① Bevington 2.3

Binomial distribution $\quad n = 6 \quad p = q = 1/2$

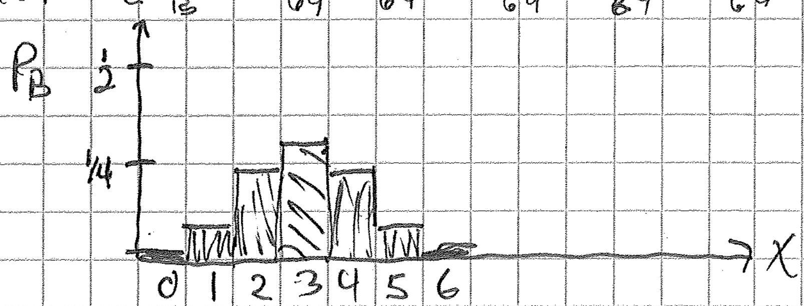
$$P_B(x; n, p) = \frac{n!}{(n-x)!\, x!}\, p^x q^{n-x}$$

Here $p = q = 1/2$ so $p^x q^{n-x} = (1/2)^6 = \frac{1}{64}$ $\left(= \left(\frac{1}{2}\right)^n\right)$

$n! = 6! = 6\cdot5\cdot4\cdot3\cdot2\cdot1 = 720$

x	$P_B =$
0	$\frac{720}{6!\ 0!}\left(\frac{1}{64}\right) = \frac{1}{64} = .0156$
1	$\frac{720}{5!\ 1!}\left(\frac{1}{64}\right) = \frac{6}{64} = \frac{3}{32} = .09375$
2	$\frac{720}{4!\ 2!}\left(\frac{1}{64}\right) = \frac{6\cdot5}{2}\frac{1}{64} = \frac{15}{64} = 0.2344$
3	$\frac{720}{3!\ 3!}\left(\frac{1}{64}\right) = \frac{6\cdot5\cdot4}{3\cdot2\cdot1}\frac{1}{64} = \frac{20}{64} = 0.3125 \quad \left(= \frac{5}{16}\right)$
4	$\frac{720}{2!\ 4!}\left(\frac{1}{64}\right) = \frac{6\cdot5}{2\cdot1}\frac{1}{64} = \frac{15}{64} = 0.2344$
5	$\frac{720}{1\cdot5!}\left(\frac{1}{64}\right) = \frac{6}{64} = \frac{3}{32} = .09375$
6	$\frac{720}{0!\ 6!}\frac{1}{64} = \frac{1}{64} = .0156$

Check: $\sum P_B = \frac{1}{64} + \frac{6}{64} + \frac{15}{64} + \frac{20}{64} + \frac{15}{64} + \frac{6}{64} + \frac{1}{64} = \frac{64}{64} = 1$ ✓



Mean $\quad \mu = np = 6\cdot\frac{1}{2} = 3$ (= mode, median)

Std Dev $\quad \sigma = \sqrt{np(1-p)} = \sqrt{6\cdot\frac{1}{2}\cdot\frac{1}{2}} = \sqrt{\frac{6}{4}} = \sqrt{\frac{3}{2}} = 1.225$

For $p = 1/6$ $q = 5/6$

$$p^x q^{n-x} = \left(\frac{1}{6}\right)^x \left(\frac{5}{6}\right)^{n-x} = \frac{1}{6^6} 5^{n-x} = \frac{5^{n-x}}{46{,}656}$$

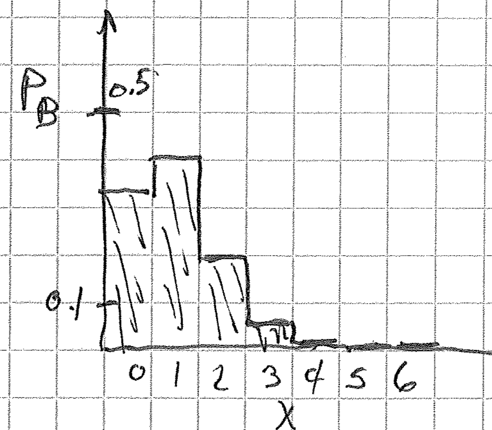
$$P_B(x; 6, 1/6) = \frac{n!}{(n-x)!\,x!} p^x q^{n-x}$$

$$= \frac{6!}{(6-x)!\,x!} \frac{5^{6-x}}{46{,}656}$$

and with $p = 1/6$ $\bar{x} = np = 6\left(\frac{1}{6}\right) = 1$ $\sigma = \sqrt{npq} = \sqrt{6 \cdot \frac{1}{6} \cdot \frac{5}{6}}$
$= 0.913$

Via matlab:

x	$P_B(x) =$
0	0.3349
1	0.4019
2	0.2009
3	0.0536
4	0.0080
5	$0.000643 = 6.4 \times 10^{-4}$
6	$2.14 \times 10^{-5} = .0000214$



② 2.9 Bevington Prob. of failure $p = 0.073$ $q = .927$

In Class of 32 students: expected # of failures $\mu = np = (.073)(32)$
(N)

$\mu = 2.34$ students
(≈ 2)

Prob that 5 or more will fail:

$$P(\geq 5) = \sum_{x=5}^{32} P(x) = 1 - \sum_{x=0}^{4} P(x)$$

In Poisson limit

$$P(\geq 5) = 1 - \sum_{x=0}^{4} \frac{e^{-\mu}(\mu)^x}{x!} \quad + \mu = 2.34$$

$P(0) = e^{-\mu} = .09633$ $P(1) = \frac{\mu}{1} \cdot P(0) = 0.2254$ $P(2) = \frac{\mu}{2} P(1) = 0.2637$ $P(3) = \frac{\mu}{3} P(2) = 0.2057$ $P(4) = \frac{\mu}{4} P(3) = 0.1203$

$\sum_{x=0}^{4} P(x) = 0.9114$ $1 - \sum_{x=0}^{4} P(x) = .0886$ prob of ≥ 5 students failing

OR using $P_{binomial}$ $P(\geq 5) = 0.0803$

#3 Error Propagation
Bevington 3.1 c, e

(c) $x = \frac{1}{u^2}$

$$\sigma_x^2 = \left(\frac{\partial x}{\partial u}\right)^2 \sigma_u^2 \quad \& \quad \frac{\partial x}{\partial u} = \frac{\partial}{\partial u}\left(u^{-2}\right) = -2u^{-3} = \frac{-2}{u^3}$$

So $\sigma_x^2 = \left(\frac{-2}{u^3}\right)^2 \sigma_u^2 = \frac{4}{u^6}\sigma_u^2$ $\qquad \frac{\partial x}{\partial u}$ evaluated at $\bar{u}$

~~then~~ $\sigma_x = \frac{2}{\bar{u}^3}\sigma_u$ $\qquad \left(\text{note } \frac{\sigma_x}{x} = \frac{2/u^3\,\sigma_u}{1/u^2} = 2\frac{\sigma_u}{u}\right)$

(e) $x = u^2 + v^2$

covariance term

$$\sigma_x^2 = \left(\frac{\partial x}{\partial u}\right)^2 \sigma_u^2 + \left(\frac{\partial x}{\partial v}\right)^2 \sigma_v^2 + 2\frac{\partial x}{\partial u}\frac{\partial x}{\partial v}\sigma_{uv}^2$$

$$\sigma_{uv}^2 = \frac{1}{N}\sum_i (u_i - \bar{u})(v_i - \bar{v})$$

$$\frac{\partial x}{\partial u} = 2u \qquad \frac{\partial x}{\partial v} = 2v$$

$$\sigma_x^2 = (2u)^2\sigma_u^2 + (2v)^2\sigma_v^2 + 2(2u)(2v)\,\sigma_{uv}^2$$

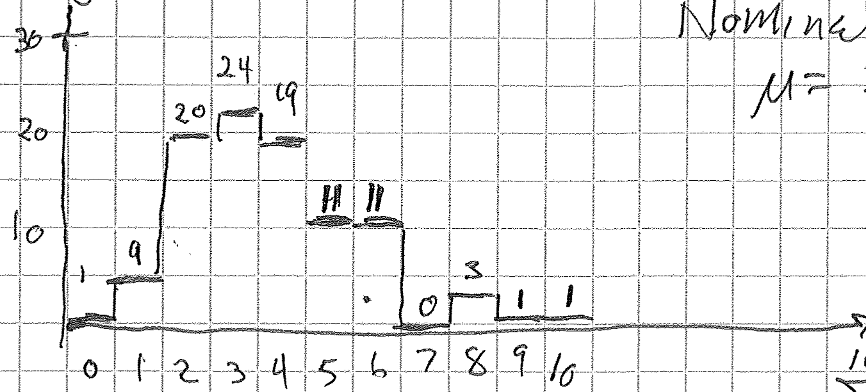
$$= 4\left(\bar{u}^2\sigma_u^2 + \bar{v}^2\sigma_v^2\right) + 8\bar{u}\bar{v}\,\sigma_{uv}^2$$

Neglecting the covariance term (u, v truly indep. and uncorrelated)

$$\sqrt{\sigma_x^2} = 2\sqrt{u^2\sigma_u^2 + v^2\sigma_v^2}$$

$$\sigma_x = 2\sqrt{\bar{u}^2\sigma_u^2 + \bar{v}^2\sigma_v^2}$$

#4. Bevington 3.9



Freq of X in 100 trials $F(x)$

Nominal mean rate $\mu = 3.7 \frac{\text{decays}}{\text{minute}}$

$x = \#$ of $\frac{\text{decays}}{\text{minute}}$

$$\sum_{x=0}^{10} F(x) = 100 \text{ Trials}$$

$$\bar{x} = \sum_x x P(x) = \sum_{x=0}^{10} x \frac{F(x)}{100}$$

$F(x)$ = freq of x in histogram

$P(x) = \frac{F(x)}{\text{total \# of trials}}$

$$= 0 \cdot 1 + 1 \cdot \frac{9}{100} + 2 \cdot \frac{20}{100} + 3 \cdot \frac{24}{100} + 4 \cdot \frac{19}{100} + 5 \cdot \frac{11}{100} + 6 \cdot \frac{11}{100}$$

$$+ 7\left(\frac{0}{100}\right) + 8 \cdot \frac{3}{100} + 9 \cdot \frac{1}{100} \cdot 10 \frac{1}{100}$$

$$= \frac{1}{100}\left[0 + 9 + 40 + 72 + 76 + 55 + 66 + 0 + 24 + 9 + 10\right]$$

$\boxed{\bar{x} = 3.61}$ Poisson expected $\sigma = \sqrt{npq} = \sqrt{3.7} = 1.92$

$= \sqrt{\mu \cdot 1}$

$$\sigma_x^2 = \left(\sum_x x^2 P(x)\right) - \left(\sum x P(x)\right)^2$$

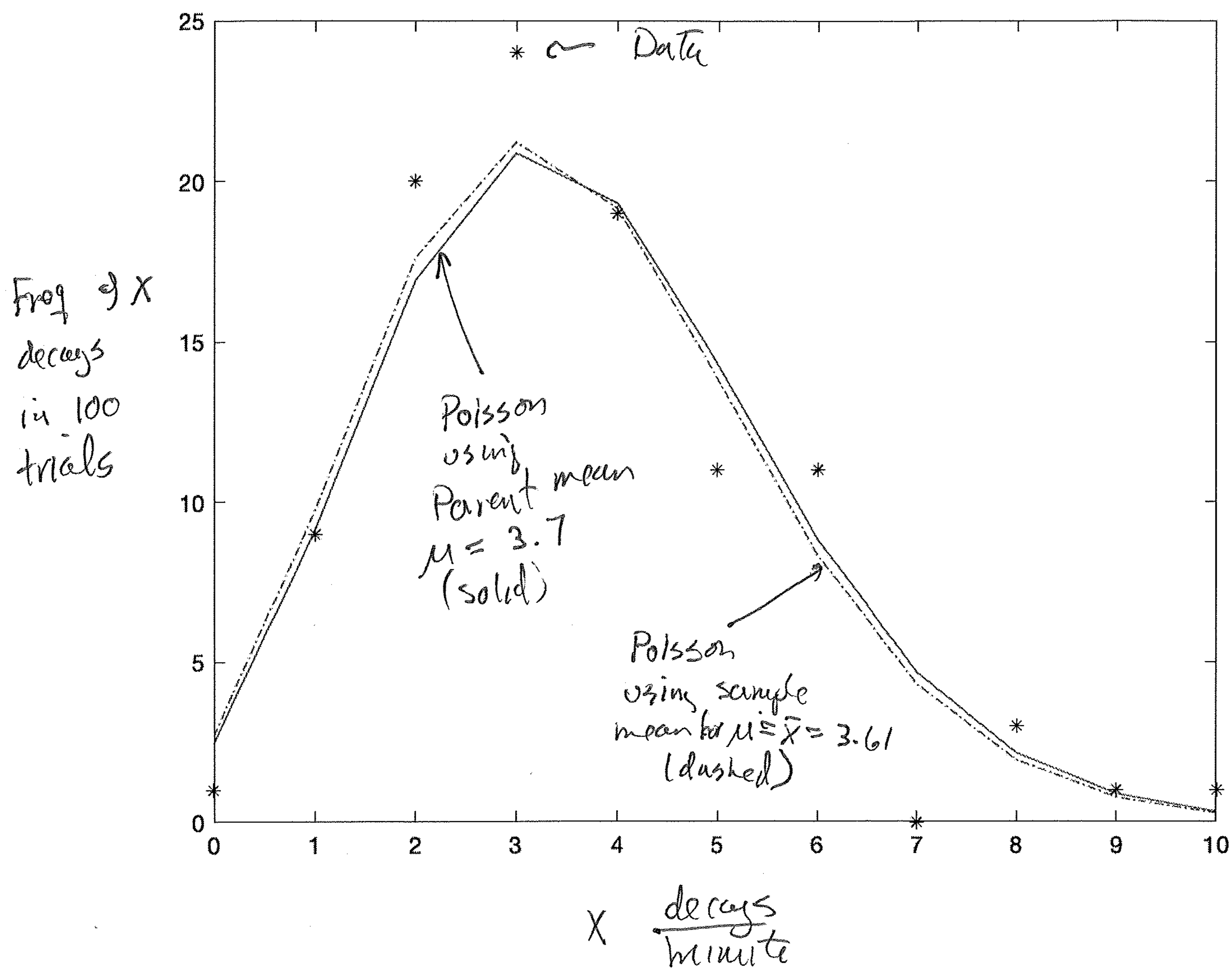
$$= \overline{x^2} - (\bar{x})^2$$

$$\sigma_x^2 = \left[0^2(.01) + 1^2(.09) + 2^2(.20) + 3^2(.24) + 4^2(.19) + 5^2(.11) + 6^2(.11)\right.$$

$$\left. + 7^2(.00) + 8^2(.03) + 9^2(.01) + 10^2(.01)\right] - (3.61)^2$$

$\sigma_x^2 = 16.53 - 13.03 = 3.50$

$\sigma_x = 1.87$ expected $\sigma_x = \sqrt{\mu} = 1.92$ via $\sqrt{3.7}$



Data
Poisson using Parent mean $\mu = 3.7$ (solid)
Poisson using sample mean for $\mu \approx \bar{x} = 3.61$ (dashed)
Freq of X decays in 100 trials
X decays/minute
0
5
10
15
20
25
0
1
2
3
4
5
6
7
8
9
10