Math 3280 Practice Midterm 2

The test will primarily cover chapters 4, 5, and 6, although some material from earlier chapters might be involved (determinants in chapter 3.6 for example). The actual midterm will have 3 or 4 required questions. One sheet of notes and a calculator are allowed - however you must indicate the use of a calculator, and you must show the steps in your calculations for full credit.

(1) Find the general solution to the ODE: $y^{(3)} - 5y'' + 12y' - 8y = 0$.

(2) Find the solution to the initial value problem $y'' - 2y' + 5y = e^{2x}$, $y'(0) = 0$, $y(0) = -1$.

(3) Write down the form of a particular solution y_p of the ODE $y''+y = x^2e^x+\cos(x)$. You do not have to determine the coefficients of the functions.

(4) If an $n \times n$ matrix A has the property that $A^3 = 2A$, what are the possible values of the determinant of A?

(5) Solve the initial value problem $y''' - 27y = e^{3x}$, $y(0) = y'(0) = y''(0) = 0$.

(6) Rewrite the initial value problem $y''' + y'' + y = t$, $y(0) = y'(0) = y''(0) = 0$ as an equivalent first-order system.

(7) Indicate whether each of the following statements is true or false.

(a) The set of solutions $(x, y, z) \in \mathbb{R}^3$ to the equation $x + y + z = 0$ is a vector subspace of $\mathbb{R}^3$ of dimension 2.

(b) The set of solutions $(x, y, z) \in \mathbb{R}^3$ to the equation $x + y = 1$ is a vector subspace of $\mathbb{R}^3$ of dimension 2.

(c) The set of solutions to the differential equation $y''+xy'+x^2y = 0$ is a vector space of dimension 2.

(d) The set of solutions $(x, y, z) \in \mathbb{R}^3$ of the system below is a vector subspace of $\mathbb{R}^3$ of dimension 1.

$$\begin{aligned} x + 2y + 3z &= 0 \\ 4x + 5y + 6z &= 0 \\ 7x + 8y + 9z &= 0 \end{aligned}$$

(e) The polynomials $1 + x$, $1 - x$, $1 + x^2$ are a basis for the vector space of polynomials with real coefficients of degree less than or equal to 2.