Math 3280 Practice Midterm 2

The test will primarily cover chapters 4, 5 and section 6.1, although some material from earlier chapters might be involved (determinants in chapter 3.6 for example). The emphasis will be on chapter 5. The actual midterm will have 3 or 4 required questions. One sheet of notes and a calculator are allowed - however you must indicate the use of a calculator, and you must show the steps in your calculations for full credit.

(1) Express \( w = (7, -6, 14, 0) \) as a linear combination of \( v_1 = (2, 3, 4, 0) \) and \( v_2 = (-1, 4, -2, 0) \), or show that it is impossible to do so.

(2) Find the general solution to the ODE: \( y^{(3)} - 5y'' + 12y' - 8y = 0 \).

(3) Find the solution to the initial value problem \( y'' - 2y' + 5y = e^{2x}, \ y'(0) = 0, \ y(0) = -1 \).

(4) Write down the form of a particular solution \( y_p \) of the ODE \( y'' + y = x^2 e^x + \cos(x) \). You do not have to determine the coefficients of the functions.

(5) If an \( n \times n \) matrix \( A \) has the property that \( A^3 = 2A \), what are the possible values of the determinant of \( A \)?

(6) Find a basis for the subspace defined by the following equations for \( (x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \):

\[
\begin{align*}
-3x_1 - 3x_2 + 2x_3 - 2x_4 &= 0 \\
x_1 - 3x_2 - 4x_3 &= 0 \\
7x_1 + 15x_2 + 2x_3 + 6x_4 &= 0
\end{align*}
\]

(7) Compute the eigenvalues and eigenvectors of the matrix \( A = \begin{pmatrix} 1 & 6 \\ 6 & 1 \end{pmatrix} \).