Math 3280 Assignment 4, due Thursday, October 7th.

Read sections 2.1-2.3 and 3.1-3.3 in the text.

1. Suppose a population of gophers, \( P \), grows at a rate proportional to the square root of the population. Initially the population size is 400 and is increasing at a rate of 40 gophers/month. What will the population be after 1 year?

2. Suppose a population of 15000 people are susceptible to a contagious disease, and that this disease spreads at a rate that is proportional to the number of infected people times the number of uninfected people. If there are initially 1000 people with the disease, and the number of infections is increasing at a rate of 140/day, how much longer will it take for half the population to be infected?

For the following three differential equations find the equilibria and determine their stability. Then solve the differential equation and use all of this information to sketch some of the typical trajectories.

3. \( y' = y - 5 \)

4. \( y' = y^2 - 3y \)

5. \( y' = (y - 1)^2 \)

6. Consider a fish population that would change according to the logistic equation, \( P' = kP(M - P) \) if it were undisturbed. Suppose that these fish will be removed at a rate \( hP \) for some \( h \geq 0 \). If \( k = 1 \) and \( M = 1000 \), find the value of \( h \) that will maximize the number of removed fish at a stable equilibrium population.

7. Use the method of elimination to solve the linear system
   
   \[
   \begin{align*}
   x + 3y + z &= 7 \\
   2x + z &= 5 \\
   2x + y - z &= 4.
   \end{align*}
   \]

8. The second-order differential equation \( y'' = -9y \) has a two-parameter general solution \( y = A \cos(3x) + B \sin(3x) \). For the initial conditions \( y(0) = 4 \), \( y'(0) = 3 \) find the values of \( A \) and \( B \).

9. A linear system of the form:

   \[
   \begin{align*}
   a_{11}x + a_{12}y &= 0 \\
   a_{21}x + a_{22}y &= 0
   \end{align*}
   \]

is said to be homogeneous. Explain in geometric terms why such a system must either have a unique solution or infinitely many solutions. Find a formula for the unique solution when it exists.