Math 3280 Worksheet 11: Matrix inverses and elementary matrices.

Group members (2 to 4): ____________________________________________________________
Don’t overlook the exciting part 2 of this worksheet on the back.

(1) Find the inverse of the following matrix $A$ by using row operations (multiplying rows, adding a multiple of one row to another, and interchanging rows) on the matrix $A$ adjoined to the $3 \times 3$ identity matrix. Indicate at each step what row operations you are using.

$$A = \begin{bmatrix}
0 & 0 & 1 \\
2 & 0 & 0 \\
0 & 1 & 2 \\
\end{bmatrix}$$
(2) Write the inverse from the previous problem as a product of elementary matrices by representing each of the row operations you used as elementary matrices. Here is an example. From the following row-reduction,

\[
\begin{pmatrix}
2 & 1 & 1 & 0 \\
4 & 1 & 0 & 1
\end{pmatrix}
\xrightarrow{-2R_1 + R_2}
\begin{pmatrix}
2 & 1 & 1 & 0 \\
0 & -1 & -2 & 1
\end{pmatrix}
\xrightarrow{-R_2}
\begin{pmatrix}
2 & 1 & 1 & 0 \\
0 & 1 & 2 & -1
\end{pmatrix}
\]

we can write the inverse

\[
\begin{pmatrix}
-1/2 & 1/2 \\
2 & -1
\end{pmatrix}
= \begin{pmatrix}
1/2 & 0 \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & -1 \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 \\
0 & -1
\end{pmatrix}
\begin{pmatrix}
1 & 0 \\
-2 & 1
\end{pmatrix}
\]