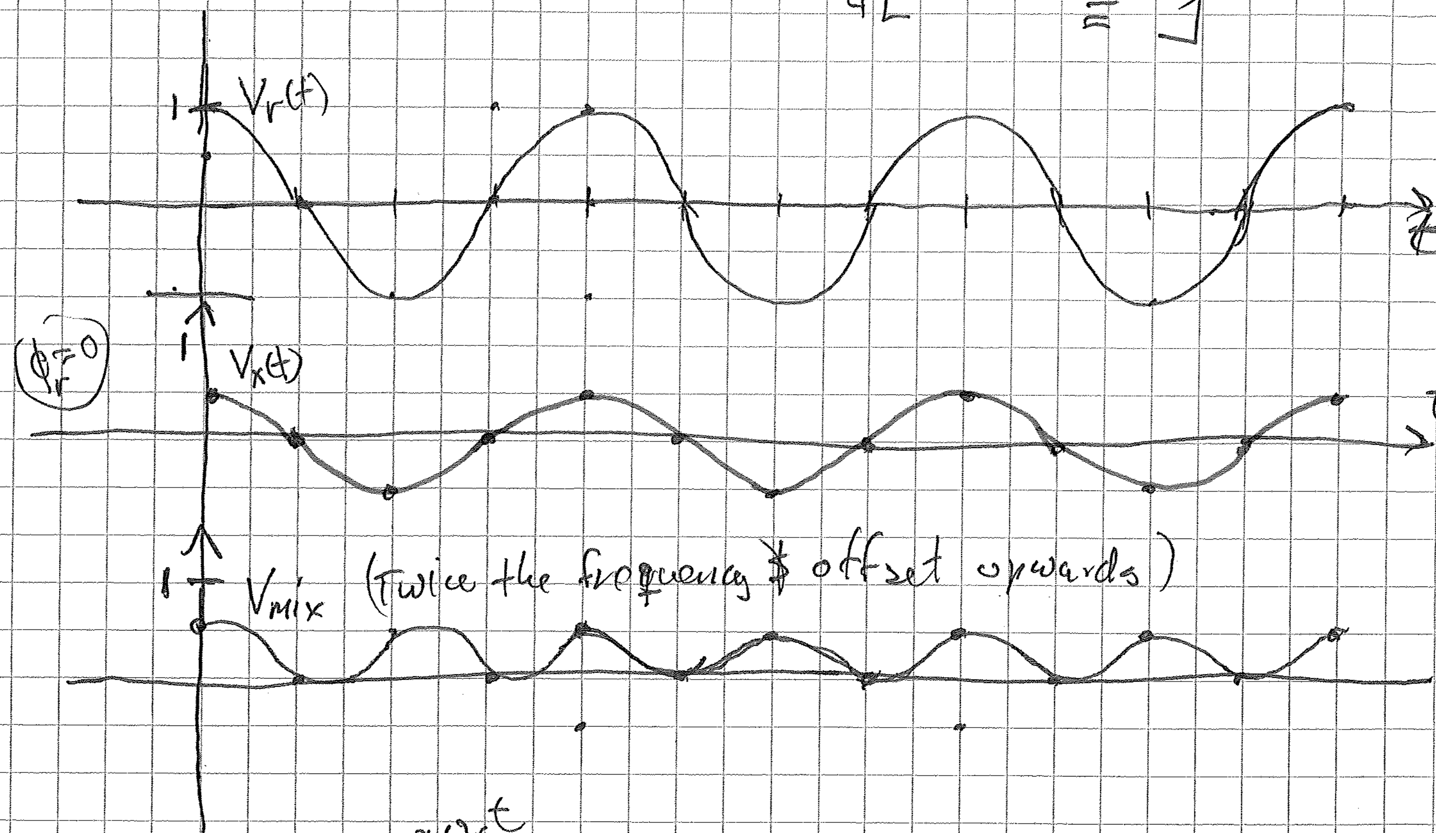
Applied Voltage = 10mV is in the middle!!

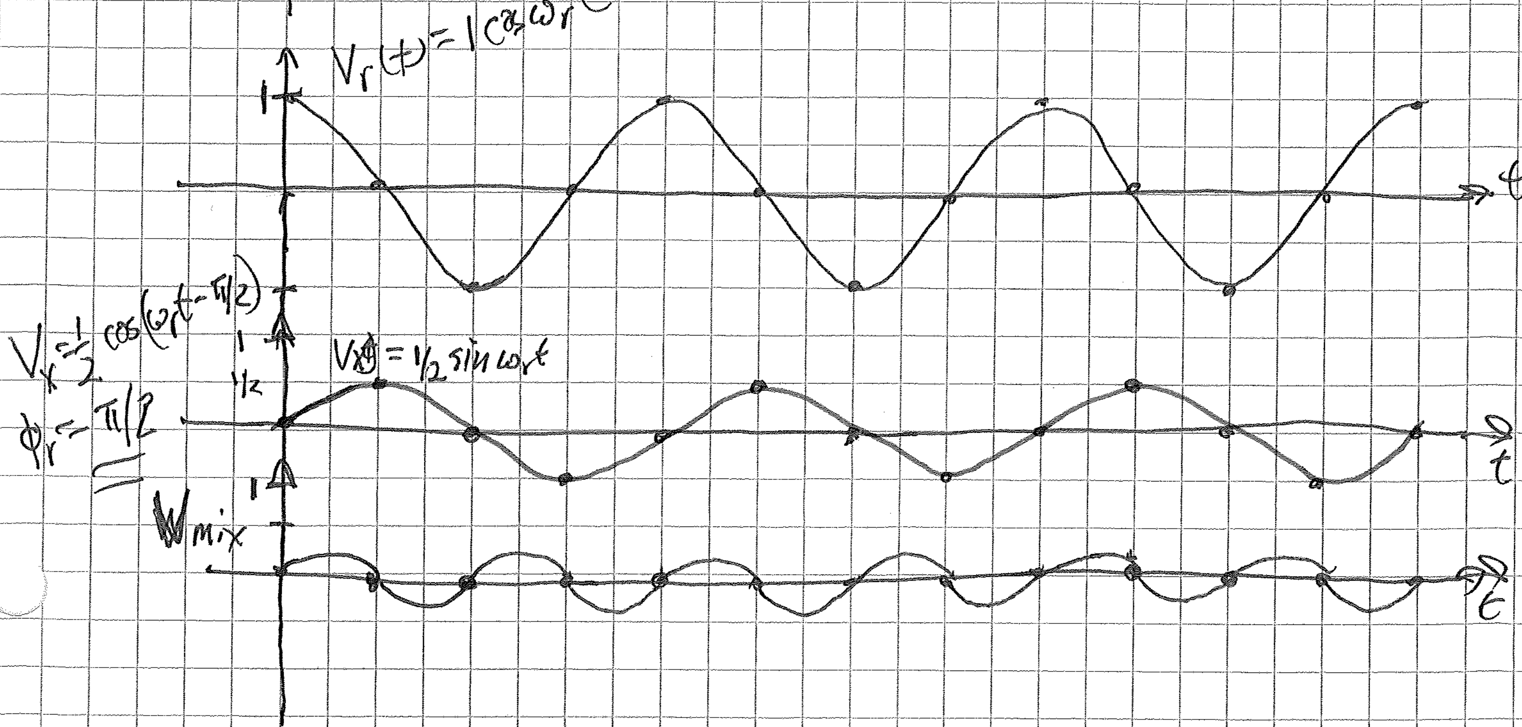
② (a) $V_r(t) = 1\cos\omega_r t$ and $V_x(t) = \frac{1}{2}\cos\omega_r t$



$$V_{mix} = \frac{1}{2}\cos^2\omega_r t = \frac{1}{2}\left[\frac{1}{2}+\frac{1}{2}\cos 2\omega_r t\right]$$

$$= \frac{1}{4}\left[1+\cos(\underline{\underline{2}}\,\omega_r t)\right]$$





② (c) With $\phi_x = 0$ the mixer/multiplier output is always ≥ 0 so it tends to charge the capacitor with the same polarity

but with $\phi_x = \pi/2$ the mixer spends equal time positive and negative, and the net charge pushed onto capacitor will be zero.

and $V_{mix}(t) = (1 \cos\omega_r t)\left(\frac{1}{2}\cos(\omega_r t - \pi/2)\right)$

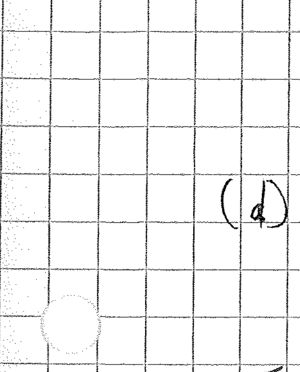
$$= \frac{1}{2}(\cos\omega_r t)(\sin\omega_r t) = \frac{1}{4}\sin(2\omega_r t)$$

$$(2\sin\theta\cos\theta = \sin 2\theta)$$

(d) After a long time the V_{out} from RC reflects the average + small wiggles

For $\phi_x = 0$ (First case) Average $V_{mix} = \frac{1}{4}V$ $\overset{DC}{\boxed{= 0.250\,V}}$

and the AC wiggles at $2\omega_r$ are attenuated by



$$\frac{1}{\sqrt{1+(\omega\tau)^2}}$$ with $\omega = 2\omega_r$ $\quad \tau = RC = 0.3s$

$\omega = 2\,2\pi\,(100Hz)$

$$\text{AC part} = \left(\frac{1}{4}V\right)\cdot\frac{1}{\sqrt{1+\left(\frac{400\pi}{s}\right)^2(0.3s)^2}} = \frac{0.25V}{377}$$

Amplitude of mixer wiggles $\quad$ filtered mixer $V_{AC} = .66\,mV$

(The AC part is ≈ 400 times smaller.)

For $\phi_x = 90°$ (2nd case) $\quad$ Output DC part $= 0$.

AC part $= 0.66\,mV$ same as first case.

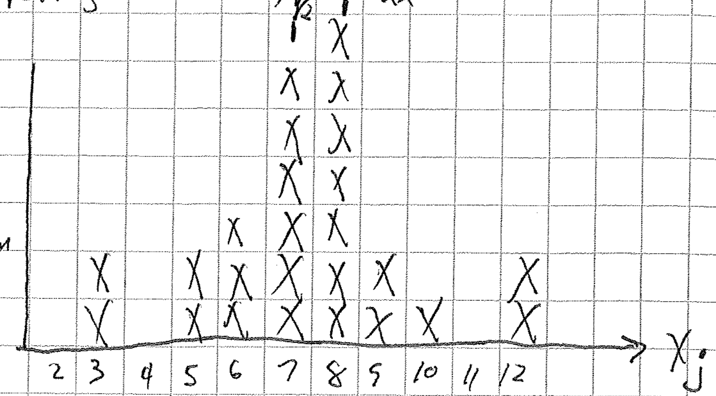
Berlington

1.4

X_i = sum of rolling 2 dice;

i	X_i
1	3
2	7
3	3
4	7
5	12
6	8
7	9
8	7
9	5
10	7
11	12
12	8
13	6
14	6
15	7
16	6
17	7
18	8
19	9
20	8
21	5
22	10
23	8
24	8
25	8

N_j = # of times sum $(X_i =) X_j$



Mean $\bar{X} = \frac{1}{N} \sum X_i \; (\cong \mu) =$

$= \frac{1}{25} \cdot 184 = 7.36$ via calculator

Median $\mu_{1/2} \cong 7$

(13 values ≤ 7 & 12 values > 7

Most Probable X_j is 8: $\mu_{max} = 8$

(7 occurences)
in 25 trials

Variances:

1.6 $s^2 = \frac{1}{N-1} \sum (X_i - \bar{X})^2 \Rightarrow s = 2.18$ via calculator

$\sigma^2 = \frac{1}{N} \sum (X_i - \bar{X})^2 \Rightarrow \sigma = 2.13$ "