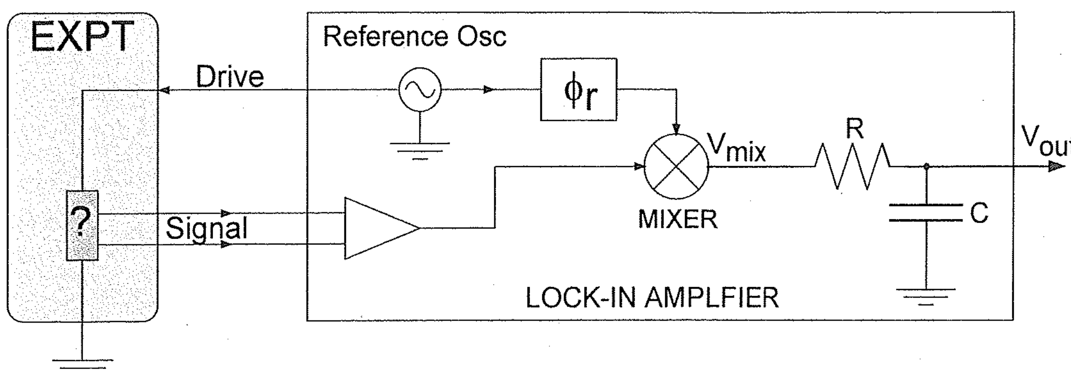
PHYS 5061 Homework 3. Due: Monday, March 25.

(1) Calculate the the rms room temperature thermal (Johnson) noise in a 100 MΩ resistor that one might encounter when connected to an oscilloscope with a 150 MHz measuring bandwidth. A steady voltage of 10 mV is applied across it. What DC current flows and what is the rms shot noise in the current? (c) Calculate the voltage noise resulting from this noise in the current and compare to the applied voltage and to the Johnson noise.

(2) A block diagram of a lock-in amplifier is shown in the figure. The driving signal to the experiment comes from a reference oscillator internal to the lock-in amplifier, $V_r(t) = V_r \cos(\omega_r t)$. The voltage measured from the experiment will be a signal ($V_s$) at the same frequency, but with a different amplitude and often a phase shift, plus some noise: $V_x(t) = V_s \cos(\omega_r t - \phi_x) + V_n(t)$.




The reference oscillator provides the drive signal to the experiment and a replica of itself that has a fixed amplitude and user-adjustable phase shift $\phi_r$ to the mixer. We'll assume for simplicity that the amplitude is simply 1.0 (volts) and the adjustable reference phase shift is $\phi_r = 0$, so $V_r(t) = 1\cos(\omega_r t)$.

The mixer, recall from class, is a multiplier – its output is the product of its two inputs $V_{mix}(t) = V_x(t) \times V_r(t)$.

(a) Ignore the presence of noise for now and assume the signal amplitude from the experiment $V_s$ is 1/2 and $\phi_x = 0$. Sketch carefully a set of graphs, one directly beneath the other with same horizontal and vertical scales, for $V_r(t)$, $V_x(t)$, and $V_{mix}(t)$ for 3 complete cycles of the reference signal.

(b) Make another set for the case $\phi_x = \pi/2$ (90 degrees).

(c) Discuss the effect of the low pass filter on the two cases of $V_{mix}(t)$.

(d) For a concrete example, assume the driving frequency to our experiment is $f_r = 100\,\text{Hz}$ (remember $\omega_r = 2\pi f_r$), and the RC time constant for the low pass filter in the lock-in amplifier is 0.3 s ($\tau = RC = \frac{1}{\omega_{3dB}} = 0.3\,\text{s}$). Assume many cycles have passed and the output voltage has reached a steady-state. Calculate the DC voltage present at the output of the low pass filter *and* the amplitude of the AC signal at 200 Hz that is also in the mixer output but attenuated by the LP filter. (The filter has a response of $G(\omega) = \frac{1}{\sqrt{(1+(\omega\tau)^2}}$.) Do this for both $\phi_x = 0$ and $\phi_x = \pi/2$ scenarios above. Comment on the relative sizes of the two contributions (DC and AC) to the output voltage of the low-pass filter for each $\phi_x$ case.

(3) Bevington: 1.4 and 1.6. Learn how to make your calculator do these calculations. Give values for both $\sigma$ and $s$.

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function $f(x)$ over all values of $x$:

$$\cdots(x_j)] = \int_{-\infty}^{\infty} f(x)P(x)\,dx$$



*Most probable value* $\mu_{max}$: $P(\mu_{max}) \geq P(x \neq \mu_{max})$

*Mean:* $\mu \equiv \langle x \rangle$

*Average deviation:* $\alpha \equiv \langle |x_i - \mu| \rangle$

*Variance:* $\sigma^2 \equiv \langle (x_i - \mu)^2 \rangle = \langle x^2 \rangle - \mu^2$

*Standard deviation:* $\sigma = \sqrt{\sigma^2}$

*Sample mean:* $\bar{x} = (1/N)\Sigma x_i$

*Sample variance:* $s^2 = \dfrac{1}{(N-1)}\Sigma(x_i - \bar{x})^2$

## EXERCISES

**1.1.** How many significant features are there in the following numbers?
(*a*) 976.45 (*b*) 84,000 (*c*) 0.0094 (*d*) 301.07
(*e*) 4.000 (*f*) 10 (*g*) 5280 (*h*) 400.
(*i*) $4.00 \times 10^2$ (*j*) $3.010 \times 10^4$

**1.2.** What is the most significant figure in each of the numbers in Exercise 1.1? What is the least significant?

**1.3.** Round off each of the numbers in Exercise 1.1 to two significant digits.

**1.4.** Find the mean, median, and most probable value of $x$ for the following data (from rolling dice).

| $i$ | $x_i$ | $i$ | $x_i$ | $i$ | $x_i$ | $i$ | $x_i$ | $i$ | $x_i$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 6 | 8 | 11 | 12 | 16 | 6 | 21 | 5 |
| 2 | 7 | 7 | 9 | 12 | 8 | 17 | 7 | 22 | 10 |
| 3 | 3 | 8 | 7 | 13 | 6 | 18 | 8 | 23 | 8 |
| 4 | 7 | 9 | 5 | 14 | 6 | 19 | 9 | 24 | 8 |
| 5 | 12 | 10 | 7 | 15 | 7 | 20 | 8 | 25 | 8 |

**1.5.** Find the mean, median, and most probable grade from the following set of grades. Group them to find the most probable value.

| $i$ | $x_i$ | $i$ | $x_i$ | $i$ | $x_i$ | $i$ | $x_i$ |
|---|---|---|---|---|---|---|---|
| 1 | 73 | 11 | 73 | 21 | 69 | 31 | 56 |
| 2 | 91 | 12 | 46 | 22 | 70 | 32 | 94 |
| 3 | 72 | 13 | 64 | 23 | 82 | 33 | 51 |
| 4 | 81 | 14 | 61 | 24 | 90 | 34 | 79 |
| 5 | 82 | 15 | 50 | 25 | 63 | 35 | 63 |
| 6 | 46 | 16 | 89 | 26 | 70 | 36 | 87 |
| 7 | 89 | 17 | 91 | 27 | 94 | 37 | 54 |
| 8 | 75 | 18 | 82 | 28 | 44 | 38 | 100 |
| 9 | 62 | 19 | 71 | 29 | 100 | 39 | 72 |
| 10 | 58 | 20 | 76 | 30 | 88 | 40 | 81 |

**1.6.** Calculate the standard deviation of the data of Exercise 1.4.