The test will primarily cover chapters 4, 5, and 6, although some material from earlier chapters might be involved (determinants in chapter 3.6 for example). The actual midterm will have 3 or 4 required questions. One sheet of notes and a calculator are allowed - however you must indicate the use of a calculator, and you must show the steps in your calculations for full credit.

(1) Find the general solution to the ODE: \( y^{(3)} - 5y'' + 12y' - 8y = 0 \).

(2) Find the solution to the initial value problem \( y'' - 2y' + 5y = e^{2x}, \ y'(0) = 0, \ y(0) = -1 \).

(3) Write down the form of a particular solution \( y_p \) of the ODE \( y'' + y = x^2e^x + \cos(x) \). You do not have to determine the coefficients of the functions.

(4) If an \( n \times n \) matrix \( A \) has the property that \( A^3 = 2A \), what are the possible values of the determinant of \( A \)?

(5) Solve the initial value problem \( y''' - 27y = e^{3x}, \ y(0) = y'(0) = y''(0) = 0 \).

(6) Rewrite the initial value problem \( y''' + y'' + y = t, \ y(0) = y'(0) = y''(0) = 0 \) as an equivalent first-order system.

(7) Indicate whether each of the following statements is true or false.

(a) The set of solutions \((x, y, z) \in \mathbb{R}^3\) to the equation \( x + y + z = 0 \) is a vector subspace of \( \mathbb{R}^3 \) of dimension 2.

(b) The set of solutions \((x, y, z) \in \mathbb{R}^3\) to the equation \( x + y = 1 \) is a vector subspace of \( \mathbb{R}^3 \) of dimension 2.

(c) The set of solutions to the differential equation \( y'' + xy' + x^2y = 0 \) is a vector space of dimension 2.

(d) The set of solutions \((x, y, z) \in \mathbb{R}^3\) of the system below is a vector subspace of \( \mathbb{R}^3 \) of dimension 1.

\[
\begin{align*}
x + 2y + 3z &= 0 \\
4x + 5y + 6z &= 0 \\
7x + 8y + 9z &= 0
\end{align*}
\]

(e) The polynomials \( 1 + x, \ 1 - x, \ 1 + x^2 \) are a basis for the vector space of polynomials with real coefficients of degree less than or equal to 2.