Math 3280 Worksheet 39: Solving initial value problems with Laplace transforms

Group members (2 to 4):
(1) Solve the initial value problem $x^{\prime \prime}+x^{\prime}-2 x=t, x(0)=x^{\prime}(0)=0$ using the Laplace transform. If you have time, check your work using other methods (characteristic equation and undetermined coefficients). A table of Laplace transforms is given on the back of this sheet.

| Function $f(t)$ | Transform $\mathcal{L}(f(t))=F(s)$ |
| :--- | :--- |
| 1 | $\frac{1}{s}$ |
| $t$ | $\frac{1}{s^{2}}$ |
| $t^{n}(n$ is a non-negative integer $)$ | $\frac{n!}{s^{n+1}}$ |
| $t^{a}(a>-1)$ | $\frac{\Gamma(a+1)}{s^{a+1}}$ |
| $e^{k t}$ | $\frac{1}{s-k}$ |
| $\cos (k t)$ | $\frac{s}{s^{2}+k^{2}}$ |
| $\sin (k t)$ | $\frac{k}{s^{2}+k^{2}}$ |
| $-t f(t)$ | $F^{\prime}(s)$ |
| $\int_{0}^{t} f(\tau) d \tau$ | $F(s) / s$ |
| $f^{\prime}(t)$ | $s F(s)-f(0)$ |
| $f^{\prime \prime}(t)$ | $s^{2} F(s)-s f(0)-f^{\prime}(0)$ |

TABLE 1. Some Laplace transforms, $\mathcal{L}(f(t))=F(s)$

