

Assignment #3 Stat 5511 Spring 2008
Due Friday, 2/22/08

(1) Use the data in problem (1) of HW#2, answer the following questions:

- (a) Test $H_0: \beta_0 = 0$ against $H_1: \beta_0 \neq 0$. Use $\alpha = .1$.
- (b) Construct a 90% confidence interval for β_0 .

(2) (Use SAS) Do Problem 2.4 on p.55.

(3) (Use SAS) Do Problem 2.12 on p.58.

(4) (Use SAS) Health risk exposed to a certain toxic agent can be studied using regression through origin. Such health risks are often expressed as relative risk, the ratio of the rate of incidence of the health problem for those exposed to the rate of incidence for those not exposed to the toxic agent. A relative risk of 1.0 implies no increased risk to the disease from exposure to the agent. The following table gives the relative risk to individuals exposed to different levels of dust in their work environment. Dust exposure (x) is measured as the average number of particles /ft³/year scaled by dividing by 10. Let $Y = (\text{relative risk} - 1)$, then regression line relating Y to x should pass through the origin.

x=dust exposure	Relative Risk	Y=relative risk - 1
75	1.10	.10
100	1.05	.05
150	.97	-.03
350	1.90	.90
600	1.33	.33
900	2.45	1.45
1300	1.70	0.70
1650	3.52	2.52
2250	4.16	3.16

- (a) Fit a simple linear regression through the origin relating Y to x .
- (b) Calculate $\hat{\sigma}^2$.
- (c)(c1) a 95% confidence interval for $E(Y_0)$ when $x_0 = 700$.
- (c2) a 95% prediction interval for Y_0 when $x_0 = 700$.

(5) Consider the model $Y_i = 6 + \beta x_i + \varepsilon_i$, where ε_i for $i=1,2,\dots,n$ are independent $N(0,\sigma^2)$.

(a) Show that the least squares estimate of β is,

$$\hat{\beta} = [\sum (Y_i - 6) x_i] / \sum x_i^2.$$

(a) Calculate $\text{Var}(\hat{\beta})$.

(b) Construct a $100(1-\alpha)\%$ confidence interval for β .

(6) (Optional): Consider the simple linear regression model $Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$, $i = 1, 2, \dots, n$, with $E(\varepsilon_i) = 0$ and $\text{Var}(\varepsilon_i) = \sigma^2$ and the ε_i 's are uncorrelated. Prove that
$$\text{COV}(\hat{\beta}_0, \hat{\beta}_1) = -\sigma^2 \bar{x} / S_{xx}$$

(7) (Optional) Do problem 2.23 on p.61.