Math 3280 Practice Midterm 2

The test will primarily cover chapters 4, and 5, although some material from earlier chapters might be involved (determinants in chapter 3.6 for example). The emphasis will be on chapter 5. The actual midterm will have 3 or 4 required questions. One sheet of notes and a calculator are allowed - however you must indicate the use of a calculator, and you must show the steps in your calculations for full credit.

(1) Express $w = (7, -6, 14, 0)$ as a linear combination of $v_1 = (2, 3, 4, 0)$ and $v_2 = (-1, 4, -2, 0)$, or show that it is impossible to do so.

(2) Find the general solution to the ODE: $y^{(3)} - 5y'' + 12y' - 8y = 0$.

(3) Find the solution to the initial value problem $y'' - 2y' + 5y = e^{2x}$, $y'(0) = 0$, $y(0) = -1$.

(4) Write down the form of a particular solution y_p of the ODE $y'' + y = x^2 e^x + \cos(x)$. You do not have to determine the coefficients of the functions.

(5) If an $n \times n$ matrix A has the property that $A^3 = 2A$, what are the possible values of the determinant of A?

(6) Find a basis for the subspace defined by the following equations for $(x_1, x_2, x_3, x_4) \in \mathbb{R}^4$:

$$-3x_1 - 3x_2 + 2x_3 - 2x_4 = 0$$
$$x_1 - 3x_2 - 4x_3 = 0$$
$$7x_1 + 15x_2 + 2x_3 + 6x_4 = 0$$

(7) Solve the initial value problem $y''' - 27y = e^{3x}$, $y(0) = y'(0) = y''(0) = 0$.