Math 3280 Practice Midterm 1

This practice test is a considerably longer than the actual exam, which will have four required questions. The material is from Chapters 1 to 4 in our text. Questions on the actual test will be chosen based on this practice test, the homework, and the worksheets done in class. On the midterm, you are required to show your work - little or no credit will be given for just the final answer. Answers should be kept in exact form whenever possible (for example, $\sqrt{2}$ instead of 1.414).

(1) Solve the initial value problem $y' = xy^3$, $y(0) = 2$.

(2) Find the general solution to the ODE $xy' = 2y + x^3/(1 + x^2)$.

(3) Draw a phase diagram showing the equilibria, their stability, and the direction of the solutions for the ODE $y' = y^3 \sin y$. Justify your answers.

(4) Suppose an object moves through a fluid with resistance proportional to its velocity $v = \frac{dx}{dt}$, so $\frac{dv}{dt} = -kv$. If the initial velocity is $v(0) = 2$ meters/second, and the initial position is $x(0) = 0$, find $x(t)$ (first find $v(t)$).

(5) Your friend Don asks for your help in a homicide investigation. The corpse of the victim was found at midnight with a body temperature of $T(0) = T_0 = 30°C$ in a room with a constant ambient temperature of $A = 25°C$. Six minutes later, the body's temperature had fallen to $T(.1) = 29.9°C$ (using time t in hours). If a normal body temperature is $37°C$, when did the victim die? (Assume that Newton's law of cooling holds: the temperature changes at a rate proportional to the difference between the ambient temperature and the body temperature. Also, use the information about the change in temperature to approximate $\frac{dT}{dt}(0)$.)

(6) Use the improved Euler's method with one, two, and four steps to approximate the $y(2)$ if $y(1) = 1$ and $\frac{dy}{dx} = x/y + 3y/x$. How many digits would you trust in your last answer? For extra credit, find an answer correct to at least five digits.

(7) Find the inverse of the matrix $A = \begin{bmatrix} a & 0 & 0 \\ 0 & -1 & 3 \\ 0 & -1 & -2 \end{bmatrix}$, where a is a nonzero real number.

(8) Compute the determinant of

$$A = \begin{pmatrix} 1 & 2 & 2 & -2 \\ 1 & 4 & 5 & -6 \\ 0 & 2 & 4 & -3 \\ -1 & -6 & -8 & 9 \end{pmatrix}.$$

(9) Express $w = (7, -6, 14, 0)$ as a linear combination of $v_1 = (2, 3, 4, 0)$ and $v_2 = (-1, 4, -2, 0)$, or show that it is impossible to do so.

(10) Find a basis for the subspace defined by the following equations for $(x_1, x_2, x_3, x_4) \in \mathbb{R}^4$:

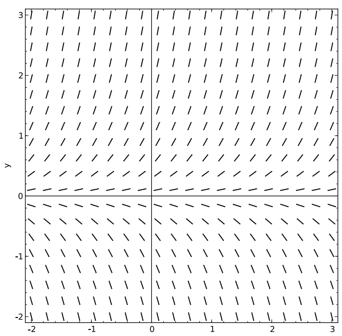
$$\begin{aligned} -3x_1 - 3x_2 + 2x_3 - 2x_4 &= 0 \\ x_1 - 3x_2 - 4x_3 &= 0 \\ 7x_1 + 15x_2 + 2x_3 + 6x_4 &= 0 \end{aligned}$$

(11) Match each of the following six differential equations with the corresponding slope field.

(a) $\dfrac{dy}{dx} = \sin(x)$ (c) $\dfrac{dy}{dx} = 2y$ (e) $\dfrac{dy}{dx} = \dfrac{x}{(1+y^2)}$

(b) $\dfrac{dy}{dx} = \dfrac{y}{(1+x^2)}$ (d) $\dfrac{dy}{dx} = 1$ (f) $\dfrac{dy}{dx} = \sin(y)$

I



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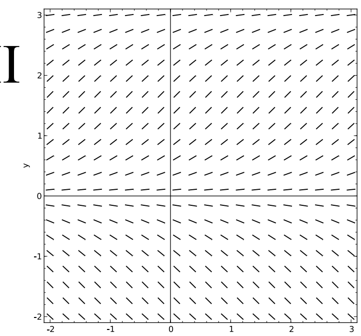

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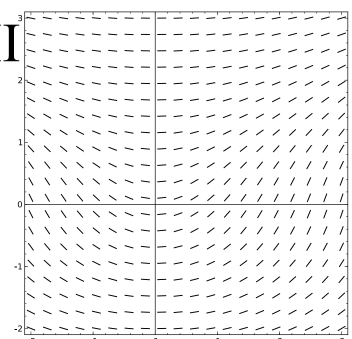

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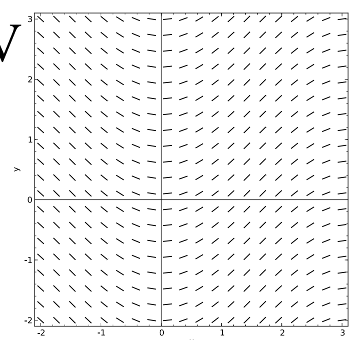

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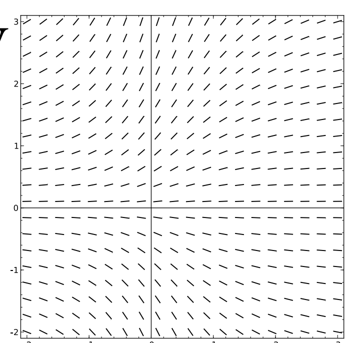

VI